



FUCHUN SECONDARY SCHOOL
Secondary 4 Express / 5 Normal Academic
Preliminary Examination 2020

CANDIDATE
NAME

CENTRE
NUMBER

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INDEX
NUMBER

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CLASS

MATHEMATICS

Paper 1

4048/01

2 Sep 2020

2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

Name of Setter: Mr Chen Xin Da

This document consists of **18** printed pages.

[Turn over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

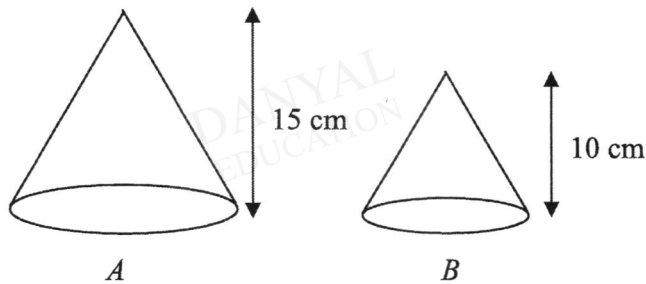
$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 Simplify $\left(\frac{16b^2}{c^6}\right)^{\frac{3}{2}}$.

Answer [2]

2 Two solid cones A and B are geometrically similar.



The heights of the two cones are 15 cm and 10 cm.

Given that the volume of A is $(3x + 1) \text{ cm}^3$ and the volume of B is $x \text{ cm}^3$, calculate the value of x .

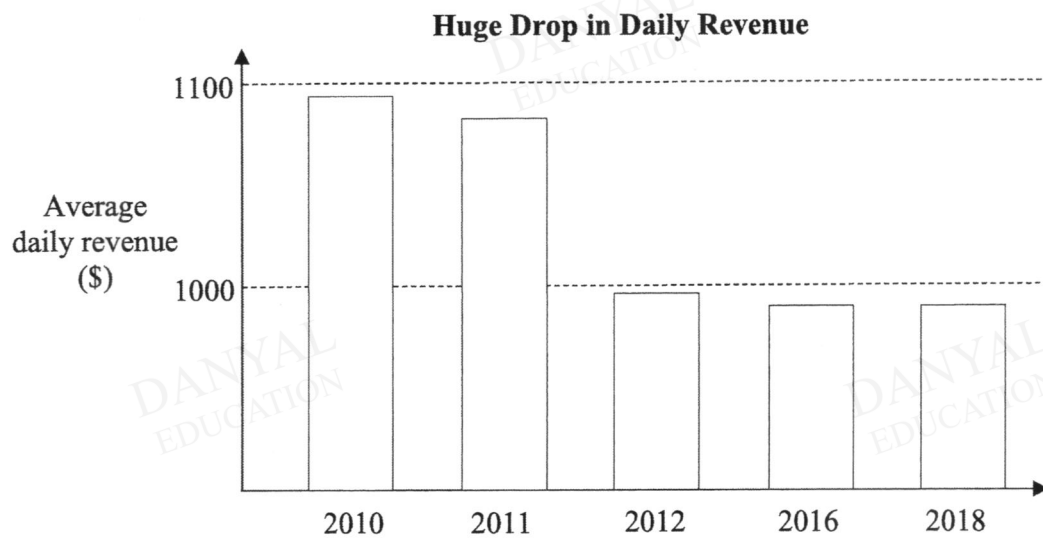
Answer $x =$ [2]

- 3 Solve the inequalities $2 \leq 3x - 1 < 2x + 8$.

Answer

[2]

- 4 The graph shows the average daily revenue for a shop over a number of years.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

.....

[2]

5 n is a positive integer.

Show that $(5n+2)^2 - 25n^2$ is a multiple of 4 for all integer values of n .

Answer

[2]

6 m is directly proportional to n^2 .

It is known that $m = 12$ for a particular value of n .

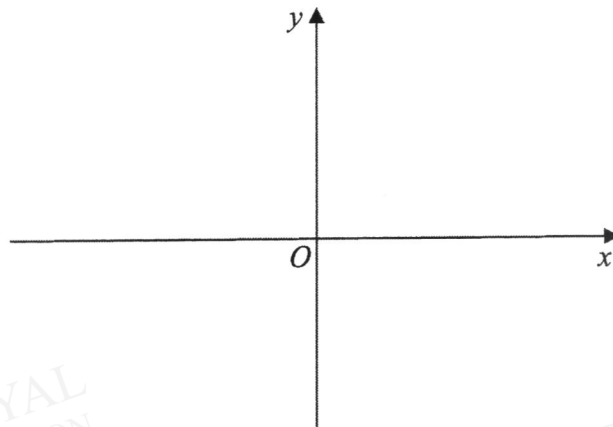
Find the value of m when this value of n is decreased by 20%.

Answer $m =$

[2]

- 7 (a) Sketch the graph of $y = -(x-1)(x+3)$ on the axes below.
Indicate clearly the x -intercepts and y -intercept of the graph.

[2]



- (b) Write down the equation of the line of symmetry of $y = -(x-1)(x+3)$.

Answer

[1]

- 8 An area of 16 cm^2 on a map represents an actual area of 4 km^2 .

Calculate

- (a) the actual area, in km^2 , that is represented by an area of 30 cm^2 on the map,

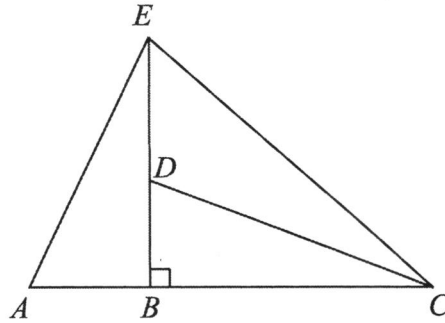
Answer km^2 [1]

- (b) the actual distance, in km, that is represented by 12 cm on the map.

Answer km [2]

- 9 In the diagram shown below, ABC and BDE are straight lines

It is given that EB is perpendicular to AC , $AE = DC$, $BC = BE$.



- (a) Prove that triangle ABE is congruent to triangle DBC .

Answer

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[2]

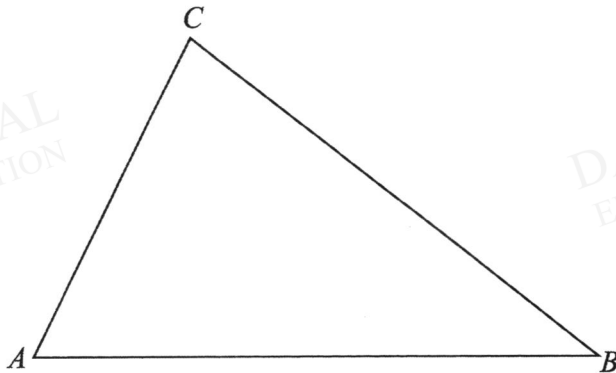
- (b) Given that the area of triangle $ABE = 10 \text{ cm}^2$ and D is the midpoint of BE , calculate the area of triangle BEC .

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Answer

..... cm^2 [1]

- 10 The diagram shows a triangle ABC .



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- (a) On the diagram, construct the bisector of angle CAB . [1]
- (b) By constructing a suitable perpendicular bisector on the same diagram, shade the region inside triangle ABC that is closer to AC than AB and closer to A than to C . [2]

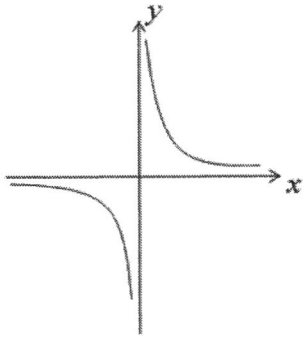


Figure 1

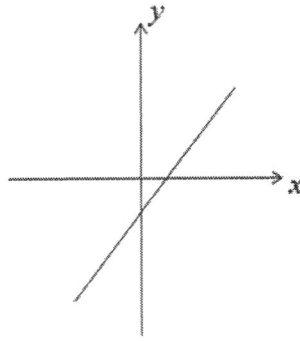


Figure 2

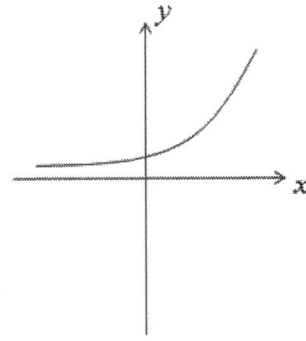


Figure 3

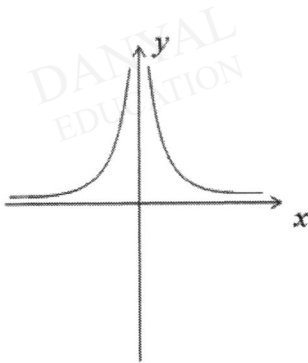


Figure 4

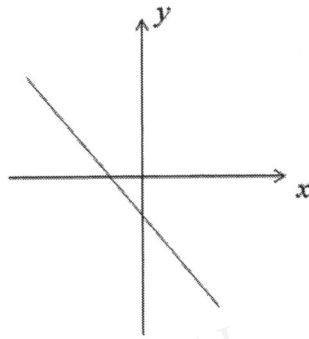


Figure 5

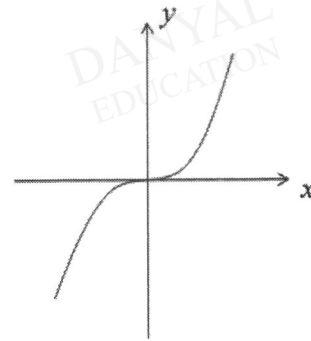


Figure 6

State the correct figure for each of the following equations.

(a) $y = 3^x$

Answer Figure [1]

(b) $y = \frac{2}{x}$

Answer Figure [1]

(c) $y = -x - 2$

Answer Figure [1]



A piece of elastic 12 cm long, hangs from a nail N , as shown in the figure above.

When a mass of m grams is attached to the lower end, the length of the elastic increases to L cm. For every 100 grams which is attached, the length of the elastic increases by 3 cm.

- (a) Calculate the length of the piece of elastic when a mass of 200 grams is attached to it.

Answer cm [1]

- (b) If the length of the piece of elastic is 24 cm, calculate the mass that is attached to it.

Answer g [1]

- (c) Write down a formula connecting the length of the elastic L , and the mass m , which is attached to it.

Answer [1]

-
- 13 A bag contains 19 marbles of which n are blue and the rest are yellow.

A marble is chosen at random and not replaced.

- (a) Write down, in terms of n , the probability that the marble is yellow.

Answer [1]

A second marble is chosen at random.

- (b) The probability that the second marble chosen is yellow is $\frac{4}{9}$.

Calculate the number of yellow marbles in the bag.

Answer [2]

- 14 Rachel invests a certain amount of money in an account.

The balance, $\$A$, of the account after t years is given by the formula $A = k \times 1.02^t$, where k is a constant.

When $t = 2$, $A = 52\,020$.

- (a) By finding the value of k , calculate the amount of money Rachel invests in the account at the start.

Answer \$ [2]

- (b) Calculate the percentage increase in the balance over 5 years.

Answer % [2]

- 15 The students of Class A and Class B took a common Mathematics test.
The table below shows the distribution of marks obtained by the students.

Class A

Marks (x)	Frequency
$0 < x \leq 5$	6
$5 < x \leq 10$	15
$10 < x \leq 15$	22
$15 < x \leq 20$	5

Class B

Mean = 10.5
Standard Deviation = 3.5

(a) For Class A, calculate an estimate for the

(i) mean marks,

Answer marks [1]

(ii) standard deviation of the marks.

Answer marks [1]

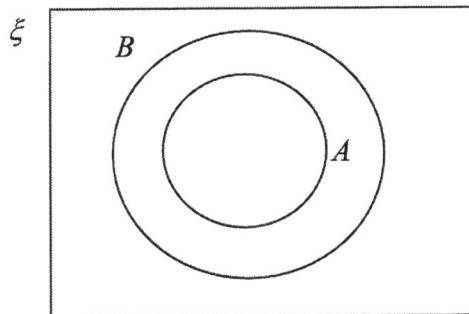
(b) Below are two statements comparing the marks obtained by the students in Class A and Class B.

For each one, state whether you agree or disagree, giving a reason for each answer.

Statement	Agree/disagree	Reason
Students in Class A scored better than students in Class B on average.		
The marks obtained by students in Class A are more consistent than the marks obtained by students in Class B.		

[2]

- 16 (a) On the Venn diagram, shade the region which represents $A' \cap B$.



[1]

- (b) $\xi = \{\text{integers } x : 1 \leq x \leq 10\}$
 $P = \{x : x \text{ is a factor of } 8\}$
 $Q = \{x : x \text{ is divisible by } 2\}$

- (i) List the elements contained in the set $P \cup Q'$.

Answer [1]

- (ii) List the elements contained in the set $P' \cap Q'$.

Answer [1]

- (iii) Is $Q \subset P$?
 Explain your answer.

Answer

[1]

17 Written as a product of its prime factors, $450 = 2 \times 3^2 \times 5^2$.

(a) (i) Express 98 as a product of its prime factors.

Answer 98 = [1]

(ii) Hence, use prime factors to explain why 98×450 is a perfect square.

Answer

.....

 [1]

(b) Given that $\sqrt[3]{450k}$ is an integer, write down the smallest integer value of k .

Answer $k =$ [1]

(c) Find the smallest positive integer m , such that $98m$ is a multiple of 450.

Answer $m =$ [1]

- 18 (a) Write as a single fraction in its simplest form $\frac{6}{(2x-1)^2} - \frac{5}{1-2x}$.

Answer

[2]

- (b) Given $4^b = \frac{1}{8} \times 32^a$, express b in terms of a .

Answer $b =$

[2]

- 19 (a) Determine whether it is possible that an interior angle of an n -sided regular polygon is 110° . Explain clearly with working.

Answer

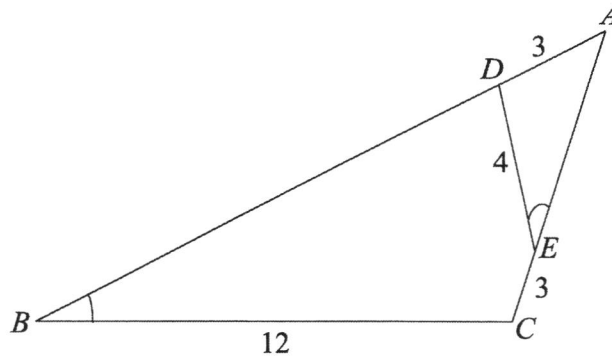
[2]

- (b) An n -sided polygon has 4 interior angles measuring 100° each. The remaining interior angles measure q° each. Find an expression for q in terms of n .

Answer $q =$

[2]

- 20 In the diagram, angle $AED = \text{angle } ABC$, $AD = 3$ cm, $DE = 4$ cm, $BC = 12$ cm and $EC = 3$ cm.



- (a) Prove that triangle ABC and triangle AED are similar.

Answer

[2]

- (b) Calculate the length of AE .

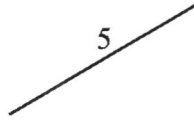
Answer

cm [2]

- (c) Write down the value of $\frac{\text{Area of } \triangle ADE}{\text{Area of } DBCE}$.

Answer

[1]



In the diagram above, A , B , C and Q are points on a circle, and angle $CAQ = 25^\circ$.
 AQ and BCT meet at the centre O . QT is a tangent to the circle at Q .

(a) Calculate

(i) angle QBC ,

Answer ° [1]

(ii) angle QOC ,

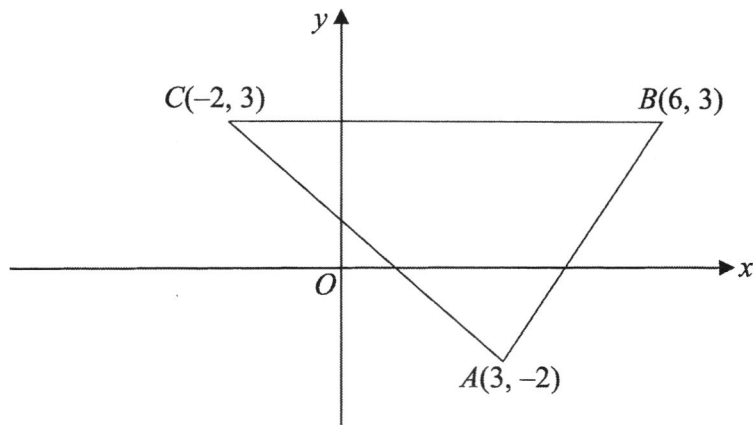
Answer ° [1]

(iii) angle CQT .

Answer ° [2]

(b) Given that $QC = 5$ cm, calculate the radius of the circle.

Answer cm [2]



- (a) Find the length of AB .

Answer units [2]

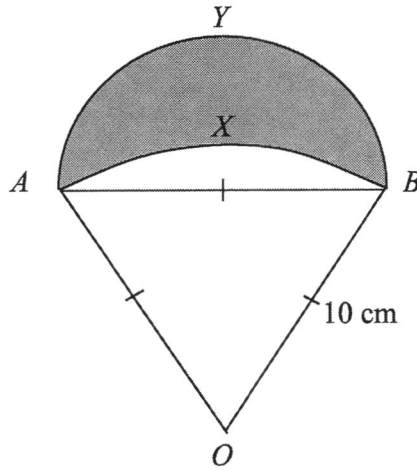
- (b) Given that the line AB cuts the y -axis at the point D , find the coordinates of D .

Answer D (.....) [2]

- (c) $ABCE$ is a trapezium with BC parallel to AE .
The area of the trapezium is 45 units^2 .
Find the coordinates of the point E .

Answer E (.....) [2]

- 23 In the diagram, OAB is an **equilateral** triangle.
 AXB is an arc of a circle with centre O and radius 10 cm.
 AYB is a semicircle with AB as the diameter.



Express the area of the shaded section as a percentage of the unshaded section.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 (a) Rearrange the formula $a = \frac{b(c^2 - d^2)}{3}$ to make c the subject.

Answer $c =$ [2]

- (b) (i) Factorise completely $50x^2 - 32$.

Answer [1]

- (ii) Hence, simplify $\frac{50x^2 - 32}{10x^2 + 3x - 4}$.

Answer [2]

-
- (c) The points $(-2, 3)$ and $(4, 12)$ lie on the curve given by the equation $y = ax^2 + bx + 2$.

Use an algebraic method to find the values of a and b .

Answer $a =$
 $b =$ [4]

(d) (i) Express $-10 + 8x + x^2$ in the form $(x + a)^2 + b$.

Answer [2]

(ii) Write down the coordinates of the minimum point of the graph of $y = -10 + 8x + x^2$.

Answer (..... ,) [1]

- 2 The first five terms in a sequence of numbers are given below.

$$T_1 = 2^2 + 1 = 5$$

$$T_2 = 3^2 + 6 = 15$$

$$T_3 = a^2 + b = c$$

$$T_4 = 5^2 + 16 = 41$$

$$T_5 = 6^2 + 21 = 57$$

- (a) Find the values of a , b and c .

Answer $a =$
 $b =$
 $c =$ [1]

- (b) Explain why the value of T_n must be odd for all values of n .

Answer
 [1]

- (c) Show that the n th term of the sequence, T_n , is given by $n^2 + 7n - 3$.

Answer

[2]

- (d) T_{p+1} and T_p are consecutive terms in the sequence.
 Find and simplify an expression, in terms of p , for $T_{p+1} - T_p$.

Answer [2]

- (e) Explain why two consecutive terms of the sequence cannot have a difference of 6.

Answer
 [1]

- 3 John cycled from his home to the town. He divided his journey into two parts.

- (a) He travelled the first 45 km at an average speed of x km/h.
Write down an expression, in terms of x , for the time taken in hours for the first 45 km.

Answer h [1]

- (b) He travelled the remaining 5 km at an average speed which was 10 km/h less than the first 45 km of his journey.
Write down an expression, in terms of x , for the time taken in hours for this part of the journey.

Answer h [1]

- (c) He took 3 hours for the whole journey.
Write down an equation in x , and show that it reduces to $3x^2 - 80x + 450 = 0$.

Answer

[3]

- (d) Solve the equation $3x^2 - 80x + 450 = 0$, giving your solutions correct to 2 decimal places.

Answer $x =$ or [4]

- (e) Hence, state the average speed for the first part and the second part of the journey, giving your answers correct to 2 decimal places.

Answer First part km/h
Second part km/h [1]

- (f) John claims that the average speed of the entire journey can be obtained by taking the mean of the two answers in (e).
Do you agree? Justify your answer with calculations.

Answer

.....

[2]

	Letters	Cards	Parcels
Kim	4	10	2
Mary	5	5	3

- (a) The information for the number of letters, cards and parcels sent by Kim and Mary can be presented by a 2×3 matrix \mathbf{N} . Write down the matrix \mathbf{N} .

Answer $\mathbf{N} = \left(\begin{array}{ccc} & & \end{array} \right)$ [1]

- (b) Postage is charged at \$0.30 for a letter, \$0.50 for a card and \$6 for a parcel. Write down a 3×1 matrix \mathbf{C} to represent each type of postage charge.

Answer $\mathbf{C} = \left(\begin{array}{c} & & \end{array} \right)$ [1]

- (c) (i) Given $\mathbf{P} = \mathbf{NC}$, evaluate \mathbf{P} .

Answer $\mathbf{P} = \left(\begin{array}{c} & & \end{array} \right)$ [2]

- (ii) State what the elements of \mathbf{P} represent.

Answer

.....
 [1]

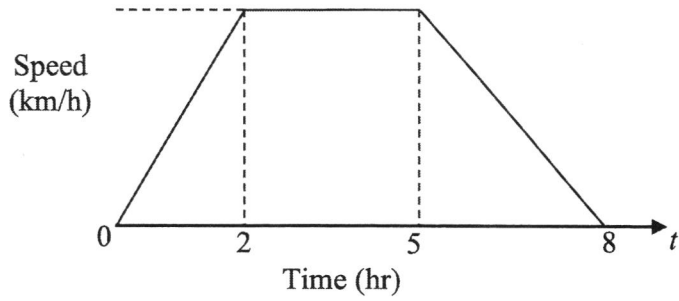
-
- (d) If the postage charge is increased by 20% for a letter, 10% for a postcard and 5% for a parcel,

- (i) formulate and write down a 3×3 matrix \mathbf{R} such that the matrix \mathbf{RC} will give the revised cost for each type of postage charge.

Answer $\mathbf{R} = \left(\begin{array}{ccc} & & \end{array} \right)$ [1]

- (ii) calculate the total amount that Kim and Mary need to pay to send out the letters, cards and parcels with the revised postage charges.

Answer \$ [3]



- (a) Given that the distance travelled in the first 5 hours is 320 km, show that $V = 80$.

Answer

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[2]

- (b) Describe the motion of the car between $t = 2$ to $t = 5$.

Answer

.....
.....

[1]

- (c) Calculate the deceleration of the car during the last 3 hours.

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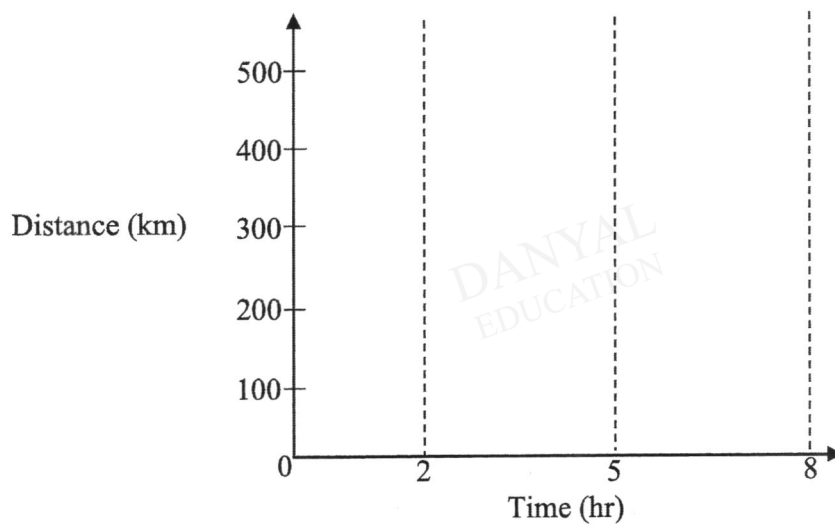
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Answer km/h² [1]

- (d) Calculate the total distance travelled by the car for the 8 hours.

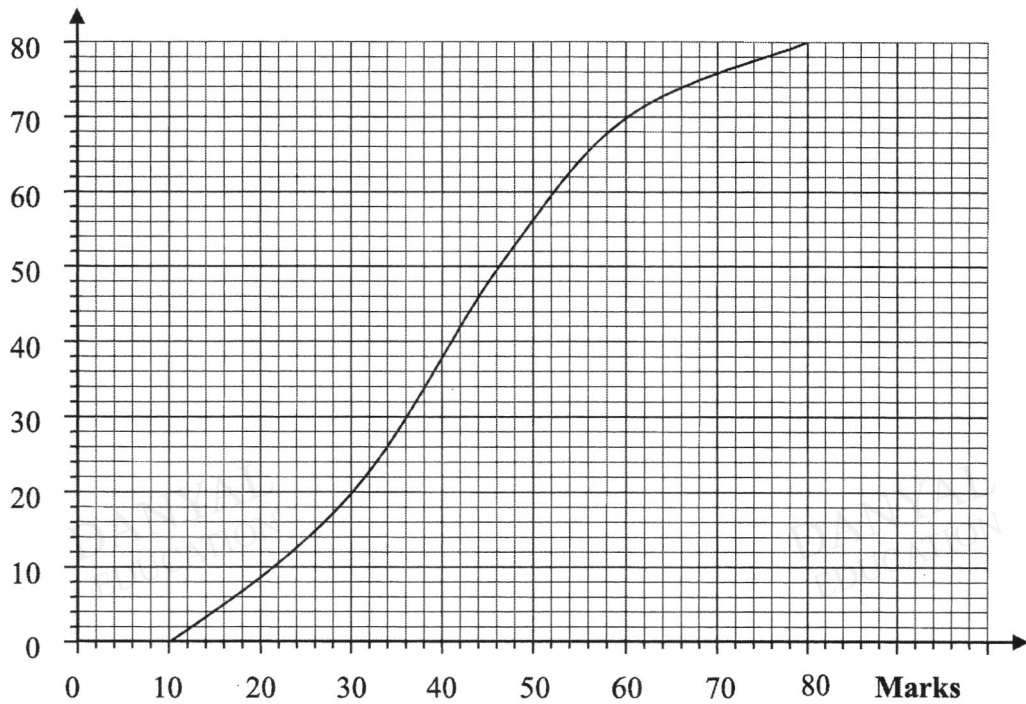
Answer km [1]

- (e) Sketch the distance-time graph for the 8 hours in the diagram below.
Label the values on the distance-axis clearly.



[3]

-
- 6 The cumulative frequency curve below illustrates the marks obtained by 80 students in School A in an examination.



- (a) Use the curve to estimate
 (i) the median mark,

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Answer marks [1]

- (ii) the interquartile range of the marks.

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Answer marks [2]

- (b) Find the number of students who obtained between 30 and 60 marks.

Answer [1]

- (c) The passing mark for the examination is 40 marks.

- (i) A student is chosen at random.

Find the probability that he or she will pass the examination.

Answer [1]

- (ii) Two students are chosen at random.
Find the probability that at least one of the students will pass the examination.

Answer [2]

- (d) The marks obtained by another 80 students in School *B* had the same median but a smaller interquartile range as compared to School *A*. Describe how the cumulative frequency curve for School *B* may differ from the curve for School *A*.

Answer [1]
.....

- 7 (a) In 2016, Anil, Ben and Cathy decided to start a small business.
Anil invested \$50 000, Ben invested \$30 000 and Cathy invested \$20 000.

They agreed that all the profits should be divided in the same ratio as the sum of money they

In 2018, the total profit was \$15 500.

- (i) What was the difference between Anil's and Cathy's share of the profit in 2018?

Answer \$ [2]

- (ii) The total profit in 2019 was 30% greater than that made in 2018.
Calculate the total profit made in 2019.

Answer \$ [2]

- (iii) The total profit in 2018 was 25% greater than that made in 2017.
Calculate the total profit made in 2017.

Answer \$ [2]

- (b) Two investment companies pay out the following interest rates as shown below.

Company A: $x\%$ simple interest per annum

Company B: $y\%$ compound interest per annum, compounded annually

[Turn over

Mr Lin invests \$50 000 in Company *A*.

Mr Tan invests \$50 000 in Company *B* and receives \$55 000 at the end of 3 years.

Both of them received the same amount of interest at the end of 3 years.

Determine which investment company offers the higher interest rate.

Show your working.

Answer

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[5]

-
- 8 The variables x and y are connected by the equation $y = \frac{x^2}{6} + \frac{2}{x}$.

Some corresponding values of x and y are given in the table below.

x	1	2	3	4	5	6	7
y	2.2	1.7	2.2	3.2	4.6	6.3	8.5

(a) On the grid opposite, draw the graph of $y = \frac{x^2}{6} + \frac{2}{x}$ for $1 \leq x \leq 7$. [3]

(b) Use your graph to write down an inequality in y to describe the range of values where $x > 1$.

Answer [1]

(c) By drawing a tangent, find the gradient of the curve at (4, 3.2).

Answer [2]

(d) The line $y + 2x = 15$ intersects the curve $y = \frac{x^2}{6} + \frac{2}{x}$.

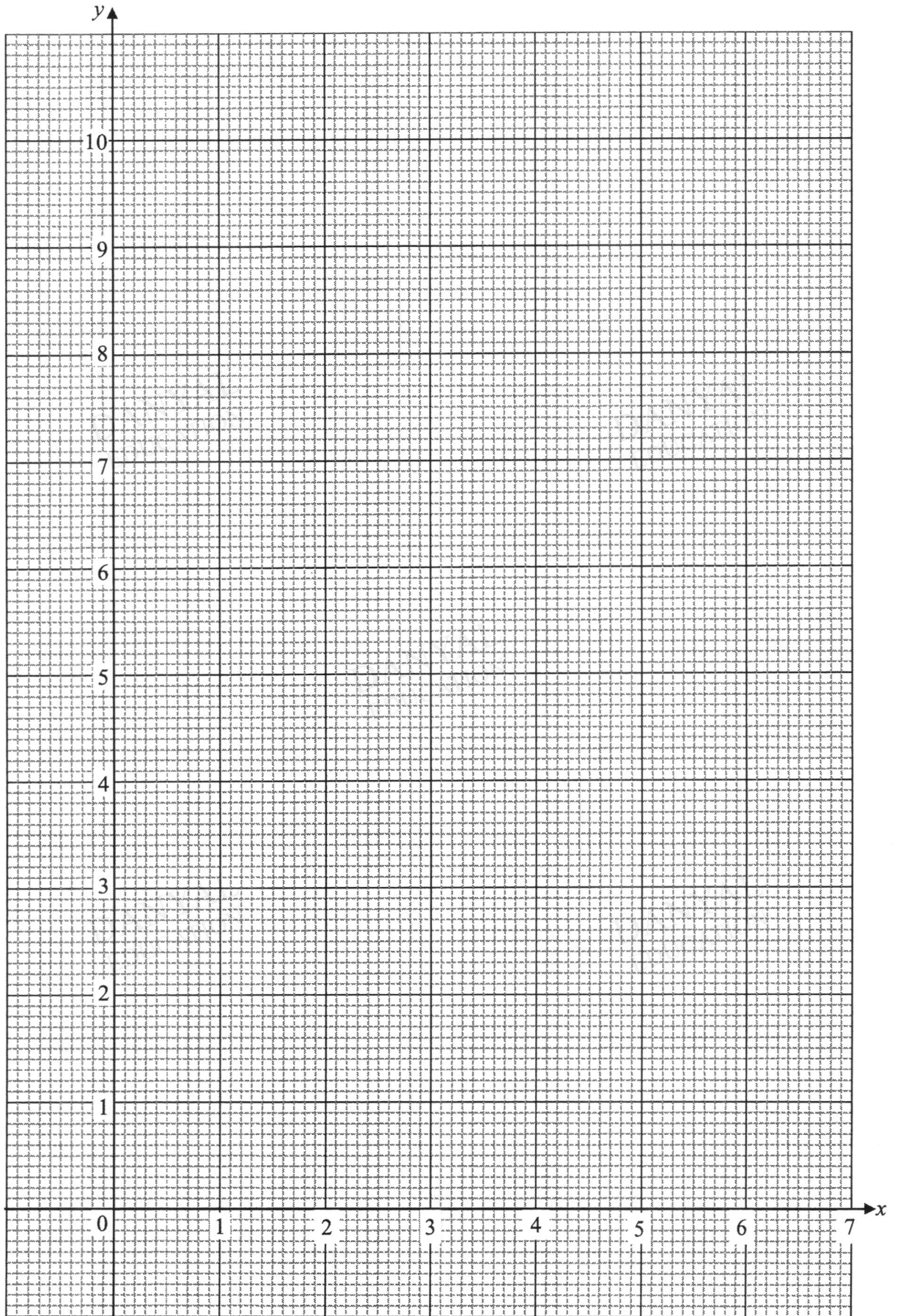
(i) On the same grid, draw the graph of the straight line $y + 2x = 15$ for $3 \leq x \leq 6$. [2]

(ii) Write down the x -coordinate of the point where this line intersects the curve.

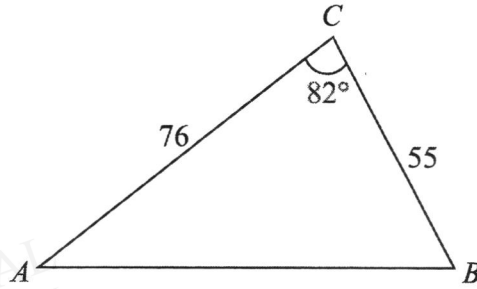
Answer $x =$ [1]

(iii) The value of x above is the solution of the equation $x^3 + ax^2 - 90x + b = 0$.
Find the value of a and the value of b .

Answer $a =$
 $b =$ [2]



- 9 The diagram shows a triangular field ABC on horizontal ground.
 B is due east of A .
 $BC = 55$ m, $AC = 76$ m and angle $ACB = 82^\circ$.



- (a) Calculate the length of AB .

Answer m [3]

- (b) Calculate the bearing of C from B .

Answer ° [2]

- (c) Calculate the area of the field ABC .

Answer m^2 [2]

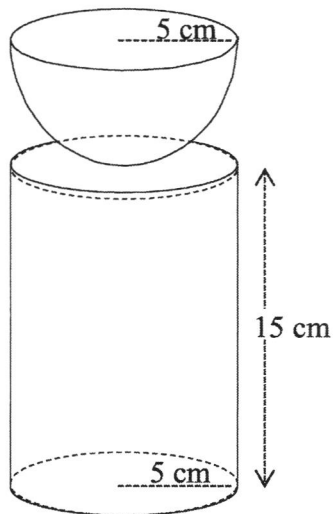
- (d) Calculate the shortest distance from C to AB .

Answer m [2]

- (e) T is the point on top of the tree vertically above C .
The angle of elevation of the top of the tree from A is 20° .
Calculate the largest angle of elevation of the top of the tree from a point on AB .

Answer $^\circ$ [3]

- 10 The diagram shows a trophy which consists of a solid hemispherical glass top that sits on top of a solid cylindrical glass base. Both the radii of the cylinder and the hemisphere are 5 cm. The height of the cylindrical base is 15 cm.



- (a) Find the total volume of the trophy.

Answer cm^3 [3]

- (b) The composition of the glass that is used to manufacture the trophy include sand, limestone and sodium carbonate in the proportion **75%, 10% and 15%** respectively.

Following are some information on sand, limestone and sodium carbonate.

	Density	Mass of one bag	Cost of one bag
Sand	1.60 g/cm ³	30 kg	\$26
Limestone	2.70 g/cm ³	25 kg	\$60
Sodium Carbonate	2.50 g/cm ³	20 kg	\$40

Sand, limestone and sodium carbonate are sold per bag.

A manufacturer claims that in making 100 such trophies, the cost of sodium carbonate involved is higher than the cost of sand.

Do you agree with the manufacturer? Support your answer with calculations.

Answer



FUCHUN SECONDARY SCHOOL
Secondary 4 Express / 5 Normal Academic
Preliminary Examination 2020

CANDIDATE
NAME

Answer scheme

CENTRE
NUMBER

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INDEX
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CLASS

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MATHEMATICS

Paper 2

4048/02

15 Sep 2020

2 hours 30 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

Name of Setter: Mr Chen Xin Da

This document consists of 21 printed pages and 1 blank page

[Turn over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

- 1 (a) Rearrange the formula $a = \frac{b(c^2 - d^2)}{3}$ to make c the subject.

$$3a = b(c^2 - d^2)$$

$$c^2 - d^2 = \frac{3a}{b} \quad (M1)$$

$$c^2 = \frac{3a}{b} + d^2$$

$$c = \pm \sqrt{\frac{3a}{b} + d^2} \quad (A1)$$

Answer $c = \pm \sqrt{\frac{3a}{b} + d^2}$ [2]

- (b) (i) Factorise completely $50x^2 - 32$.

$$2(5x+4)(5x-4) \quad (B1)$$

Answer $2(5x+4)(5x-4)$ [1]

- (ii) Hence, simplify $\frac{50x^2 - 32}{10x^2 + 3x - 4}$.

$$\frac{2(5x+4)(5x-4)}{(5x+4)(2x-1)} \quad (M1)$$

$$= \frac{2(5x-4)}{2x-1} \quad (A1)$$

Answer $\frac{2(5x-4)}{2x-1}$ [2]

- (c) The points $(-2, 3)$ and $(4, 12)$ lie on the curve given by the equation $y = ax^2 + bx + 2$. Use an algebraic method to find the values of a and b .

$$3 = a(-2)^2 + b(-2) + 2$$

$$3 = 4a - 2b + 2$$

$$4a - 2b = 1 \quad \text{--- (1)}$$

$$12 = a(4)^2 + b(4) + 2$$

$$12 = 16a + 4b + 2$$

$$16a + 4b = 10$$

$$8a + 2b = 5 \quad \text{--- (2)}$$

$$\text{(1) + (2):}$$

$$12a = 6$$

$$a = \frac{1}{2}$$

$$4\left(\frac{1}{2}\right) - 2b = 1$$

$$b = \frac{1}{2}$$

Answer

(B1)

(B1)

$a =$

$b =$

Answer (B1) if both are followed through correctly.

$\frac{1}{2}$

$\frac{1}{2}$

[4]

- (d) (i) Express $-10 + 8x + x^2$ in the form $(x+a)^2 + b$.

$$x^2 + 8x - 10 = x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 - 10$$

$$= \underbrace{(x+4)^2}_{\text{(B1)}} - \underbrace{26}_{\text{(B1)}}$$

Answer

$$(x+4)^2 - 26$$

[2]

- (ii) Write down the coordinates of the minimum point of the graph of $y = -10 + 8x + x^2$.

$$(-4, -26) \quad \text{(B1)}$$

Answer

$$(-4, -26)$$

[1]

- 2 The first five terms in a sequence of numbers are given below.

$$T_1 = 2^2 + 1 = 5$$

$$T_2 = 3^2 + 6 = 15$$

$$T_3 = a^2 + b = c$$

$$T_4 = 5^2 + 16 = 41$$

$$T_5 = 6^2 + 21 = 57$$

- (a) Find the values of a , b and c .

Answer

$a =$	4	}	(81)
$b =$	11		
$c =$	27		

[1]

- (b) Explain why the value of T_n must be odd for all values of n .

Answer The sum of an even number and an odd number is always odd. (81) [1]

- (c) Show that the n th term of the sequence, T_n , is given by $n^2 + 7n - 3$.

Answer

$$\begin{aligned}
 T_n &= (n+1)^2 + (5n-4) \quad (M1) \\
 &= n^2 + 2n + 1 + 5n - 4 \\
 &= n^2 + 7n - 3 \quad (A1)
 \end{aligned}$$

[2]

- (d) T_{p+1} and T_p are consecutive terms in the sequence. Find and simplify an expression, in terms of p , for $T_{p+1} - T_p$.

$$\begin{aligned}
 T_{p+1} - T_p &= ((p+1)^2 + 7(p+1) - 3) - (p^2 + 7p - 3) \\
 &= 2p + 8 \quad (81)
 \end{aligned}$$

Answer $2p + 8$ [2]

- (e) Explain why two consecutive terms of the sequence cannot have a difference of 6.

Answer If $2p + 8 = 6$, then $p = -1$.
Since p cannot be negative, the difference cannot be 6. (81) [1]

3 John cycled from his home to the town. He divided his journey into two parts.

- (a) He travelled the first 45 km at an average speed of x km/h.
Write down an expression, in terms of x , for the time taken in hours for the first 45 km.

$$\frac{45}{x} \quad (1)$$

Answer $\frac{45}{x}$ h [1]

- (b) He travelled the remaining 5 km at an average speed which was 10 km/h less than the first 45 km of his journey.
Write down an expression, in terms of x , for the time taken in hours for this part of the journey.

$$\frac{5}{x-10} \quad (1)$$

Answer $\frac{5}{x-10}$ h [1]

- (c) He took 3 hours for the whole journey.
Write down an equation in x , and show that it reduces to $3x^2 - 80x + 450 = 0$.

Answer

$$\frac{45}{x} + \frac{5}{x-10} = 3 \quad (1)$$

$$\frac{45(x-10) + 5x}{x(x-10)} = 3$$

$$45x - 450 + 5x = 3x^2 - 30x \quad (1)$$

$$3x^2 - 80x + 450 = 0 \quad (\text{shown}) \quad (1)$$

- (d) Solve the equation $3x^2 - 80x + 450 = 0$, giving your solutions correct to 2 decimal places.

$$\begin{aligned}
 x &= \frac{-(-80) \pm \sqrt{(-80)^2 - 4(3)(450)}}{2(3)} && \sqrt{(-80)^2 - 4(3)(450)} && (81) \\
 &= \frac{80 \pm \sqrt{1000}}{6} && \frac{80 \pm \sqrt{\text{then } 1000}}{6} && (81) \\
 &= 8.0629 \text{ or } 18.604 && && \\
 &= 8.06 \text{ or } 18.60 && && \\
 & && (81) && (81)
 \end{aligned}$$

Answer $x = \underline{8.06}$ or $\underline{18.60}$ [4]

- (e) Hence, state the average speed for the first part and the second part of the journey, giving your answers correct to 2 decimal places.

Answer First part $\underline{18.60}$ km/h
 Second part $\underline{8.60}$ km/h [1]
 (81)

- (f) John claims that the average speed of the entire journey can be obtained by taking the mean of the two answers in (e).

Do you agree? Justify your answer with calculations.

Answer $\text{Mean} = \frac{18.60 + 8.60}{2}$
 $= 13.60 \text{ km/h}$ (81) ✓
 Average speed = $\frac{45 + 5}{4}$
 $= 16 \frac{1}{2} \text{ km/h}$ (81).
 No, I do not agree. [2]

- 4 The following table shows the number of letters, cards and parcels sent by Kim and Mary.

	Letters	Cards	Parcels
Kim	4	10	2
Mary	5	5	3

- (a) The information for the number of letters, cards and parcels sent by Kim and Mary can be presented by a 2×3 matrix N . Write down the matrix N .

Answer $N = \begin{pmatrix} 4 & 10 & 2 \\ 5 & 5 & 3 \end{pmatrix}$ [1] (B1)

- (b) Postage is charged at \$0.30 for a letter, \$0.50 for a card and \$6 for a parcel. Write down a 3×1 matrix C to represent each type of postage charge.

Answer $C = \begin{pmatrix} 0.30 \\ 0.50 \\ 6 \end{pmatrix}$ [1] (B1) (cannot have units)

- (c) (i) Given $P = NC$, evaluate P .

$$P = \begin{pmatrix} 4 & 10 & 2 \\ 5 & 5 & 3 \end{pmatrix} \begin{pmatrix} 0.30 \\ 0.50 \\ 6 \end{pmatrix} = \begin{pmatrix} 18.30 \\ 22 \end{pmatrix}$$
 (M1) must show evidence of correct multiplication (A1)

Answer $P = \begin{pmatrix} 18.30 \\ 22 \end{pmatrix}$ [2]

- (ii) State what the elements of P represent.

Answer The postage cost paid by Kim and Mary respectively (B1)

[1]

- (d) If the postage charge is increased by 20% for a letter, 10% for a postcard and 5% for a parcel,
- (i) formulate and write down a 3×3 matrix \mathbf{R} such that the matrix \mathbf{RC} will give the revised cost for each type of postage charge.

Answer $\mathbf{R} = \begin{pmatrix} 1.20 & 0 & 0 \\ 0 & 1.10 & 0 \\ 0 & 0 & 1.05 \end{pmatrix}$ [1]

(B)

- (ii) calculate the total amount that Kim and Mary need to pay to send out the letters, cards and parcels with the revised postage charges.

Revised postage charge

$$\left. \begin{aligned} 1.20 \times 0.30 &= 0.36 \\ 1.10 \times 0.50 &= 0.55 \\ 1.05 \times 6 &= 6.30 \end{aligned} \right\} \text{(B1)}$$

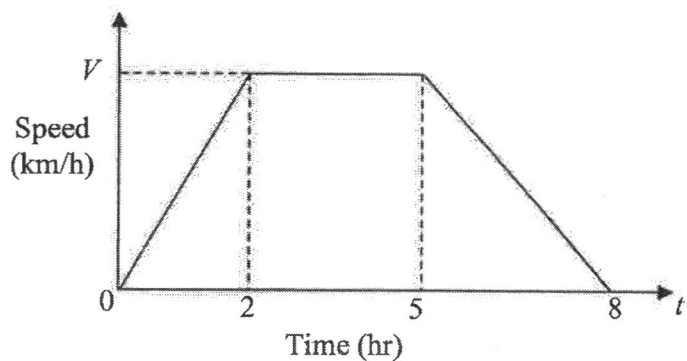
$$\begin{aligned} \text{Kim: } & 4(0.36) + 10(0.55) + 2(6.30) \\ &= \$19.54 \end{aligned}$$

$$\begin{aligned} \text{Mary: } & 5(0.36) + 5(0.55) + 3(6.30) \\ &= \$23.45 \end{aligned} \quad \left. \right\} \text{(M1)}$$

$$\begin{aligned} \text{Total} &= \$19.54 + \$23.45 \\ &= \$42.99 \end{aligned} \quad \text{(A1)}$$

Answer \$ 42.99 [3]

- 5 The speed-time graph illustrates the motion of a car during a period of 8 hours.



- (a) Given that the distance travelled in the first 5 hours is 320 km, show that $V = 80$.

Answer

$$\frac{1}{2}(5+3)(V) = 320 \quad (M1)$$

$$4V = 320$$

$$V = 80 \quad (\text{shown}) \quad (A1)$$

[2]

- (b) Describe the motion of the car between $t = 2$ to $t = 5$.

Answer zero acceleration / constant speed. (B1)

[1]

- (c) Calculate the deceleration of the car during the last 3 hours.

$$\frac{0 - 80}{3}$$

$$= -26\frac{2}{3}$$

$$a = 26\frac{2}{3} \quad (B1)$$

Answer

$26\frac{2}{3}$

km/h²

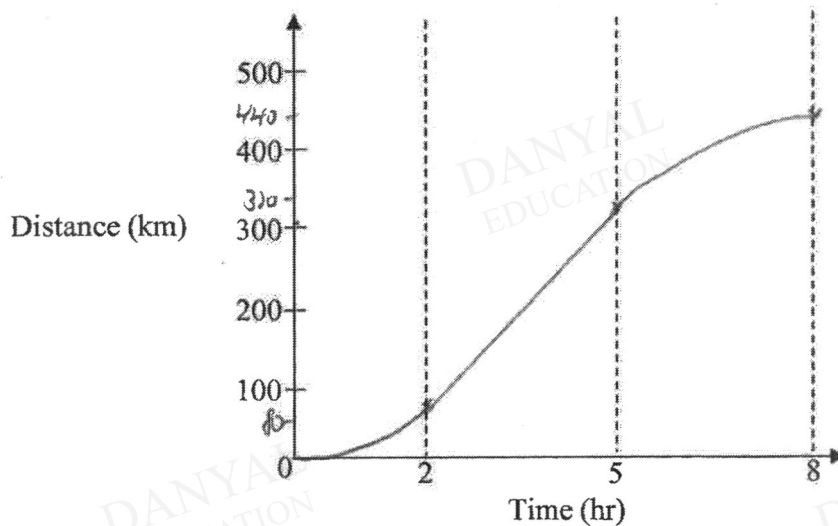
[1]

- (d) Calculate the total distance travelled by the car for the 8 hours.

$$\begin{aligned} \text{Total distance} &= \frac{1}{2} (8+3) \times 80 \\ &= 440 \text{ km} \quad (1\text{B}) \end{aligned}$$

Answer 440 km [1]

- (e) Sketch the distance-time graph for the 8 hours in the diagram below. Label the values on the distance-axis clearly.



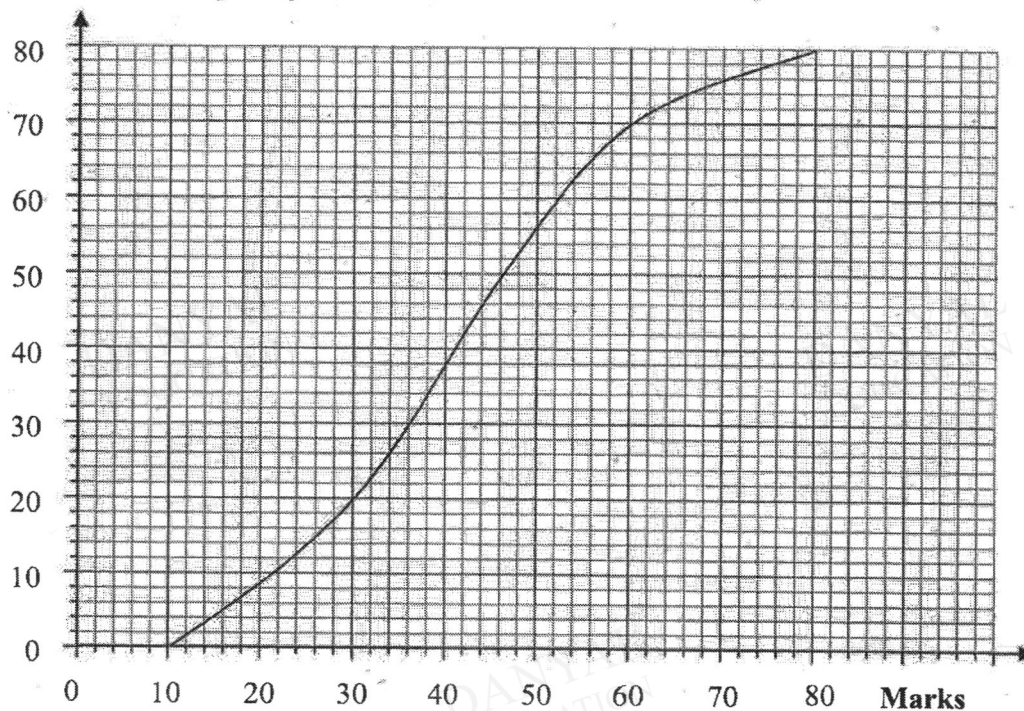
[3]

Shape of graph (1B for any part correct)
(1B for all 3 parts correct)

Label on distance - axis (1B).

- 6 The cumulative frequency curve below illustrates the marks obtained by 80 students in School A in an examination.

Cumulative Frequency



- (a) Use the curve to estimate
(i) the median mark,

Answer 41 (B1) marks [1]

- (ii) the interquartile range of the marks.

$$\begin{aligned} IQR &= 52 - 30 \\ &= 22 \end{aligned}$$

Answer 22 marks [2]

(M1) for at least 1 correct and
(A1) attempt to find the difference.

- (b) Find the number of students who obtained between 30 and 60 marks.

$$70 - 20 = 50 \quad (B1)$$

Answer 50 [1]

(c) The passing mark for the examination is 40 marks.

(i) A student is chosen at random.

Find the probability that he or she will pass the examination.

$$\frac{42}{80} = \frac{21}{40} \quad (1)$$

Answer $\frac{21}{40}$ [1]

(ii) Two students are chosen at random.

Find the probability that at least one of the students will pass the examination.

$$\begin{aligned} 1 - P(F, F) &= 1 - \left(\frac{38}{80} \times \frac{37}{79} \right) && (M1) \\ &= \frac{2457}{3160} && (A1) \end{aligned}$$

$$\begin{aligned} &P(P, F) + P(F, P) + P(P, P) \\ &= \left(\frac{42}{80} \times \frac{38}{79} \right) + \left(\frac{38}{80} \times \frac{42}{79} \right) + \left(\frac{42}{80} \times \frac{41}{79} \right) && (M1) \\ &= \frac{2457}{3160} && (A1) \end{aligned}$$

Answer $\frac{2457}{3160}$ [2]

(d) The marks obtained by another 80 students in School B had the same median but a smaller interquartile range as compared to School A. Describe how the cumulative frequency curve for School B may differ from the curve for School A.

Answer The curve for school B will be steeper
with a smaller gradient than school A (1) [1]

- 7 (a) In 2016, Anil, Ben and Cathy decided to start a small business.
Anil invested \$50 000, Ben invested \$30 000 and Cathy invested \$20 000.

They agreed that all the profits should be divided in the same ratio as the sum of money they invested.

In 2018, the total profit was \$15 500.

- (i) What was the difference between Anil's and Cathy's share of the profit in 2018?

$$\frac{30000}{100000} \times \$15500 \quad (M1) \quad \text{or} \quad 1 \text{ unit} \rightarrow \$1550 \quad (M1)$$

$$= \$4650 \quad (A1)$$

Answer \$ 4650 [2]

- (ii) The total profit in 2019 was 30% greater than that made in 2018.
Calculate the total profit made in 2019.

$$\frac{130}{100} \times \$15500 \quad (M1)$$

$$= \$20150 \quad (A1)$$

Answer \$ 20150 [2]

- (iii) The total profit in 2018 was 25% greater than that made in 2017.
Calculate the total profit made in 2017.

$$\frac{100}{125} \times \$15500 \quad (M1)$$

$$= \$12400 \quad (A1)$$

Answer \$ 12400 [2]

- (b) Two investment companies pay out the following interest rates as shown below.

Company A: $x\%$ simple interest per annum

Company B: $y\%$ compound interest per annum, compounded annually

Mr Lin invests \$50 000 in Company A.

Mr Tan invests \$50 000 in Company B and receives \$55 000 at the end of 3 years.

Both of them received the same amount of interest at the end of 3 years.

Determine which investment company offers the higher interest rate.

Show your working.

Answer

Company A

$$I = 55000 - 50000$$

$$= 5000$$

$$5000 = \frac{50000(x)(3)}{100} \quad (M1)$$

$$x = 3\frac{1}{3} \quad (A1)$$

Company B

$$55000 = 50000 \left(1 + \frac{y}{100}\right)^3 \quad (M1)$$

$$\left(1 + \frac{y}{100}\right)^3 = 1.1$$

$$\frac{y}{100} = \sqrt[3]{1.1} - 1$$

$$y = 100(\sqrt[3]{1.1} - 1)$$

$$y = 3.23 \quad (A1)$$

Company A offers the higher interest rate. (ANS)

- 8 The variables x and y are connected by the equation $y = \frac{x^2}{6} + \frac{2}{x}$.

Some corresponding values of x and y are given in the table below.

x	1	2	3	4	5	6	7
y	2.2	1.7	2.2	3.2	4.6	6.3	8.5

- (a) On the grid opposite, draw the graph of $y = \frac{x^2}{6} + \frac{2}{x}$ for $1 \leq x \leq 7$.

All points correctly plotted (B2)
4 points correctly plotted (B1)
Smooth curve (B1) [3]

- (b) Use your graph to write down an inequality in y to describe the range of values where $x > 1$.

$$y \geq 1.65 (\pm 0.2) \text{ (B1)}$$

Answer

..... [1]

- (c) By drawing a tangent, find the gradient of the curve at (4, 3.2).

Drawing of tangent (M1)

$$\text{gradient} = 1.21 (\pm 0.2) \text{ (A1)}$$

$$y > 1.65 (\pm 0.2)$$

$$1.65 < y < 8.5$$

$$1.65 < y \leq 8.5$$

Answer

..... [2]

- (d) The line $y + 2x = 15$ intersects the curve $y = \frac{x^2}{6} + \frac{2}{x}$.

- (i) On the same grid, draw the graph of the straight line $y + 2x = 15$ for $3 \leq x \leq 6$. [2]

- (ii) Write down the x -coordinate of the point where this line intersects the curve.

$$x = 5.1 (\pm 0.1) \text{ (B1)}$$

Answer

$x =$ [1]

- (iii) The value of x above is the solution of the equation $x^3 + ax^2 - 90x + b = 0$,
Find the value of a and the value of b .

$$\frac{x^2}{6} + \frac{2}{x} = -2x + 15$$

$$a = 12 \text{ (B1)}$$

$$x^3 + 12 = -12x^2 + 90x$$

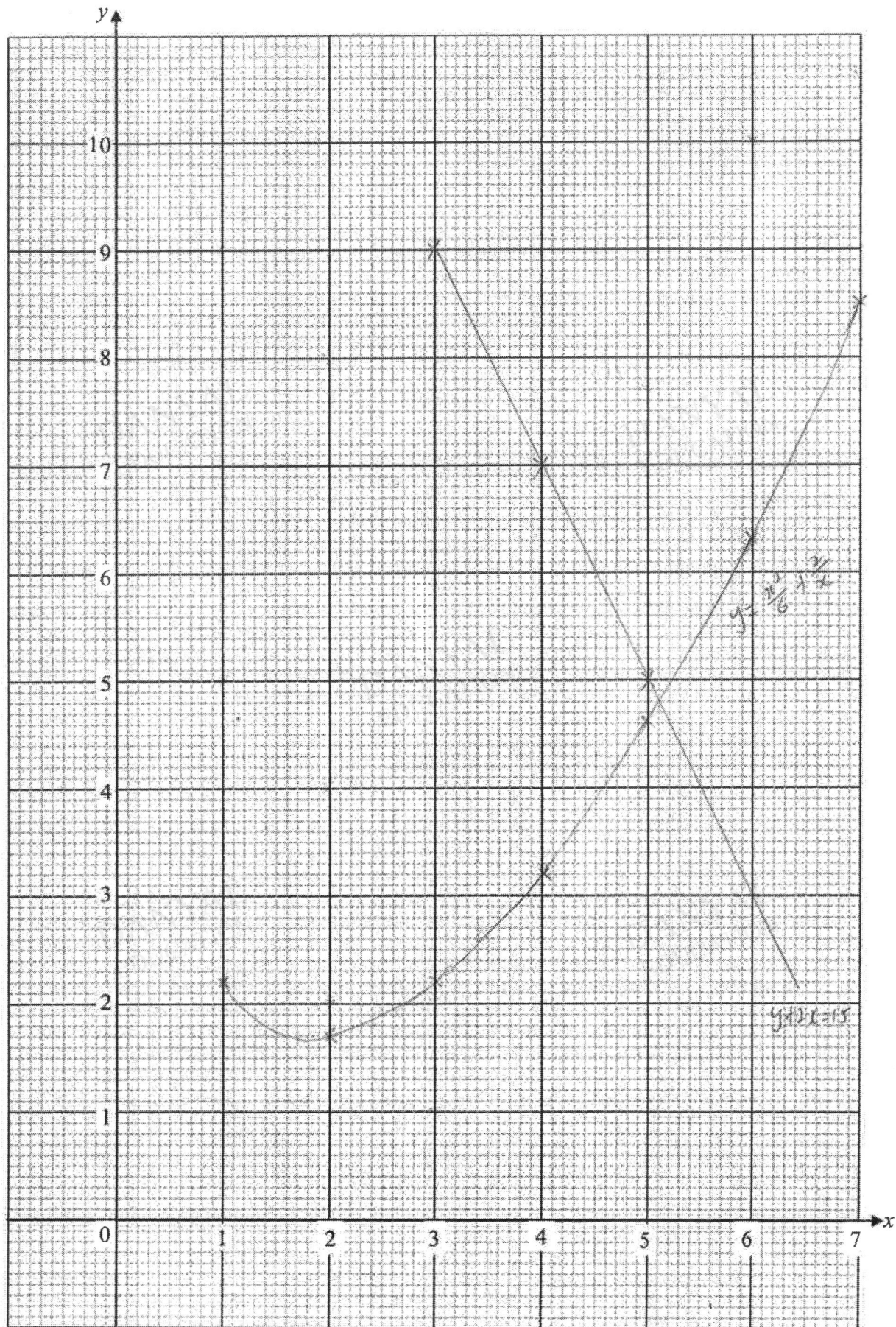
$$b = 12 \text{ (B1)}$$

$$x^3 + 12x^2 - 90x + 12 = 0$$

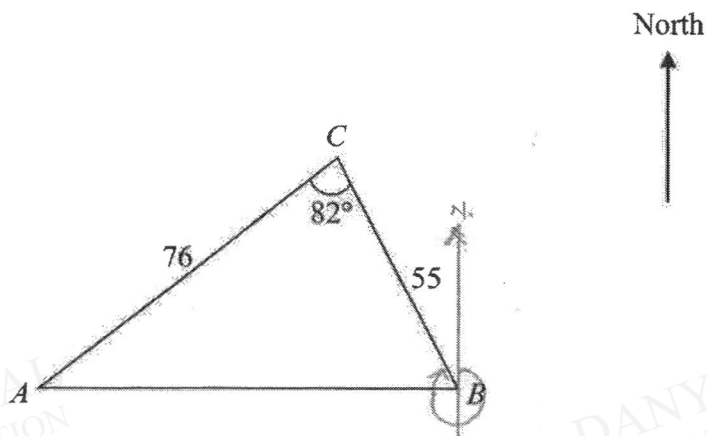
Answer

$a =$ [2]

$b =$ [2]



- 9 The diagram shows a triangular field ABC on horizontal ground.
 B is due east of A .
 $BC = 55$ m, $AC = 76$ m and angle $ACB = 82^\circ$.



- (a) Calculate the length of AB .

$$AB^2 = 76^2 + 55^2 - 2(76)(55) \cos 82^\circ \quad (M1)$$

$$AB^2 = 7637.5$$

$$AB = \sqrt{7637.5} \quad (M1)$$

$$AB = 87.393$$

$$= 87.4 \text{ (3sf)} \quad (A1)$$

Answer 87.4 m [3]

- (b) Calculate the bearing of C from B .

$$\sin \angle ABC = \frac{\sin 82^\circ}{87.393} \quad (M1)$$

$$\angle ABC = 59.45^\circ$$

$$\text{Bearing} = 270^\circ + 59.45^\circ$$

$$= 329.5^\circ \quad (A1)$$

Answer 329.5 ° [2]

- (c) Calculate the area of the field ABC .

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2}(76)(55) \sin 80^\circ \quad (\text{M1}) \\ &= 2069.7 \\ &= 2070 \text{ m}^2 \text{ (3sf)} \quad (\text{A1}) \end{aligned}$$

Answer 2070 m² [2]

- (d) Calculate the shortest distance from C to AB .

Let the shortest distance be x .

$$\begin{aligned} \frac{1}{2}(x)(87.393) &= 2069.7 \quad (\text{M1}) \\ x &= 47.365 \\ x &= 47.4 \text{ (3sf)} \quad (\text{A1}) \end{aligned}$$

Answer 47.4 m [2]

- (e) T is the point on top of the tree vertically above C .
The angle of elevation of the top of the tree from A is 20° .
Calculate the largest angle of elevation of the top of the tree from a point on AB .

$$\begin{aligned} \tan 20^\circ &= \frac{CT}{76} \\ CT &= 27.661 \quad (\text{B1}) \end{aligned}$$

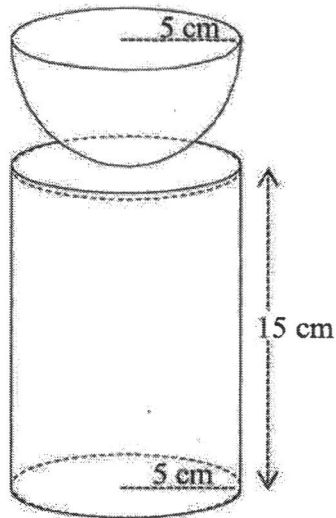
Let the largest angle of elevation be θ .

$$\tan \theta = \frac{27.661}{47.365} \quad (\text{M1})$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{27.661}{47.365}\right) \\ &= 30.3^\circ \quad (\text{A1}) \end{aligned}$$

Answer 30.3 ° [3]

- 10 The diagram shows a trophy which consists of a solid hemispherical glass top that sits on top of a solid cylindrical glass base. Both the radii of the cylinder and the hemisphere are 5 cm. The height of the cylindrical base is 15 cm.



- (a) Find the total volume of the trophy.

$$\text{Total volume} = (\pi \times 5^2 \times 15) + \frac{1}{2} \left(\frac{4}{3} \times \pi \times 5^3 \right) \quad (\text{M1, M1})$$

$$= 1439.9$$

$$= 1440 \text{ cm}^3 \quad (\text{ft}) \quad (\text{A1})$$

Answer 1440 cm³ [3]

- (b) The composition of the glass that is used to manufacture the trophy include sand, limestone and sodium carbonate in the proportion 75%, 10% and 15% respectively.

Following are some information on sand, limestone and sodium carbonate.

	Density	Mass of one bag	Cost of one bag
Sand	1.60 g/cm ³	30 kg	\$26
Limestone	2.70 g/cm ³	25 kg	\$60
Sodium Carbonate	2.50 g/cm ³	20 kg	\$40

Sand, limestone and sodium carbonate are sold per bag.

A manufacturer claims that in making 100 such trophies, the cost of sodium carbonate involved is higher than the cost of sand.

Do you agree with the manufacturer? Support your answer with calculations.

Answer

vol. of sand = $0.75 \times 100 \times 1439.9$ (M1)
 $= 107990 \text{ cm}^3$
 * Mass of sand = 1.60×107990 (M1)
 $= 172780$
 $= 172.78 \text{ kg}$
 Sand
 Bags = $172.78 \div 30$
 ≈ 6

Cost of sand = 6×26 (A1)
 $= \$156$

vol. = $0.15 \times 100 \times 1439.9$ (M1)
 $= 21599 \text{ cm}^3$
 sodium
 carbonate
 Mass = 2.50×21599 (M1)
 $= 53998 \text{ g}$
 $= 53.998 \text{ kg}$
 Bags = $53.998 \div 20$
 ≈ 3
 Cost = $3 \times 40 = \$120$ (A1)

Disagree, the cost of sodium carbonate is lower than sand. (A1)

Answer all the questions.

✓

1 Simplify $\left(\frac{16b^2}{c^6}\right)^{\frac{3}{2}}$.

$$\left(\frac{c^6}{16b^2}\right)^{\frac{3}{2}} \quad (M1)$$

OR

$$\frac{16^{-\frac{3}{2}} (b^2)^{-\frac{3}{2}}}{(c^6)^{-\frac{3}{2}}}$$

$$= \frac{c^9}{64b^3} \quad (A1)$$

$$= \frac{1}{64} \frac{b^{-3}}{c^{-9}}$$

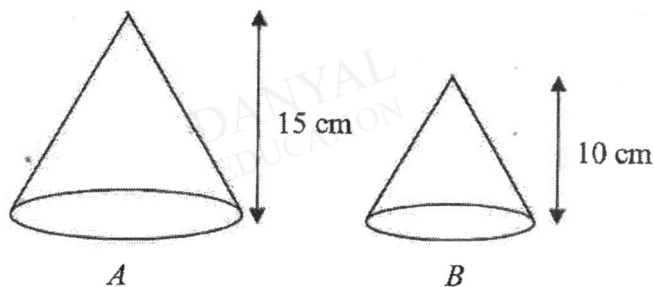
$$= \frac{c^9}{64b^3} \quad \text{M1 if } \frac{c^9}{b^3} \text{ seen}$$

Answer

$$\frac{c^9}{64b^3}$$

[2]

- 2 Two solid cones A and B are geometrically similar.



The heights of the two cones are 15 cm and 10 cm.

Given that the volume of A is $(3x + 1) \text{ cm}^3$ and the volume of B is $x \text{ cm}^3$, calculate the value of x .

$$\frac{3x+1}{x} = \left(\frac{15}{10}\right)^3 \quad (M1)$$

$$\frac{3x+1}{x} = \frac{27}{8}$$

$$24x + 8 = 27x$$

$$3x = 8$$

$$x = 2\frac{2}{3} \quad (A1)$$

Answer

 $x =$

$$2\frac{2}{3}$$

[2]

3 Solve the inequalities $2 \leq 3x - 1 < 2x + 8$.

$$2 \leq 3x - 1$$

$$3x \geq 3$$

$$x \geq 1$$

$$3x - 1 < 2x + 8$$

$$x < 4$$

(M1) show $2 \leq 3x - 1$ and $3x - 1 < 2x + 8$ and solve at least 1 correctly.

$$\therefore 1 \leq x < 4 \quad (A1)$$

Answer

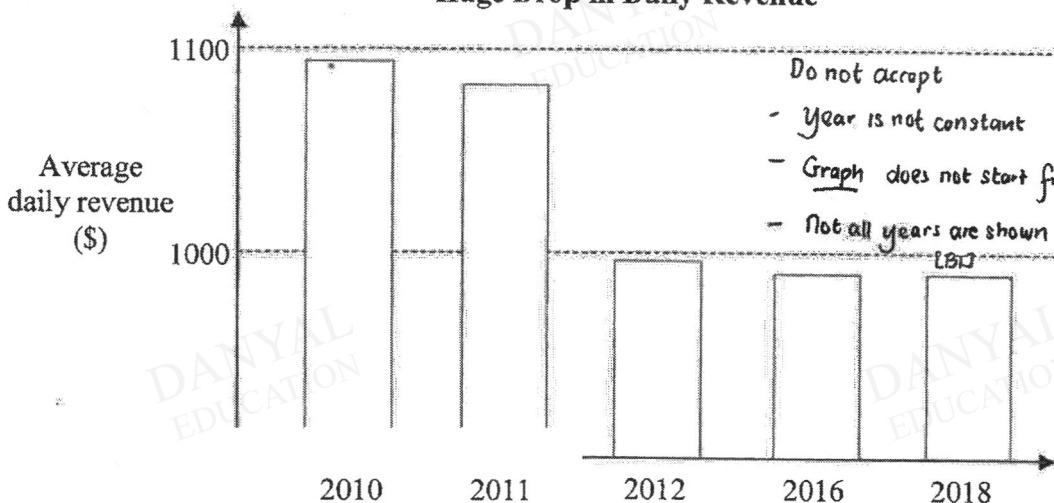
$$1 \leq x < 4$$

[2]

4 The graph shows the average daily revenue for a shop over a number of years.

"Explanation must be clear and specific"

Huge Drop in Daily Revenue



Do not accept
 - Year is not constant
 - Graph does not start from 0
 - Not all years are shown \rightarrow revenue is not known

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

Inconsistent scale on vertical axis \rightarrow exaggerates differences between years
 Not all years are shown \rightarrow revenue could have been higher in 2013 to 2015.
 Unequal spacing of years \rightarrow misrepresents the trend.
 Title is biased \rightarrow does not allow reader to make own judgement.
 Any one (B2)

[2]

5 n is a positive integer.

Show that $(5n+2)^2 - 25n^2$ is a multiple of 4 for all integer values of n .

Answer

$$25n^2 + 20n + 4 - 25n^2 \quad (M1)$$

$$= 20n + 4$$

$$= 4(5n+1) \quad (A1)$$

Since n is a positive integer, $4(5n+1)$ is a multiple of 4 for all integer values of n .

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[2]

6 m is directly proportional to n^2 .

It is known that $m = 12$ for a particular value of n .

Find the value of m when this value of n is decreased by 20%.

$$m = kn^2, \quad k \text{ is a constant} \quad (M1)$$

$$12 = kn^2$$

$$k = \frac{12}{n^2}$$

$$m = \left(\frac{12}{n^2}\right)\left(\frac{80}{100}n\right)^2$$

$$m = \left(\frac{12}{n^2}\right)\left(\frac{16}{25}n^2\right)$$

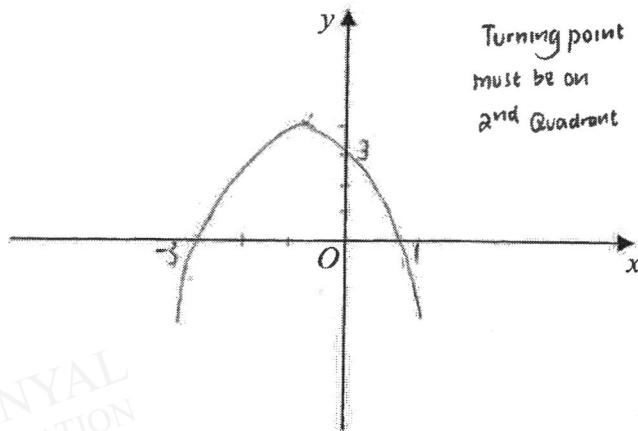
$$m = 7\frac{12}{25} \quad (A1)$$

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Answer $m = 7\frac{12}{25}$ [2]

- 7 (a) Sketch the graph of $y = -(x-1)(x+3)$ on the axes below.
Indicate clearly the x -intercepts and y -intercept of the graph.



Turning point
must be on
2nd Quadrant

Turning point +
shape of curve (B1) [2]
x- and y- intercepts (B1)

- (b) Write down the equation of the line of symmetry of $y = -(x-1)(x+3)$.

$$x = -1 \quad (B1)$$

Answer $x = -1$ [1]

- 8 An area of 16 cm^2 on a map represents an actual area of 4 km^2 .

Calculate

- (a) the actual area, in km^2 , that is represented by an area of 30 cm^2 on the map,

$$\frac{30}{16} \times 4 = 7\frac{1}{2} \quad (B1)$$

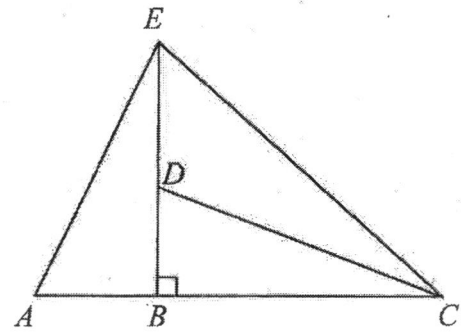
Answer $7\frac{1}{2}$ km^2 [1]

- (b) the actual distance, in km, that is represented by 12 cm on the map,

$$\begin{aligned} \text{Map} &\rightarrow \text{Actual} \\ \sqrt{16 \text{ cm}^2} &: \sqrt{4 \text{ km}^2} && (M1) \\ = 4 \text{ cm} &: 2 \text{ km} \\ = 12 \text{ cm} &: 6 \text{ km} && (A1) \end{aligned}$$

Answer 6 km [2]

- 9 In the diagram shown below, ABC and BDE are straight lines. It is given that EB is perpendicular to AC , $AE = DC$, $BC = BE$.



- (a) Prove that triangle ABE is congruent to triangle DBC .

Answer

$BC = BE$ (given)
 $AE = DC$ (given)
 $\angle ABE = \angle DBC = 90^\circ$
 $\therefore \triangle ABE \cong \triangle DBC$ (RHS)

still awarded if no " $= 90^\circ$ "

[B1]

[CB1 dep.]

[2]

- (b) Given that the area of triangle $ABE = 10 \text{ cm}^2$ and D is the midpoint of BE , calculate the area of triangle BEC .

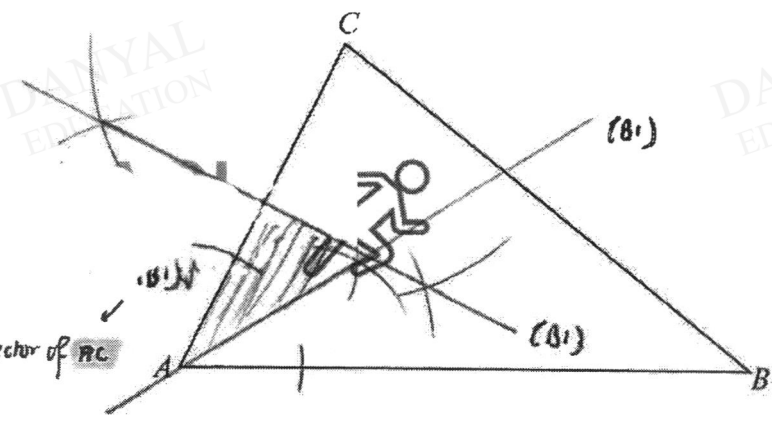
$\text{Area} = 2 \times 10$
 $= 20 \text{ cm}^2$ (B1)

Answer

..... 20 cm^2 [1]

↳ must be supported by the 3 arguments

- 10 The diagram shows a triangle ABC .



if wrong \perp bisector of AC is constructed

- (a) On the diagram, construct the bisector of angle CAB .

[1]

- (b) By constructing a suitable perpendicular bisector on the same diagram, shade the region inside triangle ABC that is closer to AC than AB and closer to A than to C .

[2]

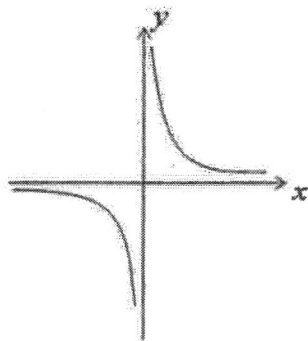


Figure 1

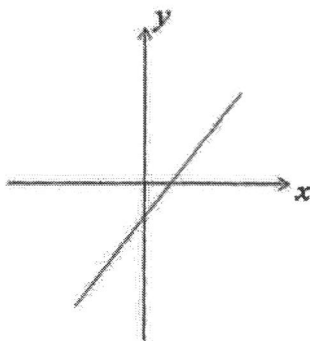


Figure 2

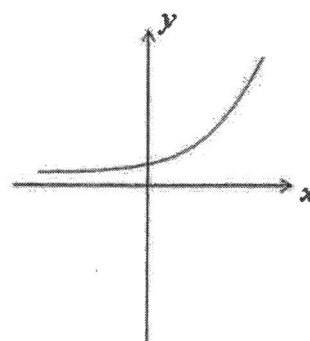


Figure 3

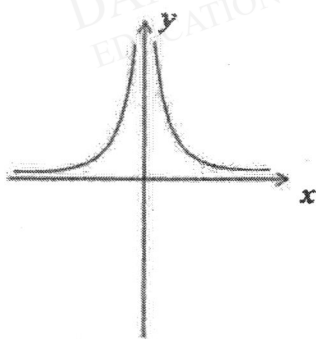


Figure 4

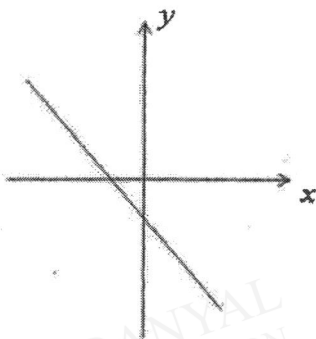


Figure 5

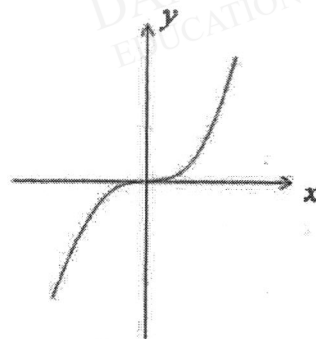


Figure 6

State the correct figure for each of the following equations.

(a) $y = 3^x$

Answer Figure 3 (B) [1]

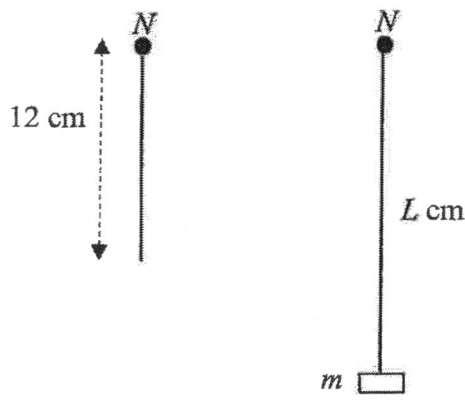
(b) $y = \frac{2}{x}$

Answer Figure 1 (B) [1]

(c) $y = -x - 2$

Answer Figure 5 (B) [1]

12



A piece of elastic 12 cm long, hangs from a nail N , as shown in the figure above.

When a mass of m grams is attached to the lower end, the length of the elastic increases to L cm. For every 100 grams which is attached, the length of the elastic increases by 3 cm.

(a) Calculate the length of the piece of elastic when a mass of 200 grams is attached to it.

$$12 + 2(3) = 18 \quad (1)$$

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Answer 18 cm [1]

(b) If the length of the piece of elastic is 24 cm, calculate the mass that is attached to it.

$$\begin{aligned} 24 - 12 &= 12 \\ 12 \div 3 &= 4 \\ 4 \times 100 &= 400 \quad (1) \end{aligned}$$

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Answer 400 g [1]

(c) Write down a formula connecting the length of the elastic L , and the mass m , which is attached to it.

$$L = 12 + \frac{3m}{100} \quad (1)$$

Answer $L = 12 + \frac{3m}{100}$ o.e. [1]

$$m = \frac{100(L-12)}{3} \quad \frac{L-12}{0.03} = m$$

$$L = 12 + 0.03m$$

- 13 A bag contains 19 balls of which n are blue and the rest are yellow.

A marble is chosen at random and not replaced.

- (a) Write down, in terms of n , the probability that the marble is yellow.

Answer $\frac{19-n}{19}$ (B1) [1]

A second marble is chosen at random.

- (b) The probability that the second marble chosen is yellow is $\frac{4}{9}$.

Calculate the number of yellow marbles in the bag.

[B1] → correct answer w/o working

$$\frac{18-n}{18} = \frac{4}{9} \quad (M1)$$

[B2] → $\frac{4}{9} = \frac{8}{18}$

∴ no of yellow : 9

$$18-n = 8$$

$$n = 10$$

$$19-10 = 9 \quad (A1)$$

Answer

9

[2]

- 14 Rachel invests a certain amount of money in an account.

The balance, \$ A , of the account after t years is given by the formula $A = k \times 1.02^t$, where k is a constant.

When $t = 2$, $A = 52\,020$.

- (a) By finding the value of k , calculate the amount of money Rachel invests in the account at the start.

$$52020 = k \times 1.02^2 \quad (M1)$$

$$k = 5000$$

$$A = 5000 \times 1.02^0$$

$$= 5000$$

Answer

\$

50000 (A1)

[2]

- (b) Calculate the percentage increase in the balance over 5 years.

$$\text{when } t = 5, A = 5000 \times 1.02^5$$

$$= 55204.0402$$

$$\% \text{ increase} = \frac{55204.0402 - 50000}{50000} \times 100\%$$

$$= 10.4\%$$

Answer

10.4 (A1)

%

[2]

- 15 The students of Class A and Class B took a common Mathematics test. The table below shows the distribution of marks obtained by the students.

Class A

Marks (x)	Frequency
$0 < x \leq 5$	6
$5 < x \leq 10$	15
$10 < x \leq 15$	22
$15 < x \leq 20$	5

Class B

Mean = 10.5
Standard Deviation = 3.5

- (a) For Class A, calculate an estimate for the

- (i) mean marks,

$\bar{x} = 10.2$ (81)

Answer 10.2 marks [1]

- (ii) standard deviation of the marks.

$\sigma_x = 4.20$ (81)

Answer 4.20 marks [1]

- (b) Below are two statements comparing the marks obtained by the students in Class A and Class B.

For each one, state whether you agree or disagree, giving a reason for each answer.

must have comparison word
E.g. higher, smaller

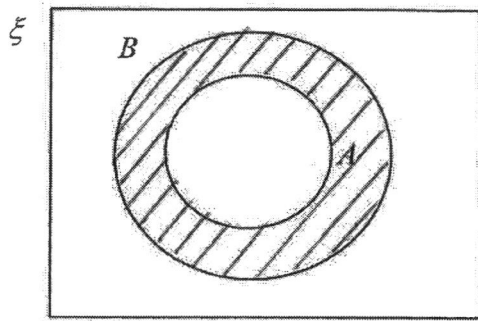
Statement	Agree/disagree	Reason
Students in Class A scored better than students in Class B on average.	Disagree	The mean marks of class B is higher than class A.
The marks obtained by students in Class A are more consistent than the marks obtained by students in Class B.	Disagree	The standard deviation in marks of class B is lower than class A.

(81)

(81)

[2]

- 16 (a) On the Venn diagram, shade the region which represents $A' \cap B$.



(B1)

[1]

- (b) $\xi = \{\text{integers } x : 1 \leq x \leq 10\}$
 $P = \{x : x \text{ is a factor of } 8\}$
 $Q = \{x : x \text{ is divisible by } 2\}$

- (i) List the elements contained in the set $P \cup Q'$.

$$P \cup Q' = \{1, 2, 3, 4, 5, 7, 8, 9\}$$

(B1)

$$P = \{1, 2, 4, 8\}$$

$$P' = \{3, 5, 6, 7, 9, 10\}$$

$$Q = \{2, 4, 6, 8, 10\}$$

$$Q' = \{1, 3, 5, 7, 9\}$$

Answer $\{1, 2, 3, 4, 5, 7, 8, 9\}$ [1]

- (ii) List the elements contained in the set $P' \cap Q'$.

$$P' \cap Q' = \{3, 5, 7, 9\}$$

(B1)

Answer $\{3, 5, 7, 9\}$ [1]

- (iii) Is $Q \subset P$?
 Explain your answer.

Answer

Accept: Q has more elements than P.

No. 6 and 10 are in Q but not in P. (B1)

Do not accept: - 6 and 10 are in Q

- {6} are in Q but not in P

" If no example is stated " Not all elements in Q are in P "

17 Written as a product of its prime factors, $450 = 2 \times 3^2 \times 5^2$.

(a) (i) Express 98 as a product of its prime factors.

$$98 = 2 \times 7^2 \quad (1)$$

Answer $98 = 2 \times 7^2$ [1]

(ii) Hence, use prime factors to explain why 98×450 is a perfect square.

Answer Since $98 \times 450 = (2 \times 3 \times 5 \times 7)^2$,
it is a perfect square. } (1)

OR $98 \times 450 = 2^2 \times 3^2 \times 5^2 \times 7^2$ All the prime factors have power that is 'even' or 'multiple of 2' or $\frac{1}{2}$.
↳ keyword

(b) Given that $\sqrt[3]{450k}$ is an integer, write down the smallest integer value of k .

$$k = 2^1 \times 3 \times 5 \\ = 60 \quad (1)$$

Answer $k = 60$ [1]

(c) Find the smallest positive integer m , such that $98m$ is a multiple of 450.

$$m = 3^2 \times 5^2 \\ = 225 \quad (1)$$

Answer $m = 225$ [1]

- 18 (a) Write as a single fraction in its simplest form $\frac{6}{(2x-1)^2} - \frac{5}{1-2x}$.

$$\begin{aligned} & \frac{6}{(2x-1)^2} + \frac{5}{2x-1} \quad \text{(M1) OR} \quad \frac{6(1-2x) - 5(2x-1)^2}{(2x-1)^2(1-2x)} \\ & = \frac{6}{(2x-1)^2} + \frac{5(2x-1)}{(2x-1)^2} \quad \text{(A1)} \\ & = \frac{10x+1}{(2x-1)^2} \quad \text{(A1)} \end{aligned}$$

$$\begin{aligned} & = \frac{6(1-2x) - 5(2x-1)^2}{(2x-1)^2(1-2x)} \\ & = \frac{6-12x - 5(4x^2 - 4x + 1)}{(2x-1)^2(1-2x)} \\ & = \frac{6-12x - 20x^2 + 20x - 5}{(2x-1)^2(1-2x)} = \frac{-20x^2 + 8x + 1}{(2x-1)^2(1-2x)} \quad \text{(M1)} \\ & \text{Answer} \quad \frac{10x+1}{(2x-1)^2} \quad \text{[2]} \end{aligned}$$

- (b) Given $4^b = \frac{1}{8} \times 32^a$, express b in terms of a .

$$2^{2b} = 2^{-3} \times 2^{5a} \quad \text{(M1: change to base 2)}$$

$$2b = -3 + 5a$$

$$b = \frac{-3 + 5a}{2} \quad \text{(A1)}$$

$$\text{Answer} \quad b = \frac{-3 + 5a}{2} \quad \text{[2]}$$

- 19 (a) Determine whether it is possible that an interior angle of an n -sided regular polygon is 110° . Explain clearly with working.

Answer

$$n = \frac{360}{70} = 5\frac{1}{7} \quad \text{(A1)}$$

$$\frac{180(n-2)}{n} = 110$$

} must show correct concept to be awarded the next 2!

Since n is not an integer, it is not possible (A1). [2]

Do not accept

Because n is decimal, ...

- (b) An n -sided polygon has 4 interior angles measuring 100° each. The remaining interior angles measure q° each. Find an expression for q in terms of n .

$$4(100) + q(n-4) = 180(n-2) \quad \text{(M1)}$$

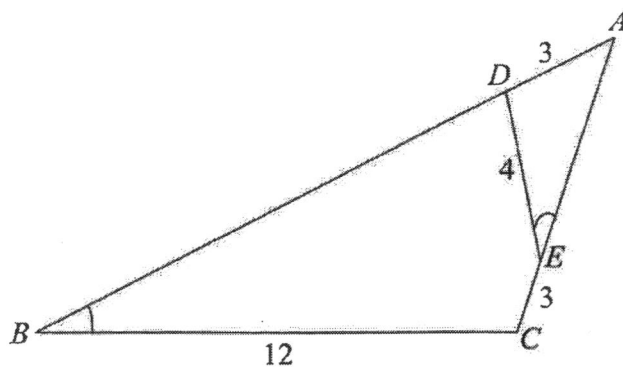
$$400 + q(n-4) = 180n - 360$$

$$q(n-4) = 180n - 760$$

$$q = \frac{180n - 760}{n-4} \quad \text{(A1)}$$

$$\text{Answer} \quad q = \frac{180n - 760}{n-4} \quad \text{[2]}$$

- 20 In the diagram, angle $AED = \text{angle } ABC$, $AD = 3 \text{ cm}$, $DE = 4 \text{ cm}$, $BC = 12 \text{ cm}$ and $EC = 3 \text{ cm}$.



- (a) Prove that triangle ABC and triangle AED are similar.

Answer

BI
 Show two pair of equal angles

$\angle ABC = \angle AED$ (Given \angle)
 $\angle BAC = \angle EAD$ (Common \angle)

$\therefore \triangle ABC$ is similar to $\triangle AED$. Bi for reasoning

[2]

- (b) Calculate the length of AE .

$$AC = \frac{12}{4} \times 3$$

$$= 9 \quad \text{(B1)}$$

$$AE = 9 - 3$$

$$= 6 \quad \text{(B1)}$$

OR $\frac{AC}{AD} = \frac{BC}{ED}$

$$\frac{AE+3}{3} = \frac{12}{4} \quad \text{(M1)}$$

$$AE = 6 \quad \text{(A1)}$$

Answer

6

cm [2]

- (c) Write down the value of $\frac{\text{Area of } \triangle ADE}{\text{Area of } DBCE}$.

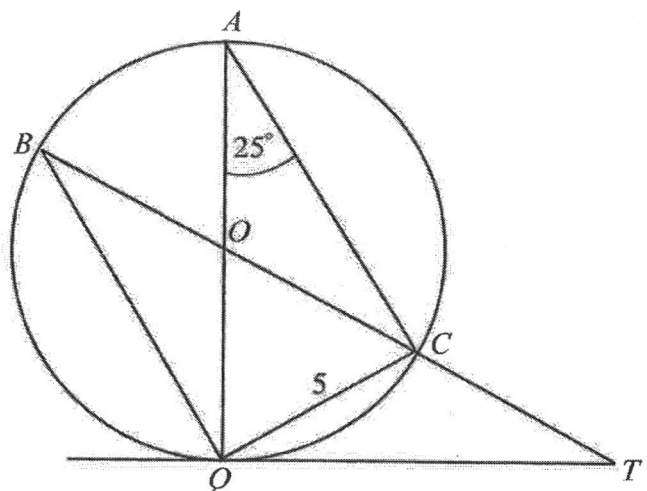
$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } DBCE} = \frac{1}{8} \quad \text{(B1)}$$

Answer

$\frac{1}{8}$

[1]



In the diagram above, A , B , C and Q are points on a circle, and angle $CAQ = 25^\circ$. AQ and BCT meet at the centre O . QT is a tangent to the circle at Q .

(a) Calculate

(i) angle QBC ,

$$\angle QBC = 25^\circ \text{ (Alt)} \quad \text{DANYAL EDUCATION}$$

Answer 25 $^\circ$ [1]

(ii) angle QOC ,

$$\angle QOC = 50^\circ \text{ (Alt)} \quad \text{DANYAL EDUCATION}$$

Answer 50 $^\circ$ [1]

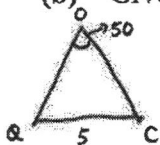
(iii) angle CQT .

$$\begin{aligned} \angle CQO &= (180 - 50) \div 2 \\ &= 65^\circ \text{ (Alt)} \end{aligned} \quad \text{DANYAL EDUCATION}$$

$$\angle CQT = 90^\circ - 65^\circ = 25^\circ \text{ (Alt)}$$

Answer 25 $^\circ$ [2]

(b) Given that $QC = 5$ cm, calculate the radius of the circle.



$$\text{In } \triangle BQC, \sin 25^\circ = \frac{5}{BC} \text{ (M1)}$$

$$BC = 11.831$$

$$\text{Radius} = 11.831 \div 2$$

$$= 5.92 \text{ (A1)}$$

$$\frac{\sin 50^\circ}{5} = \frac{\sin 65^\circ}{\text{radius}} \text{ (M1)}$$

$$\text{radius} = \frac{5 \times \sin 65^\circ}{\sin 50^\circ}$$

$$= 5.92 \text{ (A1)}$$

$$5^2 = r^2 + r^2 - 2(r)(r) \cos 50^\circ \text{ (M1)}$$

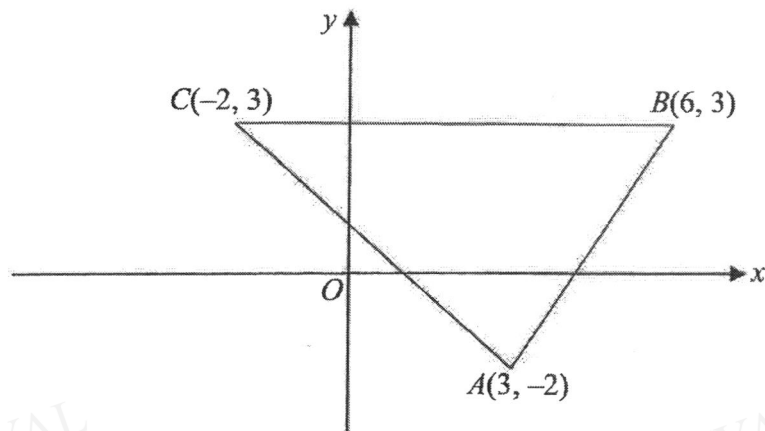
$$25 = 2r^2 - 2r^2 \cos 50^\circ$$

$$25 = 2r^2 [1 - \cos 50^\circ]$$

Answer 5.92 cm [2]

$$r = 5.92 \text{ (A1)}$$

- 22 In the figure, $A(3, -2)$, $B(6, 3)$ and $C(-2, 3)$ are the vertices of a triangle.



- (a) Find the length of AB .

$$AB = \sqrt{(6-3)^2 + (3-(-2))^2} \quad (M1)$$

$$= 5.83 \quad (A1)$$

Answer 5.83 units [2]

- (b) Given that the line AB cuts the y -axis at the point D , find the coordinates of D .

$$\text{Gradient } AB = \frac{5}{3} \quad (B1)$$

$$3 = \frac{5}{3}(6) + c$$

$$c = -7$$

$$\text{Eqn } AB: y = \frac{5}{3}x - 7$$

$$\therefore D = (0, -7) \quad (B1)$$

Answer D (0, -7) [2]

- (c) $ABCE$ is a trapezium with BC parallel to AE .
The area of the trapezium is 45 units².
Find the coordinates of the point E .

Let the length of AE be x .

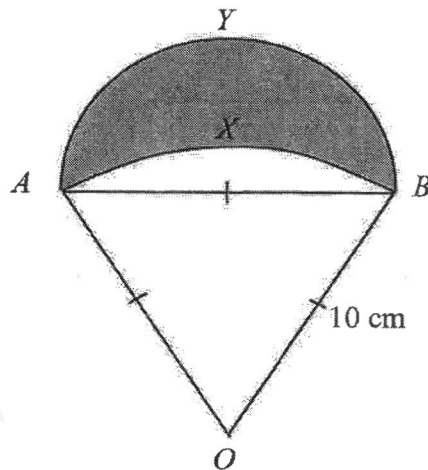
$$\frac{1}{2}(8+x)(5) = 45 \quad (M1)$$

$$x = 10$$

$$E = (-7, -2) \quad (A1)$$

Answer E (-7, -2) [2]

- 23 In the diagram, OAB is an equilateral triangle.
 AXB is an arc of a circle with centre O and radius 10 cm.
 AYB is a semicircle with AB as the diameter.



Express the area of the shaded section as a percentage of the unshaded section.

$$\angle AOB = 60^\circ$$

$$\begin{aligned} \text{Area of } \triangle AOB &= \frac{1}{2}(10)(10)(\sin 60^\circ) && \text{[M1]} \\ &= 43.301 \end{aligned}$$

$$\begin{aligned} \text{Area of sector OAB} &= \frac{60}{360}(\pi)(10)^2 && \text{[M1]} \\ &= 52.360 \end{aligned}$$

$$\begin{aligned} \text{Area of AXBA} &= 52.360 - 43.301 && \text{[M1: area of sector - area of triangle]} \\ &= 9.0590 \end{aligned}$$

If statement not written clearly,
 at most 1 mark each.

$$\begin{aligned} \text{Area of semi-circle AYB} &= \frac{1}{2}(\pi)(5)^2 && \text{[M1]} \\ &= 39.270 \end{aligned}$$

$$\begin{aligned} \text{Area of Shaded Region} &= 39.270 - 9.0590 && \text{[M1: area of semicircle - area of segment]} \\ &= 30.211 \end{aligned}$$

$$\frac{30.211}{43.301 + 9.0590} \times 100\% = 57.7\% \quad \text{[A1]}$$

Answer 57.7 % [6]