聖 嬰 中 學

HOLY INNOCENTS' HIGH SCHOOL

Name of Student

Class Index Number

/ 80

PRELIMINARY EXAMINATION 2020
SECONDARY 4 EXPRESS and 5 NORMAL ACADEMIC
MATHEMATICS 4048/01

Paper 1

Date : 28 Aug 2020

Duration: 2 hr

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is **80**.

For Examiner's use

You need to improve on your:
- PRESENTATION
- ACCURACY

Marks (max 5%)

Set by : Ms Lua Bee Hian

Vetted by : Mr Brandon Choy

Mathematical Formulae

Compound interest

$$\text{Total amount } = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean } = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation } = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

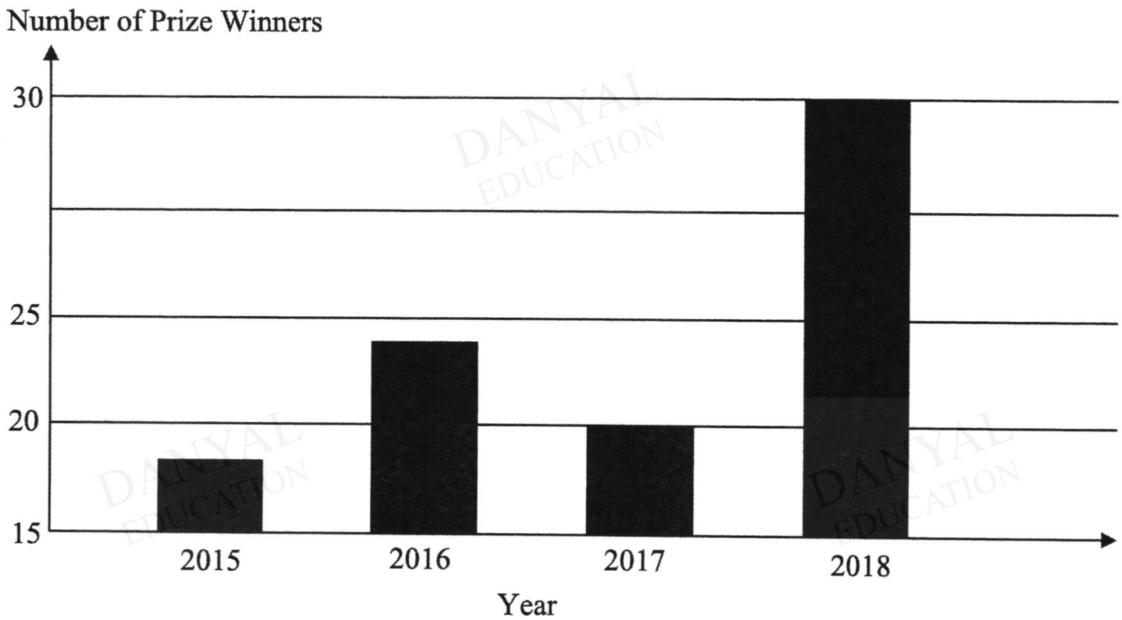
Answer **all** the questions.

1 Simplify $\left(\dfrac{d^6}{c^2}\right)^{-\frac{5}{2}}$.

Answer ………………………… [2]

2

Mathematics Olympiad Prize Winners



Explain how the graph above may be misleading.

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

…………………………………………………………………………………… [2]

3 Given that $9 \times 243^n = 1$, find n.

Answer $n =$ .... [2]

---

4 Show that $(2p+3)^2 - 1$ is divisible by 2 for all integers of p.

Answer

[2]

5 Factorise completely $6ac-14ad+3bc-7bd$.

Answer .... [2]

---

6 Given the following numbers

$$\sqrt[3]{0.027} \qquad \frac{31}{37} \qquad \frac{\pi}{22} \qquad 1.21^{\frac{1}{8}}$$

(a) write in order of size, largest first,

Answer ...., ..., ..., ... [1]

(b) find the sum of the rational numbers.

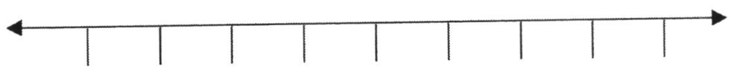
Answer .... [2]

7 **(a)** Solve the inequalities $4 < 3x - 5 \leq 13$.

Answer .... [2]

(b) Represent the solution of $4 < 3x - 5 \leq 13$ on the number line below.

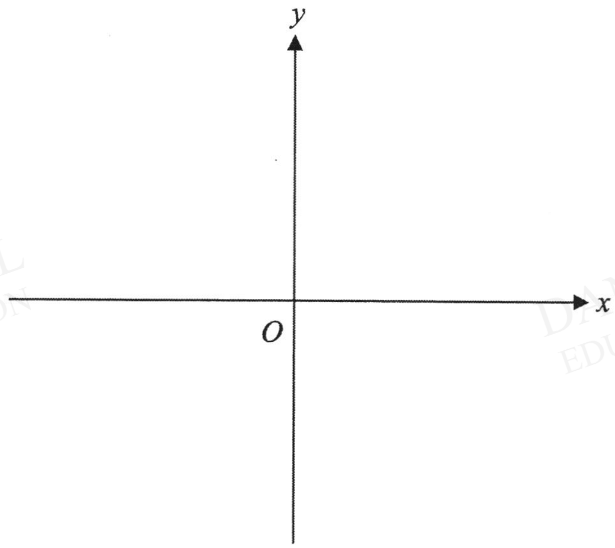
Answer



[1]

8 **(a)** **Sketch** the graph of $y=-(x+1)(x-5)$ on the axes below.
Indicate clearly the values where the graph crosses the x- and y- axes.

Answer



[2]

(b) Hence explain why there is no solution when the following equations are solved simultaneously.

$$y=-(x+1)(x-5)$$
$$y=2x+9$$

....

....

....

....

....

....

.... [1]

9 **(a)** Solve $\begin{pmatrix} 3 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ h \end{pmatrix} = 10$.

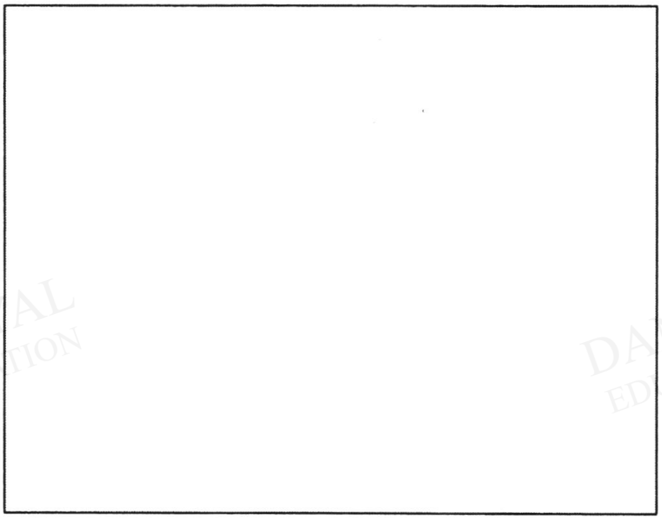
Answer $h =$ ………………………… [1]

(b) Given $G = \begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}$, find the value of G^2.

Answer $G^2 =$ ………………………… [2]

10 The diagram shows the floor plan of Wendy's rectangular bedroom.
It is drawn to a scale of 1 cm represents n metres.
The actual area of the floor is 15.75 m^2.

(a) Using the plan, find the value of n.



Scale: 1 cm represents n metres

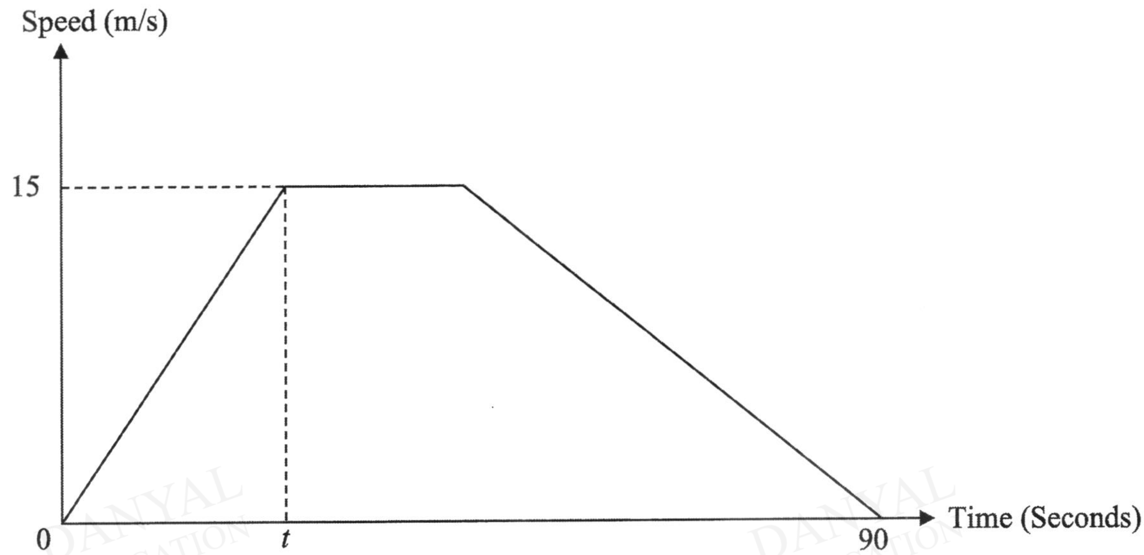
Answer n = ... [2]

(b) Wendy decides to lay square tiles on the floor in her room.
Each tile has a dimension of 0.5 m by 0.5 m.
Show that the total number of tiles required to lay the floor is 63.

Answer

[1]

11



The diagram shows the speed-time graph of a vehicle over a period of 90 s.
The vehicle reached a maximum speed of 15 m/s at time t seconds.

(a) If the acceleration of the vehicle was $0.5\ \text{m/s}^2$ in the first t seconds, calculate the value of t.

Answer $t =$ .... [1]

(b) The total distance travelled by the vehicle in 90 s was 750 m.

Calculate the duration that the vehicle was travelling at its maximum speed in seconds.

Answer .... s [2]

12 The table shows the shoe sizes of the workers in a factory.

Shoe size	38	39	40	41	42
Number of workers	10	25	15	m	35

(a) If the modal shoe size is 42, write down an inequality that must be satisfied by m.

Answer .... [1]

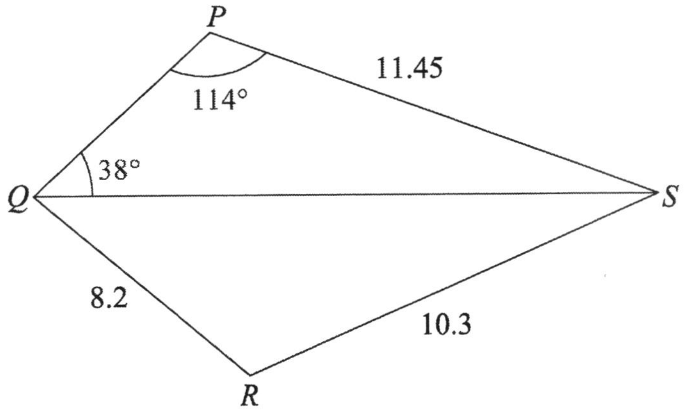
(b) If the median shoe size is 40.5, find the value of m.

Answer .... [1]

(c) Find the mean shoe size if there are 105 workers in the factory.

Answer .... [1]

13

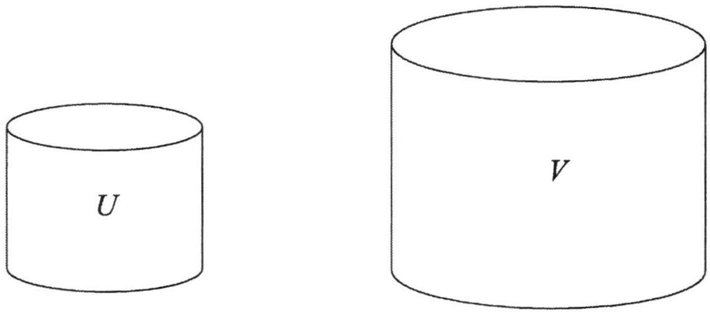


The diagram shows a quadrilateral *PQRS*.
$PS = 11.45$ cm, $QR = 8.2$ cm and $RS = 10.3$ cm.
Angle $PQS = 38°$ and angle $QPS = 114°$.

Calculate angle *QRS*.

Answer angle $QRS =$ ....° [3]

14



The diagram shows two geometrically similar closed cylindrical solids, U and V.
The volume of cylinder U is 216 cm^3 and the volume of cylinder V is 729 cm^3.
Find

(a) the value of $\dfrac{\text{height of cylinder } U}{\text{height of cylinder } V}$,

Answer .... [1]

(b) the cost of painting cylinder V if the cost of painting cylinder U with the same type of paint is \$15.90.
Assume that the cost of painting is directly proportional to the surface area.
Leave your answer correct to the nearest cent.

Answer .... [2]

15 **(a)** Express 540 as the product of its prime factor.

Answer .... [1]

(b) The number 540k is a perfect cube.
Find the smallest positive integer value of k.

Answer $k =$ .... [1]

(c) z is a number between 100 and 200.
The highest common factor of z and 540 is 30.
Find the smallest possible value of z.

Answer $z =$ .... [2]

16 Each term in this sequence is found by adding the same number to the previous term.

$$4, \quad a, \quad b, \quad 25, \quad c, \; ....$$

(a) Find the values of a, b and c.

Answer $a =$ ....

$b =$ ....

$c =$ .... [2]

(b) Write down an expression, in terms of n, for the nth term.

Answer .... [1]

(c) Explain why 101 is not a term of this sequence.

Answer

[1]

17 **(a)** $\xi = \{\text{integer } x : 2 \le x < 15\}$
$E = \{\text{prime numbers}\}$
$F = \{\text{factors of } 18\}$
$G = \{\text{multiples of } 3\}$

(i) List the elements in $E \cap F$.

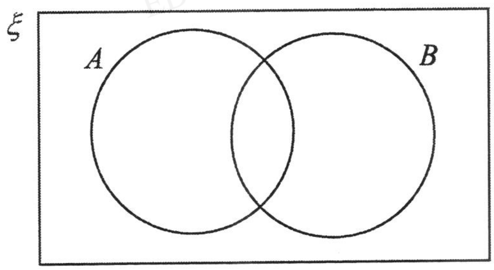
Answer .... [1]

(ii) Underline the correct statement(s) from the list below.

Answer:

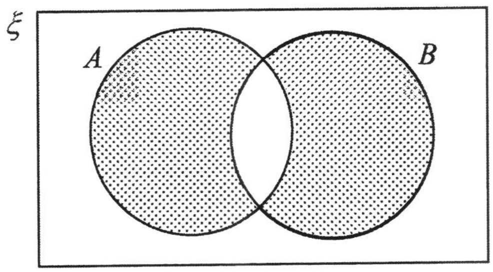
$E \cup F = \{0\}$ $\quad$ $F \subset G$ $\quad$ $11 \in E$ $\quad$ $F \cap G = \{3, 6, 9\}$ [1]

(b) On the Venn diagram, shade the region which represent AÈB'.



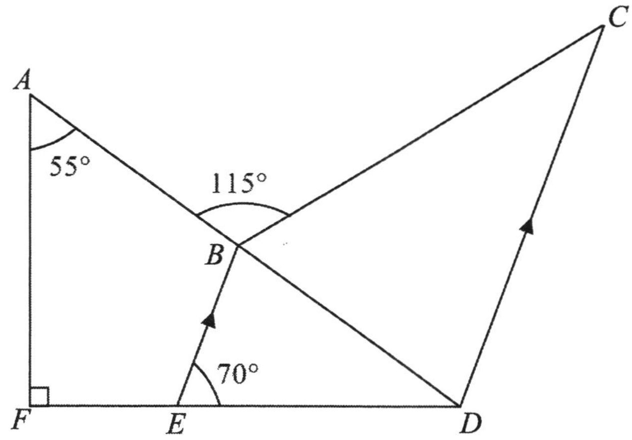
[1]

(c) Express in set notation, the shaded region represented in the Venn diagram below.



Answer .... [1]

18



In the diagram, *ABD* and *FED* are straight lines.
EB is parallel to *DC*.
Angle *FAB* = 55°, angle *ABC* = 115°, angle *EFA* = 90° and angle *DEB* = 70°.

Calculate, stating your reasons clearly,

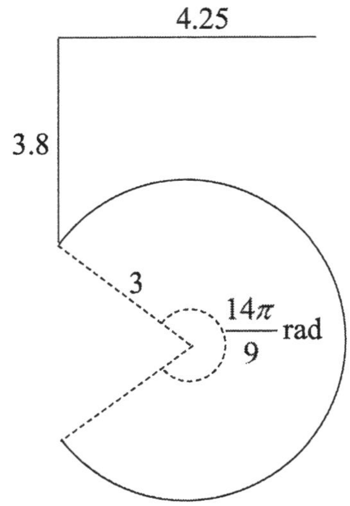
(a) angle *ABE*,

Answer angle *ABE* = ...° [2]

(b) angle *BCD*.

Answer angle *BCD* = ...° [2]

19 A thin piece of wire was bent into a shape of figure five as shown.



The shape has two straight edges of length 4.25 cm and 3.8 cm.
The curved part is the arc of the major sector of a circle with radius 3 cm.

The angle of the major sector is $\frac{14\pi}{9}$ radians.

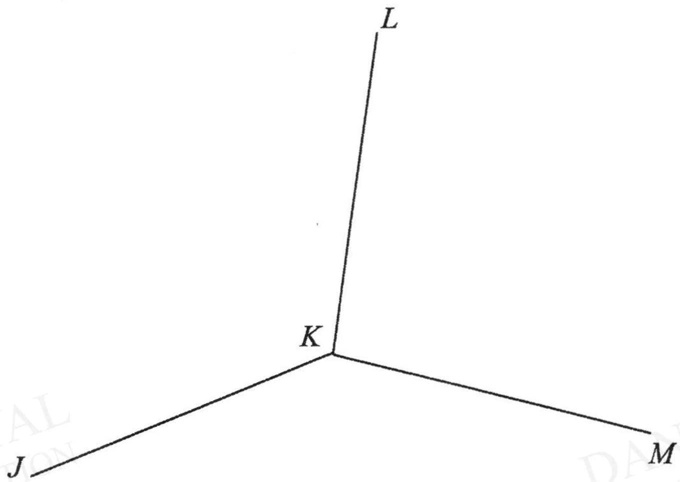
The total length of the wire used to make the figure is $(p+q\pi)$ cm.

Find the value of p and of q.

Answer $p =$ ……………………………

$q =$ …………………………… [4]

20 In the diagram, JK and KM are two sides of a regular decagon.
JK and KL are two sides of a regular hexagon



(a) Calculate the interior angle of the regular decagon.

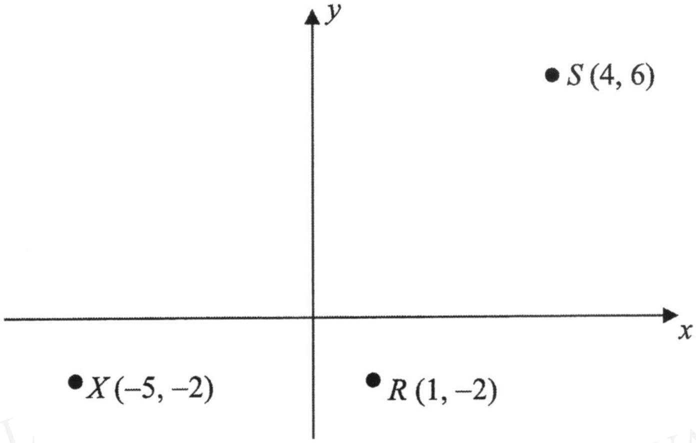
Answer ....° [1]

(b) Explain if MK and KL are two sides of a regular polygon. Show your working clearly.

Answer

[3]

21 The diagram shows the points $R\ (1,-2)$, $S\ (4, 6)$ and $X\ (-5, -2)$.



(a) The length of RS is $\sqrt{k}$. Find the value of k.

Answer $k =$ …………………………… [1]

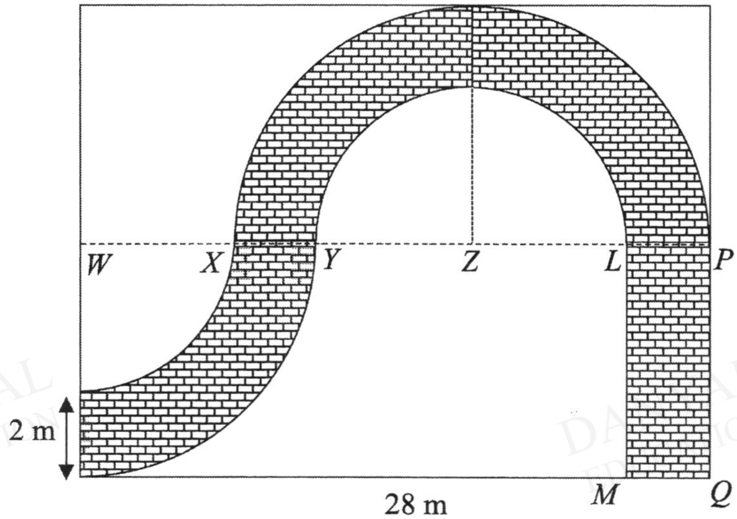
(b) Find the equation of RS.

Answer …………………………… [2]

(c) Find the value of $\cos\angle XRS$.
Give you answer correct to 4 significant figures.

Answer $\cos\angle XRS =$ ……………… [1]

22 The diagram shows a path of width 2 m in a rectangular garden of length 28 m. The outline of the path is made up of quarter circles with centre W, semicircles with centre Z and straight lines LM and PQ respectively. $WX = YZ$.



(a) Show that the width of the rectangular garden is 20 m.
Answer

[2]

(b) Calculate the area of the path.

Answer ....m^2 [3]

23 **(a)** Michael's car uses fuel at an average rate of 12.1 litres per 100 km driven.
On an average, Michael drives 15 000 km per year.
Michael currently pays \$2.55 per litre of fuel.
Assuming the price of the fuel remains the same, work out the amount Michael would expect to spend on fuel in one year.

Answer \$.............................. [2]

(b) The total sales for food and beverage services in January 2020 was estimated to be \$963 million. Online food and beverage sales made up an estimated of 9.8%.
[1 million = 10^6]

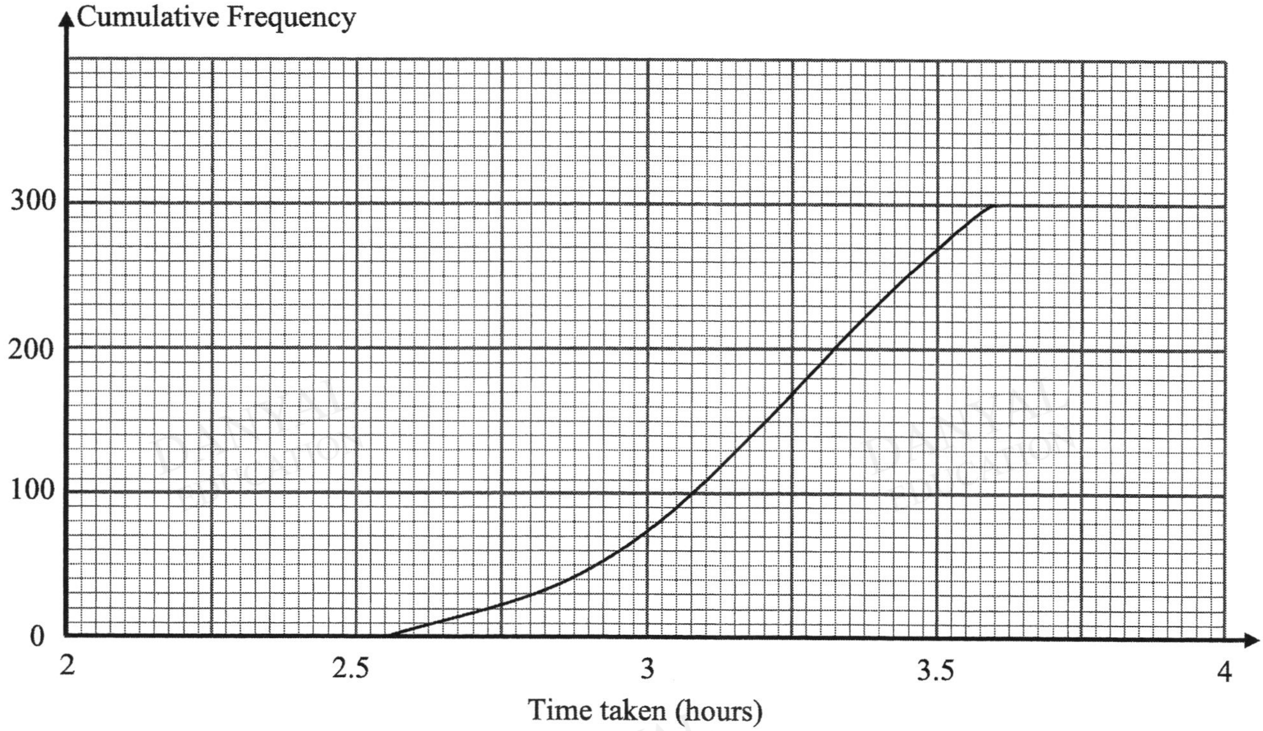
(i) Express 963 million as standard form.

Answer .... [1]

(ii) The population of Singapore in January 2020 was approximately 5.85 million.
Calculate the average amount spent on online food and beverage per person.

Answer \$.............................. [2]

24 The times taken by 300 competitors in a women's marathon race were recorded.
The cumulative frequency curve shows the distribution of their times.



(a) Use the curve to answer the following questions.

(i) One of the runners is selected at random.
Find, as a fraction in its lowest terms, the probability that the runner took more than 3.25 h.

Answer .... [1]

(ii) Find the median time.

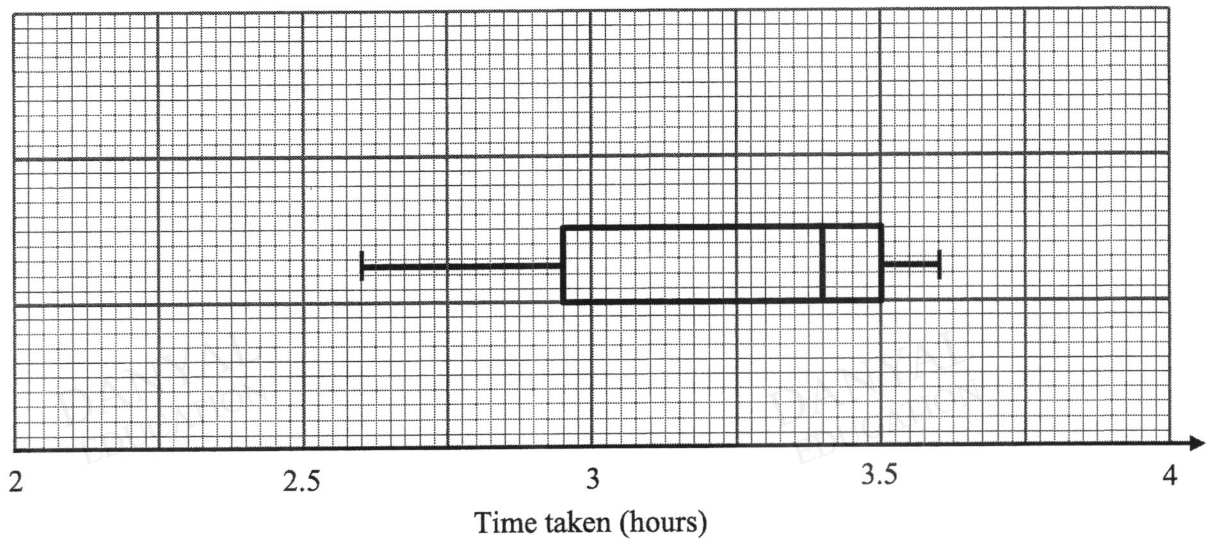
Answer .... h .... min [1]

(iii) The qualifying time for the Olympic Games was achieved by 10% of the runners.

If the race started at 11.30 am, find the time the last qualifying athlete finished the race.

Answer .... [1]

(b) The times taken by another group of 300 competitors in the same marathon race were also recorded in the previous year.
The box-and-whisker plot shows the distribution of the times.



Make two comments comparing the time taken by the two groups of competitors.

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....

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....

....

.... [2]

~ End of paper ~

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HOLY INNOCENTS' HIGH SCHOOL

Established in 1892

Name of Student

Class

Index Number

/100

PRELIMINARY EXAMINATION 2020
SECONDARY 4 EXPRESS and 5 NORMAL ACADEMIC
MATHEMATICS PAPER 2

4048/02

Date : 25 Aug 2020

Duration: 2 h 30 min

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
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Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

If you need additional answer paper or graph paper, ask the invigilator for a continuation writing paper or graph paper.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is **100**.

For Examiner's use

You need to improve on your:
- PRESENTATION
- ACCURACY

Marks (max 5%)

Set by : Mr Zoel Ng
Vetted by : Mr Brandon Choy

This document consists of 25 printed pages (including cover page) and 1 blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount } = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

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Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean } = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation } = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. A bag contains four balls, numbered 2, 3, 6, and 9
Two balls are picked from the bag at random, one after another, without replacement.

(i) Draw a possibility diagram to represent the outcomes. [2]

(ii) Find, in its simplest form, the probability that

(a) both balls have numbers less than 6, [1]

(b) both balls are odd numbers, [1]

(c) the product of the numbers is more than 10. [1]

(iii) The two balls are now picked from the bag, one after another, **with** replacement. Ken claims, "the probability that the product of the numbers is more than 10 has increased because there are more favourable outcomes."

Do you agree? Justify by showing your calculations. [2]

2. The cash price of a new car is $80000.

(a) James buys the car on hire purchase. He pays a deposit of 30% of the cash price. He then pays 72 equal monthly instalments. The interest rate charged is 2% per annum.
Calculate the total amount that James pays for the car? [3]

(b) Bryan buys the same car.
He took a loan of \$80000 from a bank which charges an interest rate of 1.8% per annum compounded yearly for 10 years.

(i) Calculate the difference in the total amount James and Bryan need to pay. [3]

(ii) The total amount owed by Bryan is paid over 120 equal monthly instalments. By showing your calculations, suggest a reason why people might want to choose Bryan's method of payment over James'. [2]

3. The number of points scored by Team A in 15 basketball matches are recorded below.

Stem	Leaf
1	0
2	
3	8
4	1 1 2 2 8
5	2 2 4 6 6
6	1 1 6

(a) Find

(i) the mean score Team A, [1]

(ii) the standard deviation of the scores of Team A. [1]

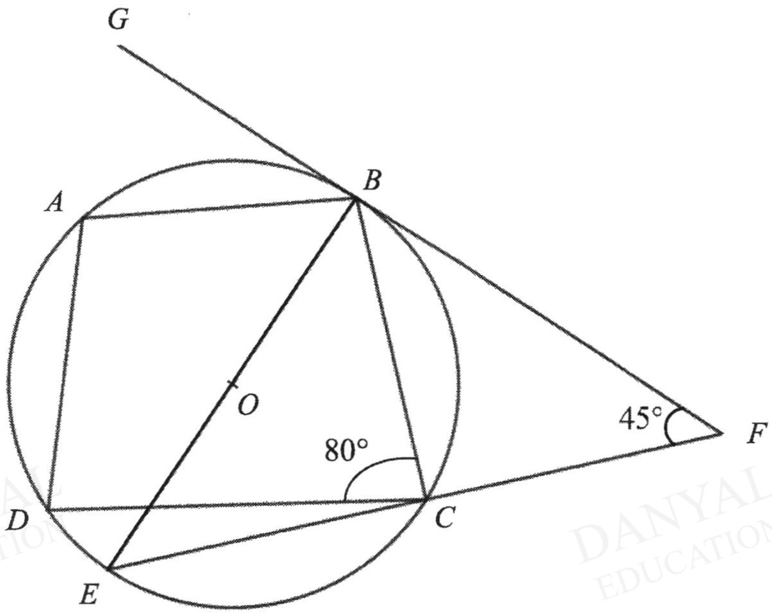
(b) The information of Team B's basketball matches are shown below.

Mean score = 45
Standard Deviation = 13.4

By comparing the mean and standard deviation, explain which team performed better. [2]

(c) Two basketball matches' scores of Team A are chosen at random.
Find the probability that both scores are greater than 55 points. [2]

4.



The diagram shows a circle $ABCED$, with centre O.
Chord EC is extended to meet tangent GF at F.
Angle $BFC = 45°$ and angle $BCD = 80°$.

(a) Find, giving reasons for each answer,

(i) angle DAB, [1]

(ii) angle FEB, [1]

(iii) angle CAB. [1]

(b) Show that

(i) triangle BEC is an isosceles triangle, [2]

(ii) triangle ECB and triangle EBF are similar. [3]

5. Some workers at a construction company are assigned to renovate type A houses. Assuming that each worker works at the same rate, this group of workers takes x days to renovate a type A house.

(a) Write down an expression, in terms of x, for the number of type A houses the group of workers can renovate in 15 days. [1]

Due to an epidemic, some workers had to be quarantined at home for the first 15 days of of the month of May.
The remaining workers took 2 more days to renovate a type A house during that 15 days.

(b) Write down an expression, in terms of x, for the number of type A houses the remaining workers can renovate for the first 15 days of May. [1]

(c) Given that 2 less houses than usual were renovated in the first 15 days of May, form an equation in terms of x and show that it reduces to $x^2+2x-15=0$. [3]

(d) Solve the equation $x^2+2x-15=0$ and find the original number of type A houses the workers were able to renovate in 15 days. [3]

6. The number of the tickets sold for a performance titled "Classic Nightingale" held at Singapore Indoor Stadium on a weekend are shown in the table below.

	Category 1	**Category 2**	**Category 3**	**Category 4**
Saturday	100	80	120	180
Sunday	90	80	70	150

The prices of tickets to this performance are stated in the table below.

Category 1	**Category 2**	**Category 3**	**Category 4**
$300	$280	$230	$200

(a) Represent the number of tickets sold for the weekend by a 2×4 matrix **A**. [1]

The price of the tickets can be represented by a matrix **B**.

(b) Write down a matrix **B**, where **AB** gives the total amount of money collected from the ticket sales on Saturday and Sunday respectively. [1]

(c) Evaluate the matrix $\mathbf{P}=\mathbf{AB}$. [2]

(d) Given that $\mathbf{C} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, evaluate $\mathbf{D} = \mathbf{AC}$ and state what the elements of **D** represents. [2]

(e) **(i)** Write down a matrix **E** such that **ED** gives the total number of ticket sold for the "Classic Nightingale" performances on both Saturday and Sunday. [1]

(ii) Hence evaluate **ED**. [1]

(f) The target number of tickets to be sold for the following weekend of "Classic Nightingale" are as follow:

Saturday tickets to be decreased to 95%.

Sunday tickets to be increased by 25%.

Write down the values of x and y such that the matric product $(x \quad y)\mathbf{D}$ gives the total number of tickets sold for the following weekend. [2]

7. (a)

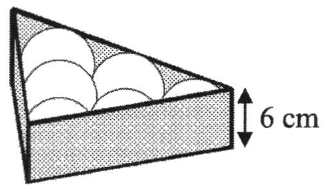


Figure 1

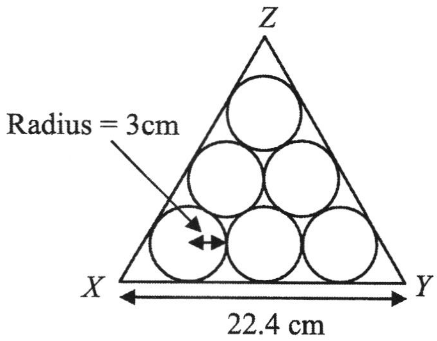


Figure 2

Figure 1 shows an equilateral triangular billiard rack of height 6cm.
It holds 6 identical billiard balls of radius 3cm that touches each other and at the sides of the rack.

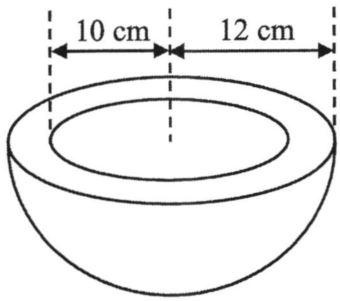
Figure 2 shows the top view of the arrangement of billiard balls in the triangular rack.
$XY = 22.4$ cm.

Find

(i) the total volume of the 6 billiard balls in the triangular rack, [2]

(ii) the volume of the unoccupied space in the triangular rack. [3]

(b)



A hollow metal hemispherical soup bowl has an external radius of 12 cm and internal radius of 10 cm.

(i) Find the total surface area of the bowl. [3]

(ii) The bowl is melted and recast into smaller cubes of side 2.5 cm.
Find the maximum number of cubes that can be made. [3]

8. **(a)** It is given that $x = \dfrac{y-x}{2y} + 6.$

(i) Express x in terms of y. [3]

(ii) Hence find the value of x when $y = 2$. [1]

(b) Given that $(a-b)^2 = 8$ and $ab = -3$, find

(i) $a^2 + b^2$, [1]

(ii) $\left(\dfrac{a+b}{2}\right)^2$. [1]

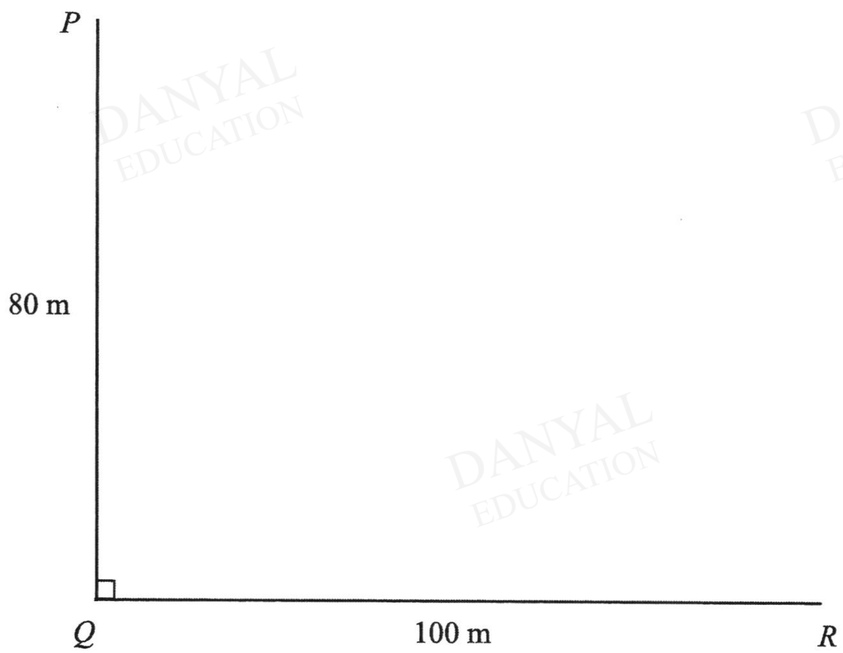
(c) Simplify $\dfrac{2x^2-12x+18}{x^2-9}$. [3]

(d) Given the graph $y=14-6x+x^2$, find the coordinates of the turning point. [3]

9. A developer bought a land in the shape of a quadrilateral *PQRS*.

$PQ = 80$ m, $QR = 100$ m, $RS = 70$m, $PS = 85$ m and angle $PQR = 90°$.

(a) Using a scale of 1 cm to represent 10 m, construct an accurate scale drawing of the plot of land. Sides *PQ* and *QR* has been drawn for you in the space below. [2]



Using the scale drawing in **part (a)**,

(b) Construct
  (i) the perpendicular bisector of *PQ*, [1]
  (ii) the bisector of angle *QRS*. [1]

(c) *Q* is due south of *P*.
State the bearing of *S* from *P*. [1]

(d) The developer intends to construct a building, *B*, where the two bisectors in **(b)** intersect.

(i) Find the actual distance of the building *B* from *R*. [1]

(ii) Jane is interested in finding the actual area of the land *RBS*.
Suggest how she can calculate this area, stating clearly the measurement(s) and method(s) required.
Numerical values and calculations are not required. [2]

(iii) Jane walks along the line connecting *R* and *S*.
Find the actual shortest possible distance between Jane and building *B*. [1]

(iv) Hence find the greatest angle of elevation from Jane to the top of building *B*, given that building *B* is 80 metres in height. [2]

10. The variables x and y are connected by the equation

$$y = 2x + \frac{30}{x} - 16$$

Some corresponding values of x and y are given in the table below.

x	1.75	2	2.5	3	4	5	6	7	8
y	4.64	p	1	0	–0.5	0	1	2.29	3.75

(a) Calculate the value of p. [1]

(b) In the grid given on page 22, draw the graph of $y = 2x + \frac{30}{x} - 16$ for $1.75 \leq x \leq 8$.

Use a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leq x \leq 8$.
Use a scale of 2 cm to represent 0.5 units, draw a vertical y-axis for $-0.5 \leq y \leq 5$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $(6,1)$. [2]

(d) Use your graph to find

(i) the range of values of x for which $y < 2.5$, [1]

(ii) the solutions to the equation $2x + \frac{30}{x} = 18$. [2]

11. COVID-19 is an ongoing novel infectious disease caused by severe acute respiratory syndrome. On March 11 2020, the World Health Organisation (WHO) declared that COVID-19 is a pandemic affecting over 100 countries around the world and the sustained risk of further global spread.

The statistics of the virus worldwide and in Singapore on 21st March is as shown below in **Figure 1** and **Figure 2** respectively.

Figure 1

Source: https://ncov2019.live/data

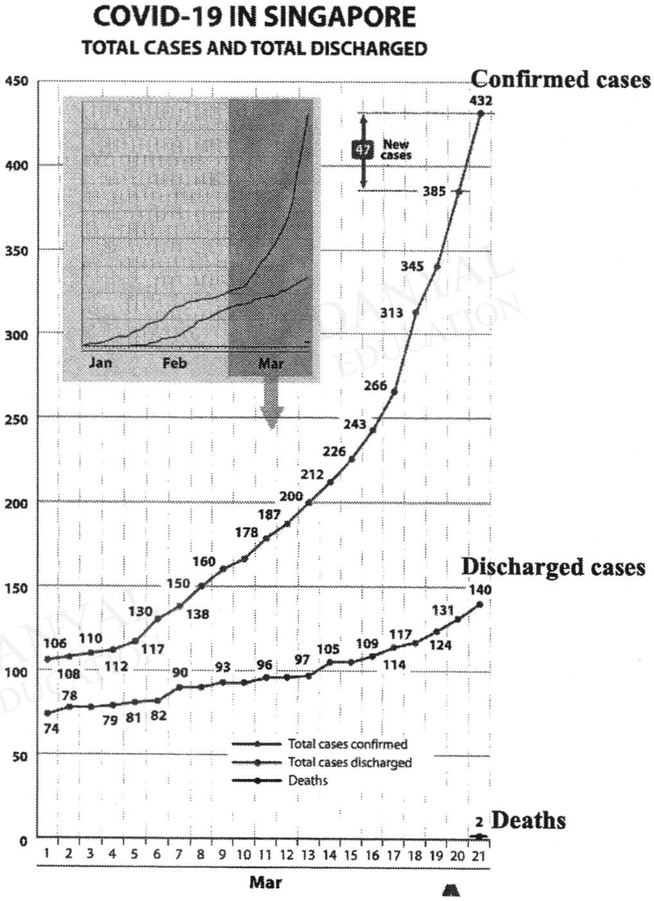


Figure 2

(Source: Ministry of Health)

(a) **(i)** The data in **Figure 1** is represented in a pie chart.
Calculate the angle that would represent the number of cases worldwide who have recovered from the illness. [2]

(ii) Calculate the percentage of deaths from the virus in Singapore as of 21st March. [2]

(b) In a mathematical model of the number of COVID-19 infections, it is assumed that each infected person can infect others.
For ***N*** infected people, the number of infected people will increase by 10% daily. For example, when ***N*** is 1 000, there will be 100 new cases the next day. When ***N*** is 10 000, there will be 1 000 new cases the next day.

The number of new infections can be modelled as follows:

$$\boxed{\begin{array}{l}\text{Number of new infections} = a \times \boldsymbol{N} \\ \text{where } a\text{, the daily infection rate, is } 0.1\end{array}}$$

The number of infected people each day can be predicted in the following formula

$$\boxed{N_{i+1} = N_i + a \times N_i}$$

where N_i and N_{i+1} is the number of people infected on the i^{th} day and the day after respectively , and $i = 1, 2, 3...$

In the model, it assumed that there is only 1 infection on day 1 where $N_1 = 1$.

The data of the number of infected people each day can be represented in the following table.

Day number (i^{th} day)	***N*, total number of people infected** ($N_{i+1} = N_i + a \times N_i$)
1 $(i = 1)$	$N_1 = 1$
2 $(i = 2)$	$N_2 = 1 + 0.1(1) = 1.1$
3 $(i = 3)$	$N_3 = 1.1 + 0.1(1.1) = 1.1(1 + 0.1) = 1.1^2$
⋮	
100 $(i = 100)$	$N_{100} = \ldots$

Turn over

To reduce the spread of COVID-19 virus, social distancing measures are put in place to lower the risk of infection from community spread. This will lower the daily infection rate, a, to 0.09.

Alex claims that the number of infections will decrease by more than 55% on day 100 if everyone practises social distancing responsibly from day 1.
Do you agree with him? Justify your answer. [6]

End Of Paper

Answers

1 $\dfrac{c^5}{d^{15}}$

2 *Possible answers:*

The **vertical axis does not start from zero** and thus it makes the number of prize winners in **2018** to be **4 times of 2017**, when it is **actually 10 more.**

The **scale of the vertical axis is inconsistent** and thus it makes the number of prize winners in **2018** to be more than **double** that in **2016**, when it is **only slightly more than 5 more**.

3 $n = -\dfrac{2}{5}$

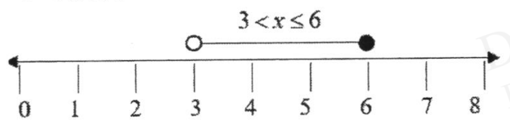
5 $(2a+b)(3c-7d)$

6(a) $1.21^{\frac{1}{8}}$, $\dfrac{31}{37}$, $\sqrt[3]{0.027}$, $\dfrac{\pi}{22}$

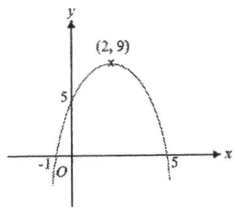
6(b) $1\dfrac{51}{370}$

7(a) $3 < x \le 6$

7(b)



8(a)



8(b) When the graph of $y = 2x + 9$ is drawn (line), there is **no intersection between the graph of** $y = -(x+1)(x-5)$ **and** $y = 2x + 9$, hence no solution.

9(a) $h = -2$

9(b) $\begin{pmatrix} -3 & 2 \\ -2 & -4 \end{pmatrix}$

10(a) Width = either 6.95 or 7 cm
Length = either 9 or 9.1 cm
Hence $62.55 \le \text{area} \le 63.7$
$\therefore 0.497 \le n \le 0.502$

11(a) 30 s

11(b) $t = 10$

12(a) $0 \le m \le 34$ or $0 \le m < 35$

12(b) 15

12(c) $40\dfrac{2}{7}$

13 $Q\hat{R}S = 133.1°$

14(a) $\dfrac{2}{3}$

14(b) \$35.78

15(a) $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

15(b) $k = 50$

15(c) $z = 150$

16(a) $a = 11$, $b = 18$, $c = 32$

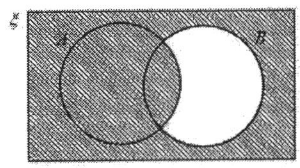
16(b) $T_n = 7n - 3$

16(c) $101 = 7n - 3$
$7n = 104$
$n = 14\dfrac{6}{7}$, which is not an integer
When the term is 101, ***n* is not an integer**, thus, 101 is not in the sequence.

17(ai) 2 and 3

17(aii) $11 \in E$ and $F \cap G = \{3, 6, 9\}$

17(b)



17(c) $(A \cap B)' \cap (A \cup B)$

18(a) 105°

18(b) 40°

19 $p = 8\dfrac{1}{20}$, $q = 4\dfrac{2}{3}$

20(a) 144°

21(a) $k = 73$

21(b) $y = \dfrac{8}{3}x - \dfrac{14}{3}$

21(c) -0.3511

22(b) 105 m^2

23(a) \$4628.25

23(b)(i) 9.63×10^8

23(b)(ii) \$16.13

24(a)(i) $\dfrac{13}{30}$

24(a)(ii) 3 h 12 min

24(a)(iii) 2.18 pm

24(b) The competitors from **this year ran faster** as they have a **shorter median time** and there is also **a smaller spread** in their timing due to the **smaller interquartile range**.

4E5N Prelim 2020 Paper 2 Answers

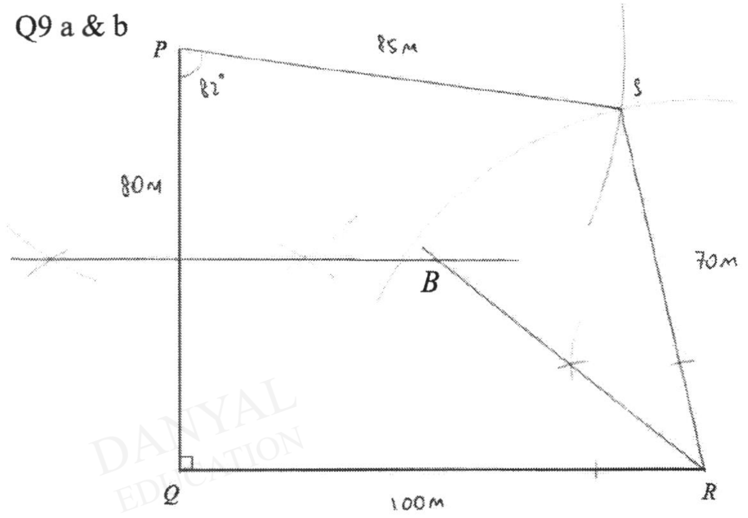
1i

Ball number	2	3	6	9
2	-	(2, 3)	(2, 6)	(2, 9)
3	(3, 2)	-	(3, 6)	(3, 9)
6	(6, 2)	(6, 3)	-	(6, 9)
9	(9, 2)	(9, 3)	(9, 6)	-

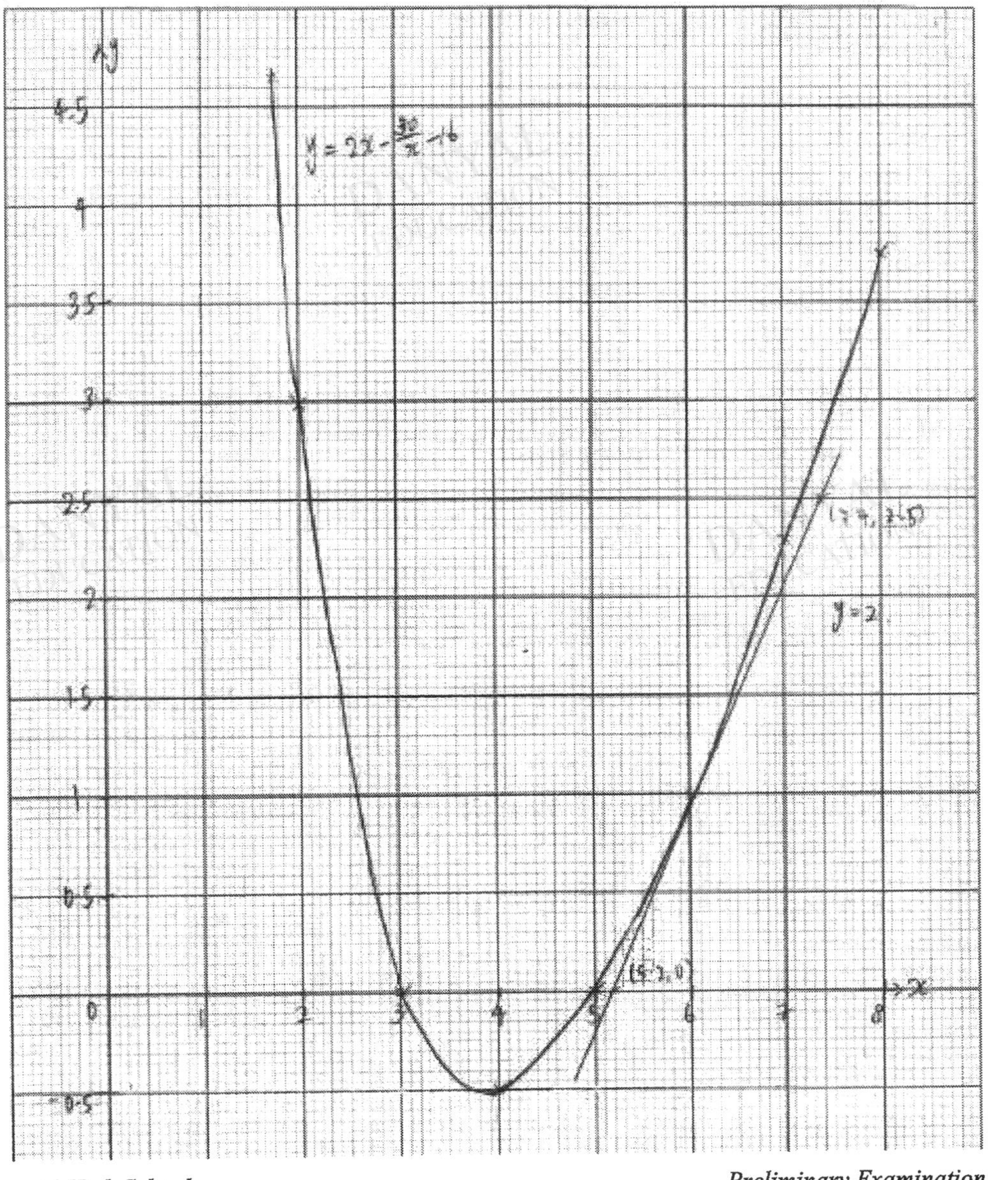
1ii(a)	1/6
1ii(b)	1/6
1ii(c)	5/6
1iii	¾ < 5/6, disagree as it has decreased
2a	\$86720
2bi	\$8904.19
2bii	796.87 < 871.11 (less monthly instalment)
3ai	48 points
3aii	13.1 points
3b	The **mean** of Team *A* scores are **higher** than Team *B*'s, hence Team *A* scored more points/better on **average.** The **standard deviation** of Team *A* scores are **smaller** than Team *B*'s hence Team *A*'s scores are **more consistent** than Team *B*'s. Hence, Team *A* performed better.
3c	2/21
4ai	100°, angles in opp segment
4aii	45°, radius perpendicular to tangent
4aiii	45°, angles in same segment
4bi	Show $\angle CBE = \angle BEC$, base angles of isosceles triangles equal
4bii	$\angle CEB = \angle BEF$ $\angle ECB = \angle EBF = 90°$ $\angle EBC = \angle EFB = 45°$ 3 pairs of corresponding angles equal
5a	No. of houses in 15 days $= \dfrac{15}{x}$
5b	No. of houses in 15 days $= \dfrac{15}{x+2}$
5d	5 houses
6a	$\mathbf{A} = \begin{pmatrix} 100 & 80 & 120 & 180 \\ 90 & 80 & 70 & 150 \end{pmatrix}$
6b	$\mathbf{B} = \begin{pmatrix} 300 \\ 280 \\ 230 \\ 200 \end{pmatrix}$
6c	$\mathbf{P} = \begin{pmatrix} 116000 \\ 95500 \end{pmatrix}$
6d	$\mathbf{D} = \begin{pmatrix} 480 \\ 390 \end{pmatrix}$ Elements of **D** represent the **total number** of tickets sold for **Saturday and Sunday respectively** for that weekend.
6ei	$\mathbf{E} = \begin{pmatrix} 1 & 1 \end{pmatrix}$
6eii	$\mathbf{ED} = \begin{pmatrix} 870 \end{pmatrix}$
6f	$x = 0.95$ $y = 1.25$
7ai	679 cm^3
7aii	625 cm^3
7bi	1670 cm^2
7bii	97 cubes
8ai	$x = \dfrac{13y}{2y+1}$
8aii	5.2
8bi	2
8bii	−1
8c	$\dfrac{2(x-3)}{x+3}$ or $\dfrac{2x-6}{x+3}$
8d	$(x-3)^2+5$; (3, 5)
9c	097.5° ±1.5°
9di	64 m ±1
9dii	$\frac{1}{2}ac\sin C$ formula, <u>**indicate specifically**</u> the two sides and one angle used. OR $\frac{1}{2}\times \text{base} \times \text{height}$ formula, <u>**indicate specifically**</u> the base and height.
9diii	40 m ±1
9div	63.4° (range from 62.9° to 64.0°)
10a	$p = 3$
10c	1.14 (range from 0.97 to 1.37)
10di	$2.1 < x < 7.2$
10dii	$y = 2$ seen, $x = 2.2$ or $x = 6.8$ (±0.1 for each)

11ai	116.0°	11b	Decrease of 59.5% > 55%, agree with Alex.
11aii	0.463%		

Q9 a & b

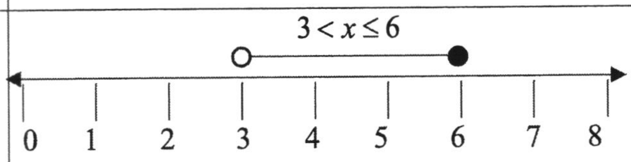


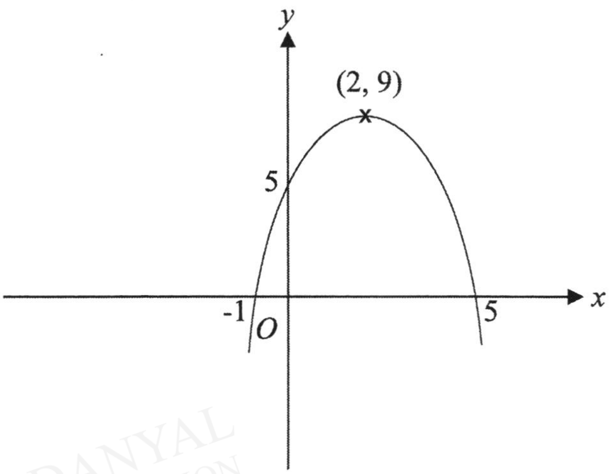
Q10)



HIHS 2020 Prelim Secondary 4E/5NA
Mathematics Paper 1 Marking Scheme

Qn. No.		Marking Scheme	Marks Allocations	Remarks
1		**Method 1:** $\left(\frac{d^6}{c^2}\right)^{-\frac{5}{2}}$ $=\frac{d^{-15}}{c^{-5}}$ $=\frac{c^5}{d^{15}}$ **Method 2:** $\left(\frac{d^6}{c^2}\right)^{-\frac{5}{2}}$ $=\left(\frac{c^2}{d^6}\right)^{\frac{5}{2}}$ $=\frac{c^5}{d^{15}}$		M1 (either by method 1 or method 2) A1
2		Possible answers: The **vertical axis does not start from zero** and thus it makes the number of prize winners in **2018** to be **4 times of 2017**, when it is **actually 10 more.** The **scale of the vertical axis is inconsistent** and thus it makes the number of prize winners in **2018** to be more than **double** that in **2016**, when it is **only slightly more than 5 more**.	B1 B1 B1 B1	Misleading feature How is it misleading Misleading feature How is it misleading
3		$9\times243^n=1$ $3^2\times\left(3^5\right)^n=3^0$ Comparing indices, $2+5n=0$ $n=-\frac{2}{5}$	M1 A1	Change to same base
4		$(2p+3)^2-1$ $=4p^2+12p+9-1$ $=4p^2+12p+8$ $=2\left(2p^2+6p+4\right)$ Since $(2p+3)^2-1$ has a **factor of 2/ factor of 4**, hence it is divisible by 2.	M1 A1	Factorise common factor 2 or 4
5		$6ac-14ad+3bc-7bd$ $=2a(3c-7d)+b(3c-7d)$ $=(2a+b)(3c-7d)$	M1 A1	

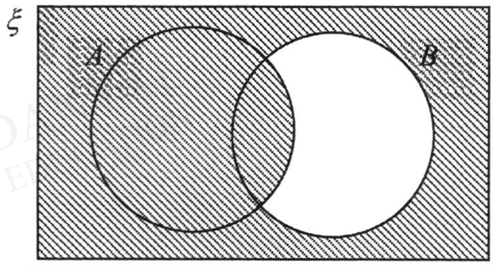
Qn. No.		Marking Scheme	Marks Allocations	Remarks
6	(a)	$\sqrt[3]{0.027}=0.3$ $\frac{31}{37}=0.\dot{8}\dot{3}\dot{7}$ $\frac{\pi}{22}=0.142799......$ $1.21^{\frac{1}{8}}=1.0241136...$ $1.21^{\frac{1}{8}},\ \frac{31}{37},\ \sqrt[3]{0.027},\ \frac{\pi}{22}$	 B1	
	(b)	Sum of rational numbers $=\sqrt[3]{0.027}+\frac{31}{37}$ $=1\frac{51}{370}$	B1 B1	Recognize the two rational numbers Must be in exact form
7	(a)	**Method 1:** $4<3x-5\le 13$ $9<3x\le 18$ $3<x\le 6$ **Method 2:** $4<3x-5\le 13$ $4<3x-5$ and $3x-5\le 13$ $x>3$ and $x\le 6$ $\therefore 3<x\le 6$	 M1 A1 M1 A1	
	(b)		B1	Scale labelled must be of equal interval No label of inequalities, do not penalise.

Qn. No.		Marking Scheme	Marks Allocations	Remarks
8	(a)		B2	B1: correct shape B1: label all interception and turning point clearly Deduct 1m for the following: • slanted/not symmetrical curve (too severe) • frizzy/multiple "line" curve
	(b)	When the graph of $y=2x+9$ is drawn (line), there is **no intersection between the graph of** $y=-(x+1)(x-5)$ **and** $y=2x+9$, hence no solution.	B1	No mark awarded for drawing of $y=2x+9$
9	(a)	$\begin{pmatrix}3 & -2\end{pmatrix}\begin{pmatrix}2\\h\end{pmatrix}=10$ $6-2h=10$ $h=-2$	B1	
	(b)	$G=\begin{pmatrix}1 & 2\\-2 & 0\end{pmatrix}$ G^2 $=\begin{pmatrix}1 & 2\\-2 & 0\end{pmatrix}\begin{pmatrix}1 & 2\\-2 & 0\end{pmatrix}$ $=\begin{pmatrix}1-4 & 2+0\\-2+0 & -4+0\end{pmatrix}$ $=\begin{pmatrix}-3 & 2\\-2 & -4\end{pmatrix}$	M1 A1	Able to recognize the property

Qn. No.		Marking Scheme	Marks Allocations	Remarks
10	(a)	By measuring, Width = 7 cm Length = 9 cm ➔Area = 63 cm^2 63 cm^2 on map represents 15.75 m^2 on floor 1 cm^2 on map represents 0.25 m^2 on floor 1cm on map represents 0.5 m on floor $\therefore n = 0.5$ **Accept the following range** Width = either 6.95 or 7 cm Length = either 9 or 9.1 cm Hence $62.55 \le \text{area} \le 63.7$ $\therefore 0.497 \le n \le 0.502$	M1 A1 M1 A1	Area falls within the range
	(b)	No. of tiles for width = $\dfrac{7\times0.5}{0.5} = 7$ No. of tiles for length = $\dfrac{9\times0.5}{0.5} = 9$ Total number of tiles $= 7\times9$ $= 63$ (shown)	B1	Do not accept $\dfrac{15.75}{0.5\times0.5}$ **Award mark for the correct method used**
11	(a)	Time to reach maximum speed = $15 \div 0.5 = 30$ s	B1	
	(b)	Let the time period for maximum speed be *t*. Total distance = 750 $\frac{1}{2}(t+90)(15) = 750$ .... $t = 10$	M1 A1	Finding distance travelled with area under graph

Qn. No.		Marking Scheme	Marks Allocations	Remarks
12	(a)	$0 \le m \le 34$ or $0 \le m < 35$	B1	
	(b)	Median = 40.5 $\Rightarrow$ median at 50^{th} and 51^{st} positions $\therefore m = 10 + 25 + (15 - 1) - 35 + 1 = 15$ Or $m = 50 - 35 = 15$	B1	Working not necessary though encouraged
	(c)	Since there are 105 workers, m = 20 $\therefore$ Mean $= \dfrac{380+975+600+820+1470}{105}$ $= \dfrac{4245}{105} = 40.4$ (3 s.f.)	B1	Accept if students calculate from calculator using Statistics mode. Accept $40\frac{2}{7}$
13		$\dfrac{QS}{\sin 114°} = \dfrac{11.45}{\sin 38°}$ $\Rightarrow QS = \dfrac{11.45 \sin 114°}{\sin 38°} = 16.99001$ $\cos QRS = \dfrac{8.2^2 + 10.3^2 - 16.99001^2}{2 \times 8.2 \times 10.3}$ $= -0.68275...$ $\therefore Q\hat{R}S = 133.1°$	M1 M1 A1	Finding QS with Sine Rule Applying Cosine Rule to find angle QRS -1 from question if premature approximation in intermediate values
14	(a)	$\dfrac{\text{Volume of cylinder } U}{\text{Volume of cylinder } V} = \left(\dfrac{\text{Height of cylinder } U}{\text{Height of cylinder } V}\right)^3$ $\Rightarrow \dfrac{216}{729} = \left(\dfrac{\text{Height of cylinder } U}{\text{Height of cylinder } V}\right)^3$ $\therefore \dfrac{\text{Height of cylinder } U}{\text{Height of cylinder } V} = \sqrt[3]{\dfrac{216}{729}} = \dfrac{2}{3}$	B1	Working will still be expected
	(b)	$\dfrac{\text{Cost of painting cylinder } U}{\text{Cost of painting cylinder } V} = \dfrac{\text{Surface area of cylinder } U}{\text{Surface area of cylinder } V}$ $\Rightarrow \dfrac{15.90}{\text{Cost of painting cylinder } V} = \left(\dfrac{2}{3}\right)^2$ $\therefore$ Cost of painting cylinder $V = \dfrac{9}{4} \times 15.90 = \$35.78$	M1, A1	Applying ratios of areas of similar solids

Qn. No.		Marking Scheme	Marks Allocations	Remarks
15	**(a)**	$540 = 2\times2\times3\times3\times3\times5$	B1 o.e.	Accept index notation
	(b)	$k = 2\times5\times5$ $k = 50$	B1	
	(c)	$30 = 2\times3\times5$ Since $30 = 2\times3\times5$ is the H.C.F., z can consist of another prime factor of 5 or above. Due to the constraint of "z is a number between 100 and 200", choose the smallest prime factor 5. $z = 2\times3\times5\times5$ $z = 150$ (*check: falls within the range given*) Alternative Method Since HCF = 30, 30 can be the other number z but since $100 < z < 200$, $z \neq 30$, $\therefore z = 30\times5 = 150$	M1 A1 M1 A1	
16	**(a)**	Common difference $=\frac{25-4}{3}$ $=7$ $a = 11$ $b = 18$ $c = 32$	B2	Deduct 1 mark for 1 mistake
	(b)	$T_1 = 4$ $T_2 = 4+7 = 11$ $T_3 = 4+2(7) = 18$ $T_4 = 4+3(7) = 25$ $T_n = 4+(n-1)(7)$ $T_n = 7n-3$	B1 (o.e.)	
	(c)	$101 = 7n-3$ $7n = 104$ $n = 14\frac{6}{7}$, which is not an integer When the term is 101, ***n* is not an integer**, thus, 101 is not in the sequence. **OR** As ***n* must be an integer**, 101 cannot be a term of this sequence.	B1	

Qn. No.		Marking Scheme	Marks Allocations	Remarks
17	**(a)(i)**	$\xi=\{2,3,4,5,6,7,8,9,10,11,12,13,14\}$ $E=\{2,3,5,7,11,13\}$ $F=\{2,3,6,9\}$ $G=\{3,6,9,12\}$ The elements in $E\cap F$ are 2 and 3 (or can write as 2, 3)	B1	Accept $E\cap F=\{2,3\}$
	(a)(ii)	**Correct:** $11\in E$ $\quad F\cap G=\{3,6,9\}$	B1	Must identify both
	(b)		B1	
	(c)	$(A\cap B)'\cap(A\cup B)$, $(A\cap B')\cup(A'\cap B)$, $(A'\cap B')'\cap(A\cap B)'$, $(A'\cup B')'\cap(A\cup B)$, $((A\cap B)\cup(A'\cap B'))'$, $((A\cap B)\cup(A\cup B)')'$	B1	Accept any
18	**(a)**	Angle $FEB=180°-70°$ (adj. $\angle$s on a str. line) $=110°$ $\therefore$ Angle $ABE=360°-90°-55°-110°$ ($\angle$ sum in quad) $=105°$ **<u>Alternative Method</u>** Angle $FDB=180°-90°-55°$ ($\angle$ sum in triangle) $=35°$ Angle $ABE=70°+35°$ (ext. angles of triangle) $=105°$	M1 A1	Deduct 1 mark from question if NO or WRONG angle properties are given
	(b)	Angle $EBC=360°-105°-115°$ ($\angle$ sum in quad) $=140°$ $\therefore$ Angle $BCD=180°-140°$ (int. $\angle$s) $=40°$	√M1 A1	√using (a)
19		Length of wire $=4.25+3.8+\dfrac{14\pi}{9}\times 3$ $=\dfrac{161}{20}+\dfrac{14\pi}{3}$ $\therefore p=8\dfrac{1}{20},\ q=4\dfrac{2}{3}$	M1 M1 B1 each	Arc length Sum of all length Accept $p=8.05$

Qn. No.		Marking Scheme	Marks Allocations	Remarks
20	**(a)**	Interior angle of decagon $\frac{(10-2)\times 180°}{10}=144°$	B1	
	(b)	Angle $LKM=360°-144°-120°$ ($\angle$ sum at a pt.) $=96°$ $\frac{(n-2)\times 180°}{n}=96°$ $96n=180n-360$ $n=\frac{360}{84}=4.285..$ Since the number of sides is **not an integer**, KL and MK will **not be 2 sides of a regular polygon**.	M1 M1 B1	Justification supported with working
21	**(a)**	$RS=\sqrt{(4-1)^2+(6+2)^2}=\sqrt{73}$ $\therefore k=73$	B1	
	(b)	$m_{RS}=\frac{6-(-2)}{4-1}=\frac{8}{3}$ $\therefore$ Eqn. of RS is $y-6=\frac{8}{3}(x-4)$ $y=\frac{8}{3}x-\frac{14}{3}$	M1 A1	Improper fractions used. (accept integer coefficients)
	(c)	$\cos X\hat{R}S=-\frac{3}{\sqrt{73}}$ $\cos X\hat{R}S\approx -0.3511$	B1	
22	**(a)**	$3\times WY=28+2$ $\Rightarrow WY=10$ m $\therefore$ Breadth of rectangular garden $=2WY=20$ m (shown) **Alternative method** $4+3WX=28$ $WX=8$ Width $=4+2WX$ $=4+16$ $=20$ m (Shown)	M1 A1 M1 A1	Finding WY
	(b)	$WY=10$ m $\Rightarrow WX=8$ m $\therefore$ Area of path $=\frac{3}{4}\pi(10^2-8^2)+2\times 10$ $=105$ m^2	√M1, M1 A1	√ Using value of WY Area of 'curved' paths, area of rectangle

Qn. No.		Marking Scheme	Marks Allocations	Remarks
23	**(a)**	Total number of litres $=\frac{15000}{100}\times 12.1$ $=1815$ Total amount $=1815\times \$2.55$ $=\$4628.25$	M1 A1	
	(b)(i)	963 million $=9.63\times 10^8$	B1	
	(b)(ii)	Amount spent on online food and beverages $=\$\left(9.63\times 10^8\right)\times 9.8\%$ $=\$94374000$ Average amount spent on food and beverages per person $=\$\left(\frac{94374000}{5.85\times 10^6}\right)$ $=\$16.13$ (2 d.p.)	M1 (s.o.i) A1	
24	**(a)(i)**	Probability $=\frac{13}{30}$	B1	
	(a)(ii)	Median time = 3.2 hrs = 3 h 12 min	B1 c.a.o.	
	(a)(iii)	Time taken by last qualifying athlete = 2.8 hrs = 2 h 48 min $\therefore$ Time last qualifying athlete finished the race = 2.18 pm	B1 c.a.o	
	(b)	For the group from previous year, Median = 3.4 hrs, IQR = 3.5 – 2.95 = 0.55 hrs For the group this year, Median = 3.2 hrs, IQR = 3.375 – 3 = 0.375 hrs The competitors from **this year ran faster** as they have a **shorter median time** and there is also **a smaller spread** in their timing due to the **smaller interquartile range**. OR The competitors this year **generally ran faster** due to lower median time and their timing is **more consistent** due to a **smaller interquartile range**.	 B1 B1	No need to quote the values s.o.i. comparison using median and IQR respectively

<table>
<tr><td></td><td><u>4E5N Prelim Math Paper 2 Marking Scheme</u></td><td></td></tr>
<tr><td>Qn</td><td>Solution</td><td>Marks</td></tr>
<tr><td>1i</td><td>
<table>
<tr><td>Ball number</td><td>2</td><td>3</td><td>6</td><td>9</td></tr>
<tr><td>2</td><td>-</td><td>(2, 3)</td><td>(2, 6)</td><td>(2, 9)</td></tr>
<tr><td>3</td><td>(3, 2)</td><td>-</td><td>(3, 6)</td><td>(3, 9)</td></tr>
<tr><td>6</td><td>(6, 2)</td><td>(6, 3)</td><td>-</td><td>(6, 9)</td></tr>
<tr><td>9</td><td>(9, 2)</td><td>(9, 3)</td><td>(9, 6)</td><td>-</td></tr>
</table>
</td><td>B2
Deduct 1m if "–" diagonal is included
Deduct 1m for any <u>two</u> mistakes in paired-values</td></tr>
<tr><td>1ii(a)</td><td>1/6</td><td>B1</td></tr>
<tr><td>1ii(b)</td><td>1/6</td><td>B1</td></tr>
<tr><td>1ii(c)</td><td>5/6</td><td>B1</td></tr>
<tr><td>1iii</td><td>New probability (with replacement)
$=\frac{12}{16}$
$=\frac{3}{4}$
$<\frac{5}{6}$ from 2(ii)(c)

Therefore, I do not agree as the probability has decreased</td><td>M1

A1 (must show comparison to 5/6)</td></tr>
<tr><td>2a</td><td>Loan amount $=\frac{70}{100}\times 80000$
$=\$56000$
Interest earned $=56000\times\frac{2}{100}\times\frac{72}{12}$
$=\$6720$
Total Amount $=\$80000+\$6720$
$=\$86720$
OR
Total Amount $=\frac{30}{100}\times 80000+\left(1+\frac{72}{12}\left(\frac{2}{100}\right)\right)\times 56000$
$=\$86720$</td><td>M1 (70% of 80 000)

M1 ($\frac{PRT}{100}$ substituted correctly)

A1</td></tr>
<tr><td>2b(i)</td><td>Total Amount (Bryan)$=\$80000\left(1+\frac{1.8}{100}\right)^{10}$
$=\$95624.189$</td><td>M1 (correct application of formula)

M1 (total amount)

A1</td></tr>
</table>

	Difference $= \$95264.189 - \$86720$ $= \$8904.19$	
2b(ii)	James' monthly instalment $= \dfrac{56000+6720}{72}$ $= \$871.111$ Bryan's monthly instalment $= \dfrac{95624.189}{120}$ $= \$796.868$ $< \$871.111$ (James') Bryan's **monthly instalment is less** than James', hence might be preferred. ------------------------- Award 1m for "Bryan's method **does not require a deposit**" (must show calculation of monthly instalment to award another mark)	M1 (use their previous two answers to find monthly instalment) A1 (comparison + explanation)
3ai	Mean score = 48 points	B1
3aii	Standard Deviation = 13.1 points (3sf)	B1
3b	The **mean** of Team *A* scores are **higher** than Team *B*'s, hence Team *A* scored more points (better) on **average (or in general)**. The **standard deviation** of Team *A* scores are **smaller** than Team *B*'s hence Team *A*'s scores are **more consistent (or has smaller spread)** than Team *B*'s. Hence, Team *A* performed better.	B1 B1 (Must answer the question)
3c	P(both scores greater than 55 points) $= \dfrac{5}{15} \times \dfrac{4}{14}$ $= \dfrac{2}{21}$	M1 A1
4ai	$\angle DAB = 180° - 80°$ (**angles in opposite segments**) $= 100°$	B1
4aii	$\angle FEB = 180° - 90° - 45°$ (**radius $\perp$ tangent**) $= 45°$	B1

4aiii	$\angle CAB = \angle FEB$ **(angles in the same segment)** $= 45°$	B1
4bi	$\angle BCE = 90°$ (angles in semi-circle) $\angle CBE = 180° - 90° - 45°$ (angle sum in triangle) $= 45°$ $= \angle BEC$ $\therefore \Delta BEC$ is an isosceles triangle.	M1 (90° "angles in semi-circle" seen) *No marks for angle $EBC = 45°$ only A1 (show base angles equal with conclusion)
4bii	$\angle CEB = \angle BEF$ (common) $\angle ECB = \angle EBF$ $= 90°$ $\angle EBC = \angle EFB$ $= 45°$ Since **3 pairs of corresponding angles are equal,** ΔECB is similiar to ΔEBF.	M2 (any 2 pairs of corr. angles shown) A1
5a	x days for 1 house 1 day for $\frac{1}{x}$ house No. of houses in 15 days $= \frac{15}{x}$	B1
5b	$(x+2)$ days for 1 house 1 day for $\frac{1}{x+2}$ house No. of houses in 15 days $= \frac{15}{x+2}$	B1
5c	$\frac{15}{x} - \frac{15}{x+2} = 2$ $15(x+2) - 15x = 2x(x+2)$ $2x^2 + 4x - 30 = 0$ $x^2 + 2x - 15 = 0$	M1 (formulating eqn) M1 (removing denominator) A1 (algebraic manipulation and simplifying)
5d	$(x+5)(x-3) = 0$ $x = -5$ or $x = 3$	M1 (factorizing or using quadratic formula) M1 (show both solutions)

	No. of houses $= \dfrac{15}{3}$ $= 5$	A1
6a	$\mathbf{A} = \begin{pmatrix} 100 & 80 & 120 & 180 \\ 90 & 80 & 70 & 150 \end{pmatrix}$	B1
6b	$\mathbf{B} = \begin{pmatrix} 300 \\ 280 \\ 230 \\ 200 \end{pmatrix}$	B1
6c	$\mathbf{P} = \begin{pmatrix} 100 & 80 & 120 & 180 \\ 90 & 80 & 70 & 150 \end{pmatrix} \begin{pmatrix} 300 \\ 280 \\ 230 \\ 200 \end{pmatrix}$ $= \begin{pmatrix} 30000 + 22400 + 27600 + 36000 \\ 27000 + 22400 + 16100 + 30000 \end{pmatrix}$ $= \begin{pmatrix} 116000 \\ 95500 \end{pmatrix}$	M1 (soi) A1
6d	$\mathbf{D} = \begin{pmatrix} 100 & 80 & 120 & 180 \\ 90 & 80 & 70 & 150 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 100 + 80 + 120 + 180 \\ 90 + 80 + 70 + 150 \end{pmatrix}$ $= \begin{pmatrix} 480 \\ 390 \end{pmatrix}$ Elements of **D** represent the **total number** of tickets sold for **Saturday and Sunday respectively** for that weekend.	B1 B1
6ei	$\mathbf{E} = \begin{pmatrix} 1 & 1 \end{pmatrix}$	B1
6eii	$\mathbf{ED} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 480 \\ 390 \end{pmatrix}$ $= \begin{pmatrix} 480 + 390 \end{pmatrix}$ $= \begin{pmatrix} 870 \end{pmatrix}$	B1
6f	$x = 0.95$ $y = 1.25$	B1 B1

7ai	Volume of 6 balls $= 6\times\frac{4}{3}\pi(3)^3$ $= 678.5840$ $= 679 \text{ cm}^3$	M1 ($\frac{4}{3}\pi(3)^3$ seen) A1
7aii	**Method 1 (Trigonometric formula)** Base area $= \frac{1}{2}\times 22.4\times 22.4\times \sin 60°$ $= 217.268$ **Method 2 (Pythagoras' Theorem)** Base area $= \frac{1}{2}\times 22.4\times\sqrt{22.4^2-11.2^2}$ $= 217.268$ Volume of triangular prism $= 217.268\times 6$ $= 1303.61 \text{ cm}^3$ Volume of unoccupied space $= 1303.61 - 678.5840$ $= 625.026$ $= 625 \text{ cm}^3$	M1 ($\frac{1}{2}ab\sin C$ seen, correct substitution) **OR** M1 (1/2 base x height seen using Pythogoras) M1 (their base area x height) A1
7bi	Total surface area of bowl $= 2\pi(10)^2 + 2\pi(12)^2 + [\pi(12)^2 - \pi(10)^2]$ $= 1671.327$ $= 1670 \text{ cm}^2$	M1 ($\pi(12)^2 - \pi(10)^2$ seen) M1($2\pi(10)^2$ and $2\pi(12)^2$ seen) A1
7bii	Volume of bowl $= \frac{2}{3}\pi(12^3) - \frac{2}{3}\pi(10^3)$ $= \frac{1456}{3}\pi \text{ cm}^3$ Volume of each cube $= 2.5^3$ $= 15.625 \text{ cm}^3$ Number of cubes $= \frac{1456}{3}\pi \div 2.5^3$ $= 97.58$ ≈ 97 cubes Maximum number of cubes = 97	M1 (vol. of bowl) M1 (vol. of bowl divide by vol. of cube) A1 (rounded down)

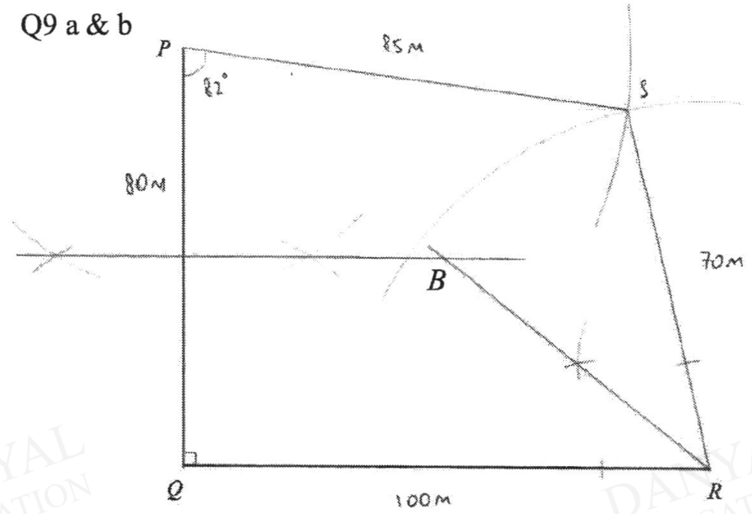
8ai	$x=\frac{y-x}{2y}+6$ $x=\frac{y-x+12y}{2y}$ or $x-6=\frac{y-x}{2y}$ $2xy=y-x+12y$ or $2xy-12y=y-x$ $2xy+x=13y$ $x(2y+1)=13y$ $x=\frac{13y}{2y+1}$	M1 (remove denominator) M1 (factorise) A1
8aii	Sub $y=2$, $x=\frac{13(2)}{2(2)+1}$ $=5.2$ or $5\frac{1}{5}$	B1
8bi	$(a-b)^2=a^2-2ab+b^2$ $8=a^2+b^2-2(-3)$ $a^2+b^2=2$	B1
8bii	$\left(\frac{a+b}{2}\right)^2=\frac{a^2+2ab+b^2}{4}$ $=\frac{2+2(-3)}{4}$ $=-1$	B1
8c	$\frac{2x^2-12x+18}{x^2-9}=\frac{2(x-3)(x-3)}{(x+3)(x-3)}$ $=\frac{2(x-3)}{x+3}$ or $\frac{2x-6}{x+3}$	M2 (1m for each complete factorization) A1
8d	$y=14-6x+x^2$ $=(x^2-6x+9)-9+14$ $=(x-3)^2+5$ *must use $\left(-\frac{6}{2}\right)^2$ or $(-3)^2$ as part of correct step Therefore coordinates of turning point are **(3, 5)**	M1 ($(x-3)^2$ seen) M1 (+5 seen) Deduct 1m for any incorrect step A1
9a	See attached	B1 (all construction lines shown + accuracy)

		B1 (labelling of S and all lengths accurately)
9b	See attached	B2 (all construction lines must be shown + accuracy)
9c	Bearing $= 097.5°$	B1 (accept $\pm 1.5°$)
9di	Actual distance $= 6.4 \times 10$ $= 64$ m	B1 (accept ± 1)
9dii	**<u>Method 1 (using trigonometry formula)</u>** **Using $\frac{1}{2}ac\sin C$ formula.** Measure or use ***BR* as *a***, ***RS* as *b***, and **angle *BRS* as *C***. *must relate sides and angle used to formula **can use other sides and angle, as long as angle is included angle **<u>Method 2 (using shortest distance)</u>** Using **area of triangle formula** $\frac{1}{2} \times \text{base} \times \text{height}$ Measure or use **the perpendicular/shortest distance from *B* to *RS* as height**, and ***RS* as the base** (or vice versa). *must relate the sides used to formula **can use other base / height pairing	B2 (correct formula used, with sides and angle clearly stated) B2 (correct formula used, with sides clearly stated) *student must state perpendicular or shortest distance, not just B to RS
9diii	Actual distance $= 4 \times 10$ $= 40$ m OR Student can calculate area of triangle RBS using their measurements (about 1380 m^2). Then letting distance be h, equate $\frac{1}{2} \times 70 \times h = 1400$ to find h. Value of h must be within 40 m (± 1 m)	B1 (accept ± 1 m) B1
9div	Let the greatest angle of elevation be θ.	

	$\tan\theta = \frac{80}{40}$ $\theta = \tan^{-1} 2$ $= 63.4°$	M1 (correct trigo ratio formed using their **d(iii)**) A1 *62.9° if 41m; 64.0° if 39m)
10a	$p = 2(2) + \frac{30}{2} - 16$ $= 3$	B1
10b	See attached graph	B1 – **all** points plotted accurately B1 – axes, scale & label B1 – smooth curve passing through **all** points
10c	Draw tangent at $x = 6$ $\text{gradient} = \frac{2.5 - 0}{7.4 - 5.2}$ $= 1.14$	B1 (accurate at (6, 1)) B1 (accept 0.97 to 1.37)
10di	$2.1 < x < 7.2$	B1 (accept ± 0.1 for each end)
10dii	$2x + \frac{30}{x} = 18$ $2x + \frac{30}{x} - 18 = 0$ $2x + \frac{30}{x} - 16 = 2$ Draw $y = 2$ to find intersection points. $x = 2.2$ (± 0.1) or 6.8 (± 0.1)	M1 (draw $y = 2$) A1
11ai	$\frac{92592}{287379} \times 360° = 115.99°$ $= 116.0°$ (1 dp)	M1 A1
11aii	Percentage of deaths $= \frac{2}{432} \times 100\%$ $= 0.463\%$ (3 sf) or $\frac{25}{34}\%$	M1 A1
11b	Without Social Distancing, $a = 0.1$	

	$N_1 = 1$ $N_2 = 1 + 0.1(1) = 1.1$ $N_3 = 1.1 + 0.1(1.1) = (1.1)(1 + 0.1) = (1.1)^2$ $N_4 = (1.1)^2 + 0.1(1.1)^2 = (1.1)^2(1 + 0.1) = (1.1)^3$ $\vdots$ $N_{100} = (1.1)^{99} = 12527.8294$	M1 (for N_{100} without social distancing)
	With Social Distancing, $a = 0.09$ $N_1 = 1$ $N_2 = 1 + 0.09(1) = 1.09$ $N_3 = 1.09 + 0.09(1.09) = (1.09)(1 + 0.09) = (1.09)^2$ $N_4 = (1.09)^2 + 0.09(1.09)^2 = (1.09)^2(1 + 0.09) = (1.09)^3$ $\vdots$ $N_{100} = (1.09)^{99} = 5072.5144$	M1 (formulating the pattern for a=0.09) S.O.I M1 (for N_{100} with social distancing)
	Percentage Decrease $= \dfrac{12527.8294 - 5078.5144}{12527.8294} \times 100\%$ $= 59.5\%$ (3 sf) I **agree** with Alex's claim as the percentage decrease is 59.5% which is **more than 55%.**	M1 (% decrease formula using their N_{100} values) M1 (correct % decrease) A1 (with comparison)
	OR Number of new infections $= \dfrac{1.09^{99}}{1.1^{99}} \times 100\%$ $= 40.5\%$ (3 sf)	M1 (fraction of their N_{100} values seen, **correctly labelled)**
	Percentage Decrease $= 100\% - 40.5\%$ $= 59.5\%$ (3 sf)	M1 (correct % decrease)
	I **agree** with Alex's claim as the percentage decrease is 59.5% which is **more than 55%.**	A1 (with comparison)

Q9 a & b



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Q10)