Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School
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Parent's Signature

PRELIMINARY EXAMINATION 2020
SECONDARY 4

MATHEMATICS

4048/01

Paper 1

Monday 14 September 2020

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use	
Total	**/ 80**

This document consists of **16** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

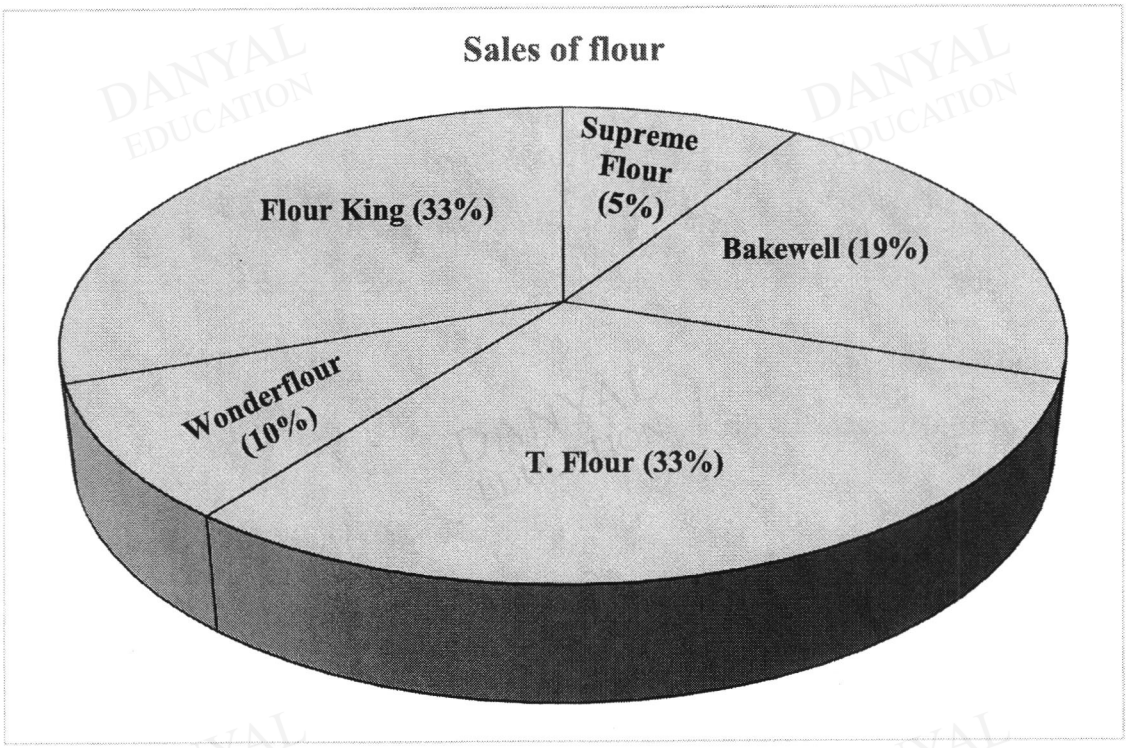
1 Write the following in order of the size, starting with the **smallest**.

$$\frac{23}{45} \quad \sqrt[3]{-64} \quad (-0.86)^2 \quad -0.7^2$$

Answer ....., ..., ..., ... [1]

---

2 The following pie chart represents the percentage of the sales of five different brands of flour in a supermarket.



Explain how the chart above may be misleading.

.....

..... [1]

---

3 **(a)** Simplify $\left(\frac{p^6}{q^3}\right)^{-\frac{2}{3}}$, leaving your answer in positive indices.

Answer ..... [2]

(b) Given that $8^{1+b} = 16^{b-2}$, find the value of b.

Answer $b =$ ..... [2]

---

4 **(a)** Calculate $\frac{18.6^3}{\sqrt{23} - 2.59}$.
Write down the first five digits of your answer.

Answer ..... [1]

(b) Write your answer to **part (a)** correct to 2 significant figures.

Answer ..... [1]

---

5 Factorise $5qr + 6s - 2r - 15qs$ completely.

Answer ..... [2]

6 The area of an 8 km^2 park is represented on a map by an area of 200 cm^2.

(a) If the map has a scale of $1:n$, find the value of n.

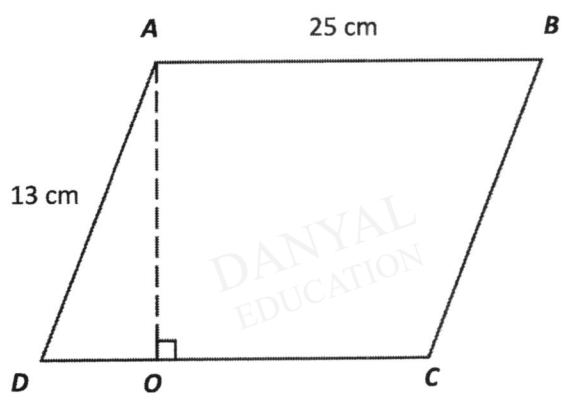
Answer $n =$ ..... [2]

(b) If the perimeter of the park on the map is 13 cm, calculate the actual perimeter in kilometres.

Answer ..... km [1]

(c) A renovation was done on the park and its area on the map is now represented by the following parallelogram $ABCD$, with $AB = 25$ cm, $AD = 13$ cm and $DO = \frac{1}{5}DC$.

Find the percentage change in the area of the park.



Answer ..... [3]

7 Write as a single fraction in its simplest form $\frac{5x}{1-x^2}+\frac{3}{x-1}$.

Answer ..... [3]

---

8 Without using a calculator, show that $5^{2018}-5^{2017}$ is an even number.

Answer

[2]

---

9 Written as a product of its prime factors, $3024 = 2^x \times 3^y \times 7$.

(a) Find the values of x and y.

Answer $x =$ .....

$y =$ ..... [2]

(b) Explain if 3024 is a square number.

Answer .....

..... [1]

---

10 A cylindrical container has a radius of 5.4 cm and a capacity of 1.8 litres.
Calculate the height of the container.

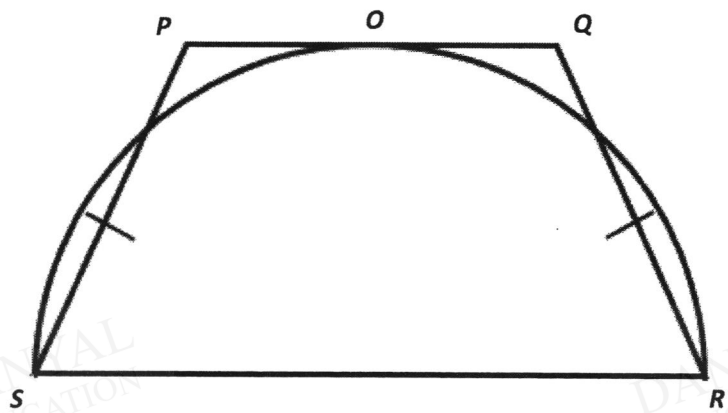
Answer ..... cm [2]

---

11 Solve the equation $\frac{2x-3}{3} - \frac{x+2}{5} = 4$.

Answer $x =$ ..... [2]

12 The following diagram shows a semicircle ROS and an isosceles trapezium $PQRS$, where $PS = (2x+2y)$ cm, $QR = (3y-2x+5)$ cm, $RS = (5y+x-1)$ cm. Given that $PS = QR$ and that the height of the trapezium is $(x+2y)$ cm, find the exact area of the semicircle ROS, giving your answer in terms of π.



DANYAL EDUCATION

Answer ..... cm^2 [4]

13 Mrs Lai would like to deposit \$1080 in a bank which pays 7 % interest per annum compounded half-yearly. Calculate the amount of money in the account at the end of 2 years.

Answer \$ ..... [2]

---

14 5 men are hired to paint a house. If an additional man is hired, the painting can be completed 4 days earlier. Calculate the number of additional men to be hired if the painting is to be completed 18 days earlier.

Answer ..... [3]

---

15 A polygon has n sides. Two of its exterior angles are $24°$ and $86°$, while the other $(n-2)$ exterior angles are $50°$ each. Calculate the value of n.

Answer $n =$ ..... [2]

16 The table below shows the number of enrichment lessons attended by students.

Number of lessons	0	1	2	3	4	5
Number of students	1	3	x	8	5	2

(a) Write down the largest possible value of x given that 3 is the only mode.

Answer $x =$ ..... [1]

(b) Write down the largest possible value of x given that the median is 3.

Answer $x =$ ..... [1]

(c) Calculate the value of x given that the mean is 2.5.

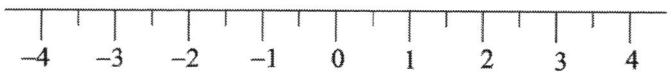
Answer $x =$ ..... [2]

---

17 (a) Solve the inequalities $x-9 \leq 3x-2 < \frac{2}{3}x+5$.

Answer ..... [2]

(b) Represent your answer to part **(a)** on the number line below.



[1]

18 Given that $5y = \sqrt{\dfrac{6+xz^2}{x}}$,

(a) find the value of y when $x = 2$ and $z = -1$.

Answer $y =$ ..... [2]

(b) express x in terms of y and z.

Answer $x =$ ..... [2]

---

19 $\xi = \{x \text{ is an integer: } 1 \leq x \leq 10\}$
$A = \{\text{factors of } 24\}$
$B = \{\text{prime numbers}\}$

(a) List the elements in

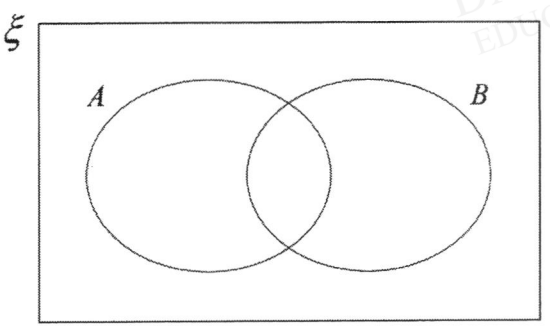
(i) A',

Answer ..... [1]

(ii) $A \cap B$.

Answer ..... [1]

(b) On the Venn diagram, shade the region which represents $(A \cup B')'$.



[1]

---

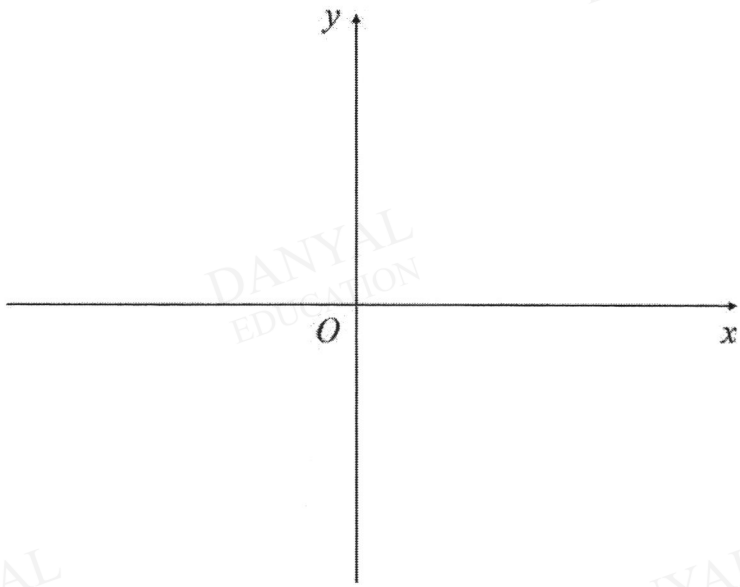
20 (i) Express $x^2 - 6x + 10$ in the form $(x - p)^2 + q$.

Answer ..... [2]

(ii) Write down the coordinates of the minimum point of the graph of $y = x^2 - 6x + 10$.

Answer (..... , ...) [1]

(iii) Sketch the graph of $y = x^2 - 6x + 10$ on the axes below.
Indicate clearly the value where the graph crosses the y-axis.



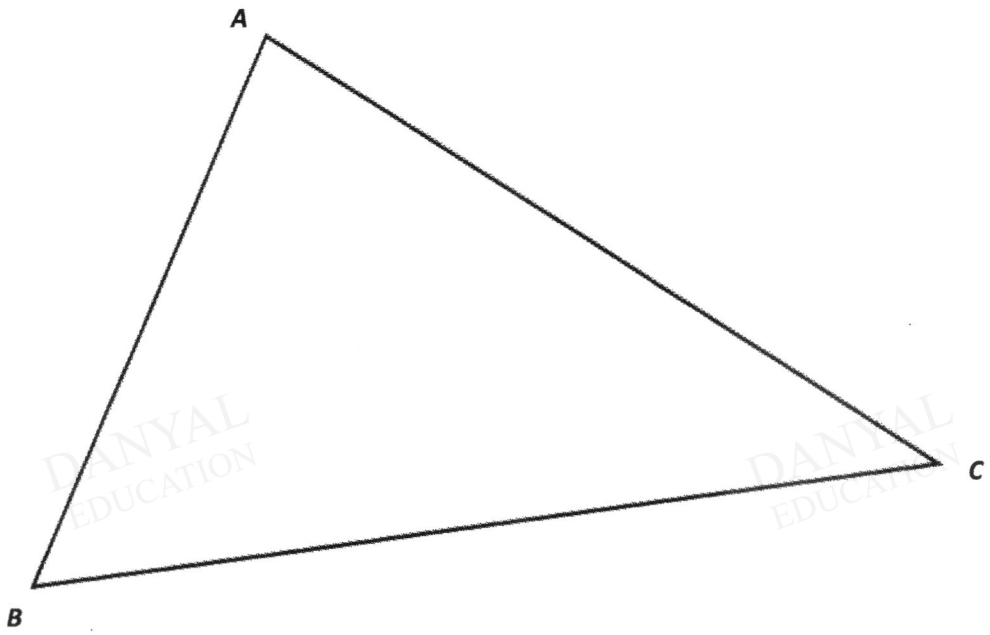
[2]

(iv) Explain why the equation $x^2 - 6x + 10 = k$ does not have solutions for some values of k.

Answer .....

..... [1]

21 The diagram is a plan of a triangular field ABC, drawn to a scale of 1 cm to 100 m.



DANYAL EDUCATION

(a) Draw the perpendicular bisector of BC. [1]

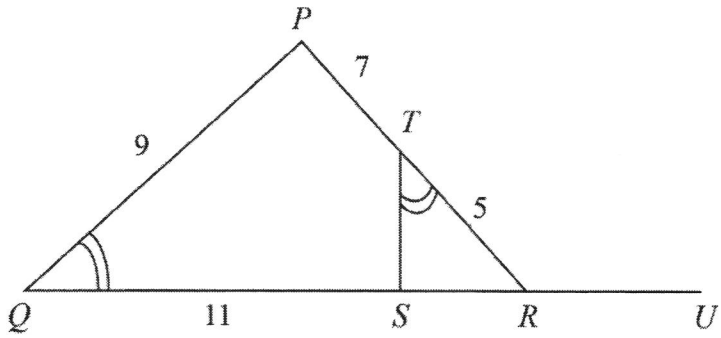
(b) A structure, S, in the field is 450 m from A and is equidistant from BA and BC.

By making appropriate constructions on the diagram, indicate clearly the position of S. [2]

(c) Another structure, T, is to be built equidistant from B and C and from BA and BC. By locating T, find the actual distance, in metres from S to T.

Answer ..... m [1]

22 In the diagram, $QSRU$ is a straight line. $PQ = 9$ cm, $PT = 7$ cm, $TR = 5$ cm and $QS = 11$ cm. Angle PQR = Angle STR.



(a) Show that triangles PQR and STR are similar.

Answer

[2]

(b) Find

(i) the length of SR,

Answer $SR =$ ..... cm [3]

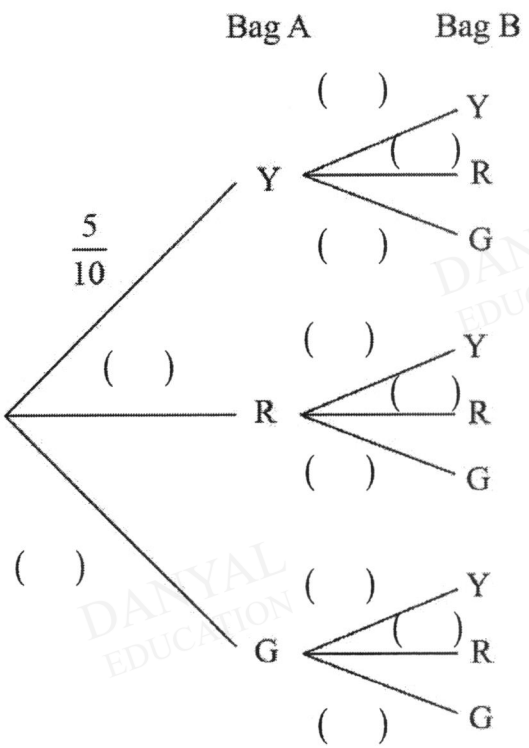
(ii) $\cos \angle PRU$, given that angle TSR is a right angle.

Answer ..... [1]

23 Bag A contains 10 coloured balls of which 5 are yellow, 3 are red and the remaining balls are green. Bag B contains 3 yellow balls and 6 red balls.

A ball is drawn at random from Bag A and placed in Bag B. A second ball is then drawn from Bag B.

(i) Complete the tree diagram to show this information.



[2]

(ii) Find the probability that

(a) both balls drawn are of the same colour,

Answer ..... [2]

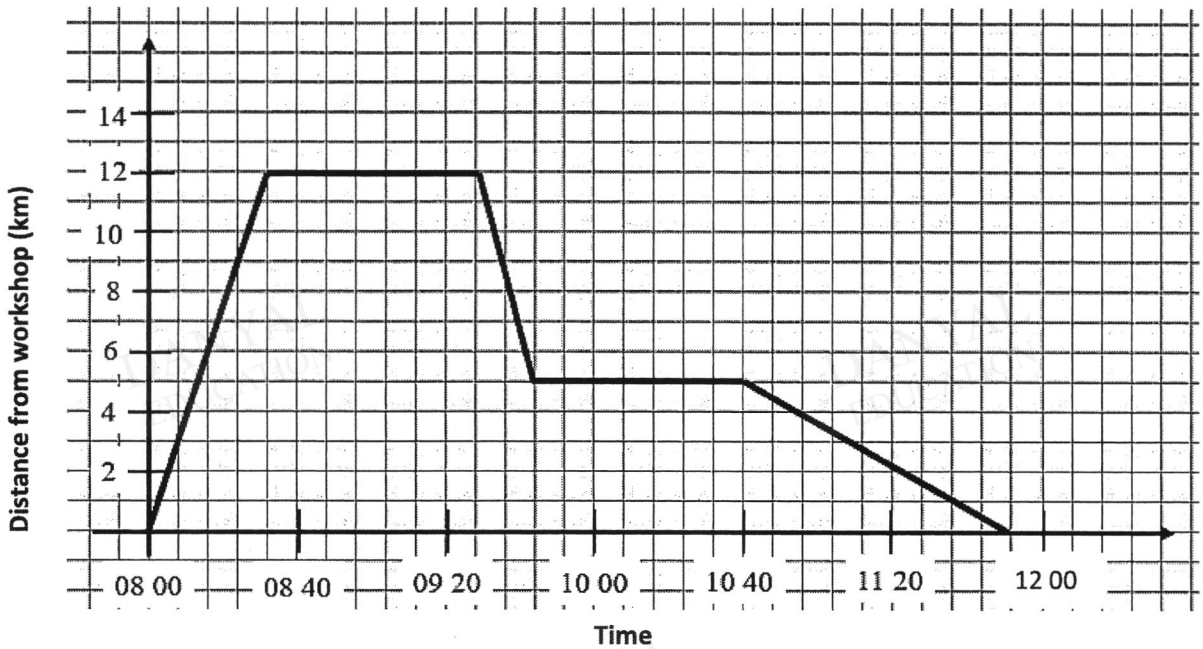
(b) both balls drawn are of different colours,

Answer ..... [1]

(c) a yellow ball is drawn from Bag B.

Answer ..... [2]

24 David drove from his workshop to repair customer A's computer. On his way back, he stopped to repair customer B's computer. The graph shows his entire journey.



(a) How long did he take to repair both computers in total?

Answer ..... minutes [1]

(b) How far was he from customer A at 0824?

Answer ..... km [1]

(c) Find the speed of his travel from Customer A to Customer B.

Answer ..... km/h [1]

(d) David realised he had forgotten one of his repair tools. His sister, Sarah, left his workshop at 08 16 to bring him his tool. She drove towards customer A's place at a constant speed of 36 km/h.

Show her journey on the above graph. [1]

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PRELIMINARY EXAMINATION 2020

SECONDARY 4

MATHEMATICS 4048/02

Paper 2

Tuesday 1 September 2020

2 hours 30 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

	For Examiner's Use
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total Marks	

This document consists of **23** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

DANYAL EDUCATION

1 A is the point $(-20, -11)$ and B is the point $(8, 10)$.

(a) Find the equation of the line AB.

Answer ..... [2]

(b) Show that the line AB does not pass through the point $(1, 5)$.

Answer

[2]

(c) A line l, perpendicular to the line AB, passes through the point $(-15, 10)$.
The product (gradient of l) × (gradient of AB) equals -1.

Use this information to find the equation of the line l.

Answer ..... [2]

(d) The equation of another line h is $4x + 3y - 15 = 0$.

Without solving for x and y, explain whether line l intersects line h.

Answer

[2]

2 **(a)** Each term in this sequence is found by adding the same number to the previous term.

$a,\ 1,\ b,\ c,\ 13,\ .....$

(i) Find the values of a, b and c.

Answer $a =$ ....., $b =$ ....., $c =$ ..... [2]

(ii) Write down, in terms of n, a formula for the n^{th} term.

Answer ..... [1]

(iii) Explain why the terms of the sequence are all odd numbers.

Answer .....

.....

.....

..... [1]

(b) Observe the following difference of unit fractions.
(Unit fractions : fractions with numerator equal to 1)

1st line : $1-\frac{1}{2}=\frac{1}{2}$

2nd line : $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

3rd line : $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$

.
.
.

10th line : $\frac{1}{10}-\frac{1}{11}=\frac{1}{110}$

.
.
.

(i) Write down the 4th line and the 5th line.

Answer

[2]

(ii) Write down the n^{th} line.

Answer

[1]

(iii) A student claims that $\frac{1}{421}$ exists in row n.
Is he correct? Justify your answers with working.

Answer

[2]

(iv) By adding the first 3 lines, we obtain $1-\frac{1}{4}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}$.

Thus, $1=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$.

Using this information, express 1 as a sum of seven different unit fractions.

Answer

[1]

3 As part of a Values in Action project, three Secondary 4 classes from Brightgrove Secondary School collected old newspapers and clothes to raise funds for charity.
The collection was done over two weeks.

The following table shows the weight of the collections made in kilograms (kg), by the three classes, 4E, 4F and 4G, in Week 1.

	Week 1		
	4E	4F	4G
Newspapers (kg)	390	300	350
Clothes (kg)	150	200	180

(a) Represent the weights of the newspapers and clothes collected in Week 1 in a 2×3 matrix **A**.

Answer **A** = [1]

(b) 1 kg of newspapers is sold at \$0.15 and 1 kg of clothes is sold at \$0.45. Represent this information in a 1×2 row matrix **H**.

Answer **H** = [1]

DANYAL EDUCATION

(c) The collection done by the same classes in Week 2 is given by the matrix **B.**

$$\mathbf{B} = \begin{pmatrix} 220 & 250 & 200 \\ 260 & 230 & 170 \end{pmatrix}$$

(i) Evaluate the matrix $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

Answer ..... [1]

(ii) Given that $\mathbf{L} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, evaluate the matrix $\mathbf{M} = \mathbf{RL}$.

Answer ..... [2]

(iii) State what the elements of **M** represent.

Answer

[1]

(iv) Evaluate the matrix **HM.**

Hence, state the total amount of money raised by the 3 classes.

Answer $ ..... [2]

4 A shop owner bought some essential oil for \$500.
She paid \$$x$ for each litre of essential oil.

(a) Find, in terms of x, an expression for the number of litres she bought.

Answer ..... [1]

(b) Due to a leak, she lost 3 litres of essential oil.
She sold the remainder of the essential oil for \$1 per litre more than she paid for it.

Write down an expression, in terms of x, for the sum of money she received.

Answer \$ ..... [2]

(c) She made a profit of \$20.

Write down an equation in x to represent this information and show that it reduces to

$$3x^2 + 23x - 500 = 0 \ .$$

Answer

[3]

DANYAL EDUCATION

(d) Solve the equation $3x^2 + 23x - 500 = 0$, giving your solutions correct to 2 decimal places.

Answer

Answer $x =$ ..... or $x =$ ..... [4]

(e) Find, correct to the nearest whole number, how many litres of essential oil she sold.

Answer ..... [2]

5 When x number of books are printed by a book store, the printing cost, \$$y$, of each book can be modelled by the equation

$$y = \frac{240}{x} + 12.$$

The table below gives some values of x and the corresponding values of y.

x	10	20	30	40	60	80	120
y	36	24	20	18	16	p	14

(a) Calculate the value of p.

Answer $p =$ ..... [1]

(b) On the grid opposite,

use a scale of 2 cm to represent 20 units, draw a horizontal x-axis for $0 \le x \le 120$,
use a scale of 2 cm to represent 5 units, draw a vertical y-axis for $0 \le y \le 40$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate the number of books to be printed if the book store wishes to achieve the printing cost of \$19 for each book.

[1]

Answer .....

(d) By drawing a tangent, find the gradient of the curve at (40, 18).

[2]

Answer .....

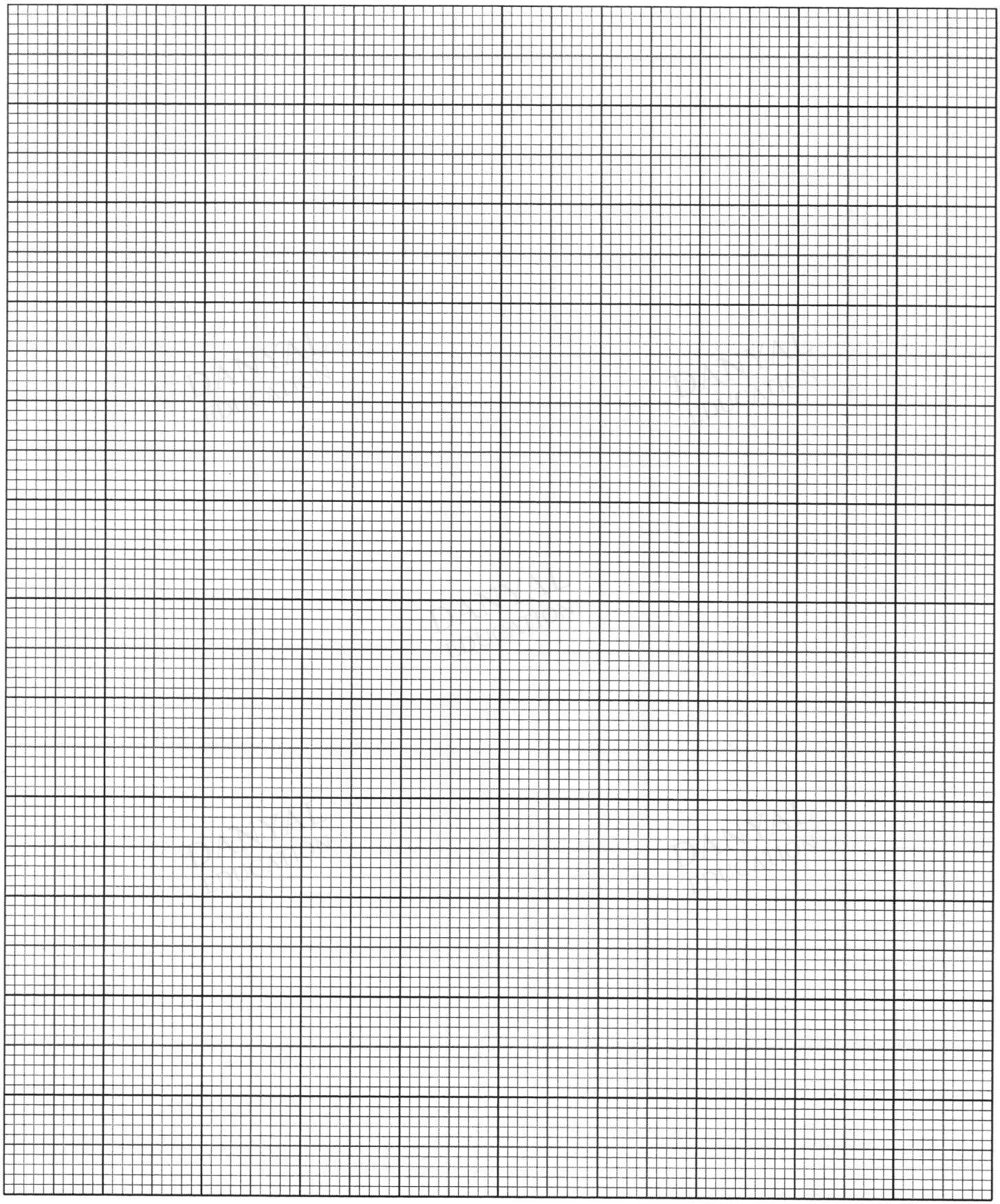
(e) The selling price of each book is given to be $\$\left(30 - \frac{x}{5}\right)$.

(i) On the same grid used in part **(b)**,
draw the graph of $y = \left(30 - \frac{x}{5}\right)$ for $0 \le x \le 120$. [2]

(ii) Assuming that all printed books are sold, write down a possible value of x such that the book store will make a profit.

Answer ..... [1]

DANYAL EDUCATION



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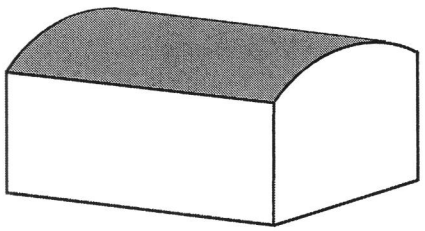


Diagram 1

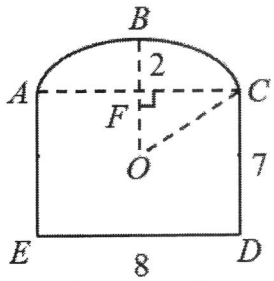


Diagram 2

Diagram 1 shows a garage and Diagram 2 shows the cross section of its end.
The owner needs to order a new roof, represented by the shaded area, for his garage.
The roof is represented by arc ABC, of a circle with centre O and radius r m. $ACDE$ is a rectangle. AC intersects OB at F and $\angle OFC = \frac{\pi}{2}$.

The owner has these measurements : $ED = 8$ m, $BF = 2$ m, $CD = 7$ m, and the length of the garage is 12 m.

(a) By expressing OF in terms of r, show that $r = 5$ m.
Answer

[3]

(b) Show that angle AOC is approximately 1.855 radians.

Answer

[2]

(c) The material for the roof costs \$12.50 per m^2.

Find the cost of the new roof. Give your answer to the nearest dollar.

DANYAL EDUCATION

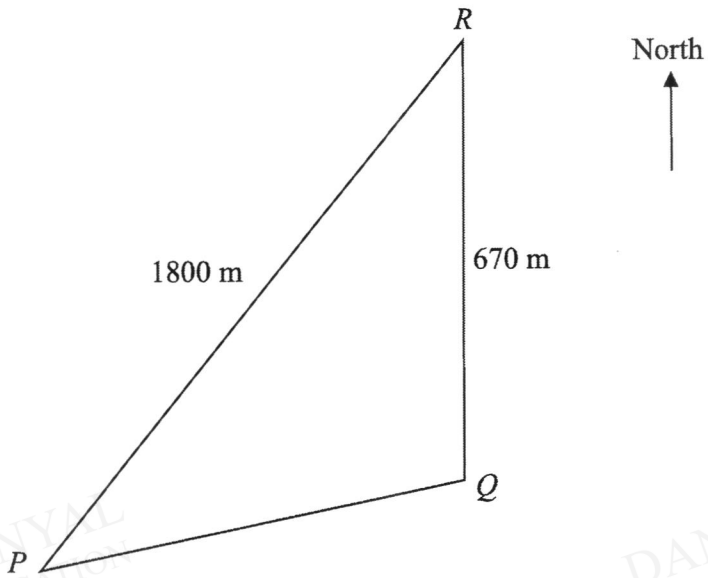
Answer \$ ..... [3]

(d) Calculate the volume of the garage.

DANYAL EDUCATION

Answer .....m^3 [4]

7



P, *Q* and *R* represent three points on an island.
Q is 670 m from *R* and *Q* is due south of *R*.
P is 1800 m from *R*.

(a) Given that the bearing of *P* from *R* is 235°, show that angle $PRQ = 55^\circ$.

Answer

[1]

(b) Calculate *PQ*.

Answer ..... m [2]

(c) Find the bearing of *Q* from *P*.

Answer ..... [3]

(d) A hiker walks in a straight line from P to R and stops at a rest point X, where X is closest to Q. Calculate PX.

DANYAL EDUCATION

Answer ..... m [2]

(e) At X, the hiker spots an eagle hovering vertically above R.

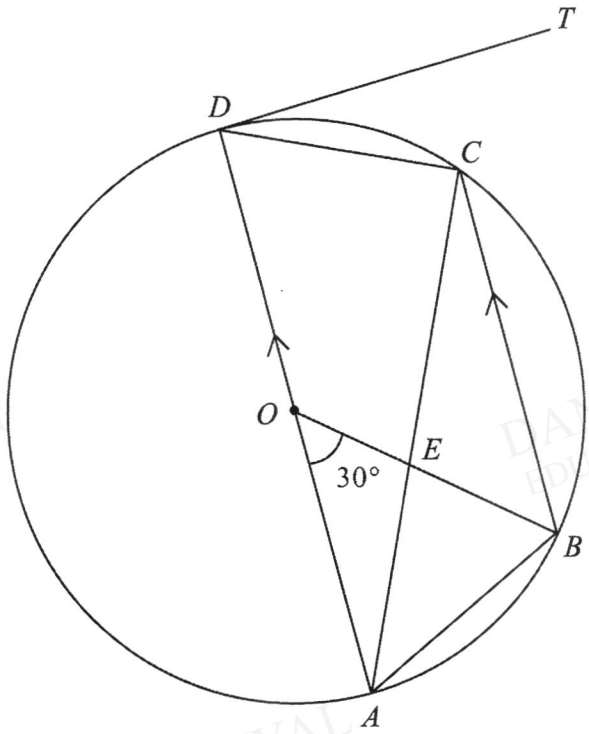
The angle of elevation of the eagle from X is 23.1°.

Calculate the height of the eagle from the ground.

DANYAL EDUCATION

Answer ..... m [2]

8 In the diagram, A, B, C and D are points on the circumference of a circle with centre O.
AD is a diameter of the circle and TD is a tangent to the circle.
OB intersect AC at E, angle $AOB = 30°$ and AD is parallel to BC.



(a) Find, giving reasons to your workings,

(i) angle BCA,

Answer ..... [1]

(ii) angle CDA,

Answer ..... [2]

(iii) angle ABC,

Answer ..... [2]

(iv) angle TDC.

Answer ..... [2]

(b) It is given that angle BXA = angle BCA.

Explain whether point X should lie inside, on our outside the circle.

Answer

[2]

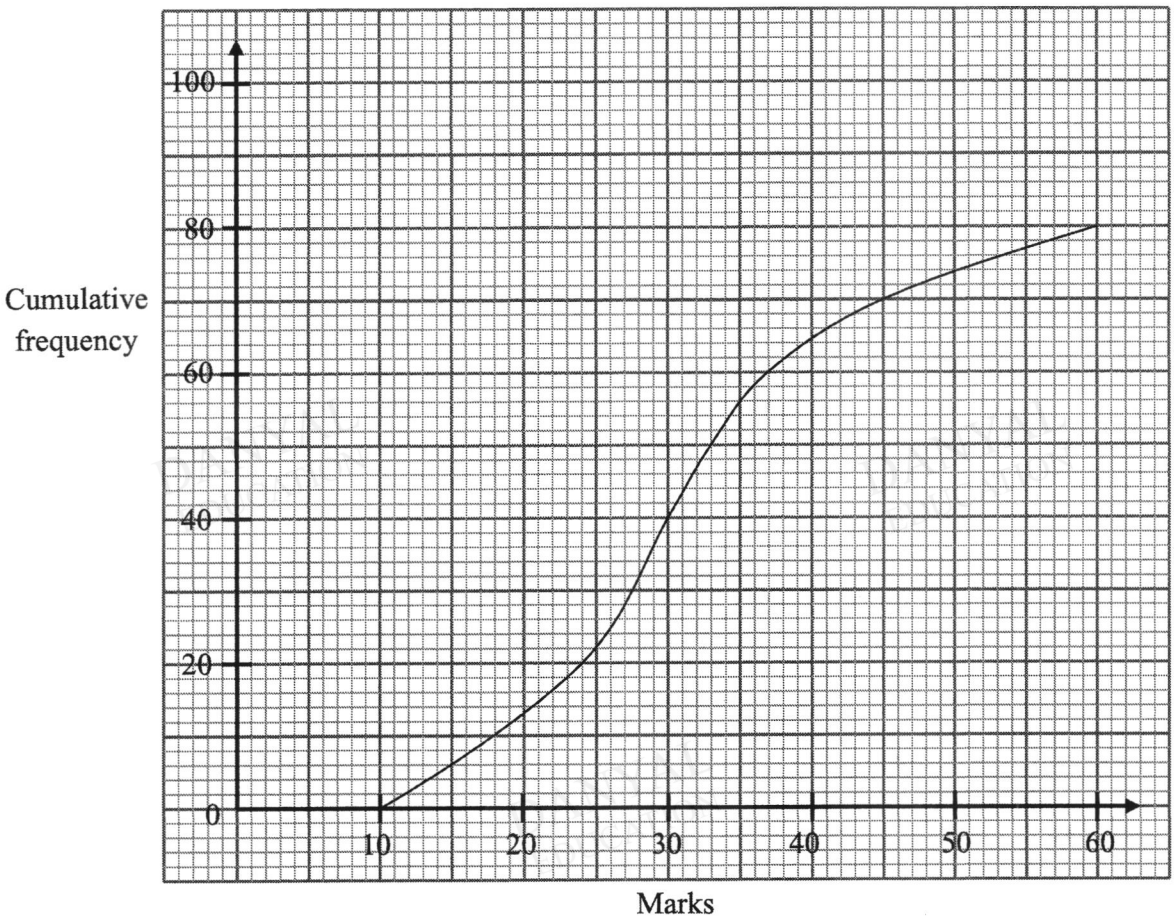
(c) "A circle with diameter AC can be drawn through the points A, B and C."

Determine, with clear explanation, if the above statement is true or false.

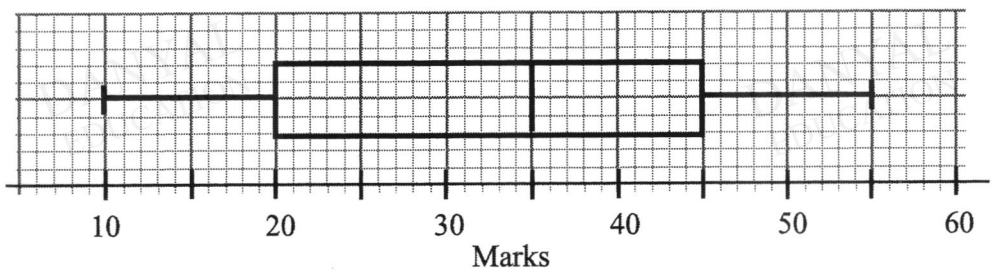
Answer

[2]

9 (a) The cumulative frequency graph shows the distribution of the scores of a Geography test (Test 1) taken by 80 students. Test 1 was marked out of 60.



This box-and-whisker plot represents the distribution of the scores of the same group of students for another Geography test (Test 2). Test 2 was marked out of 60.



(i) Use the two diagrams to complete this table for the two tests.

Test	Lower Quartile	Median	Upper Quartile	Interquartile range
1		30		
2	20		45	25

[3]

To obtain a distinction for Test 1 and Test 2, a student needs to score at least x marks.

(ii) If 12.5% of the students scored distinction for Test 1, find x.

Answer ..... [2]

(iii) "There is a higher proportion of students who scored distinction for Test 1."

Do you agree with this statement? Give a reason for your answer.

Answer

[2]

(b) The table below summarises the speeds of 95 cars on a stretch of road.

Speed (x km/h)	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$	$65 < x \leq 75$	$75 < x \leq 85$
Number of cars	13	29	40	8	5

Calculate an estimate of

(i) the mean speed,

Answer ..... km/h [1]

(ii) the standard deviation.

Answer .....

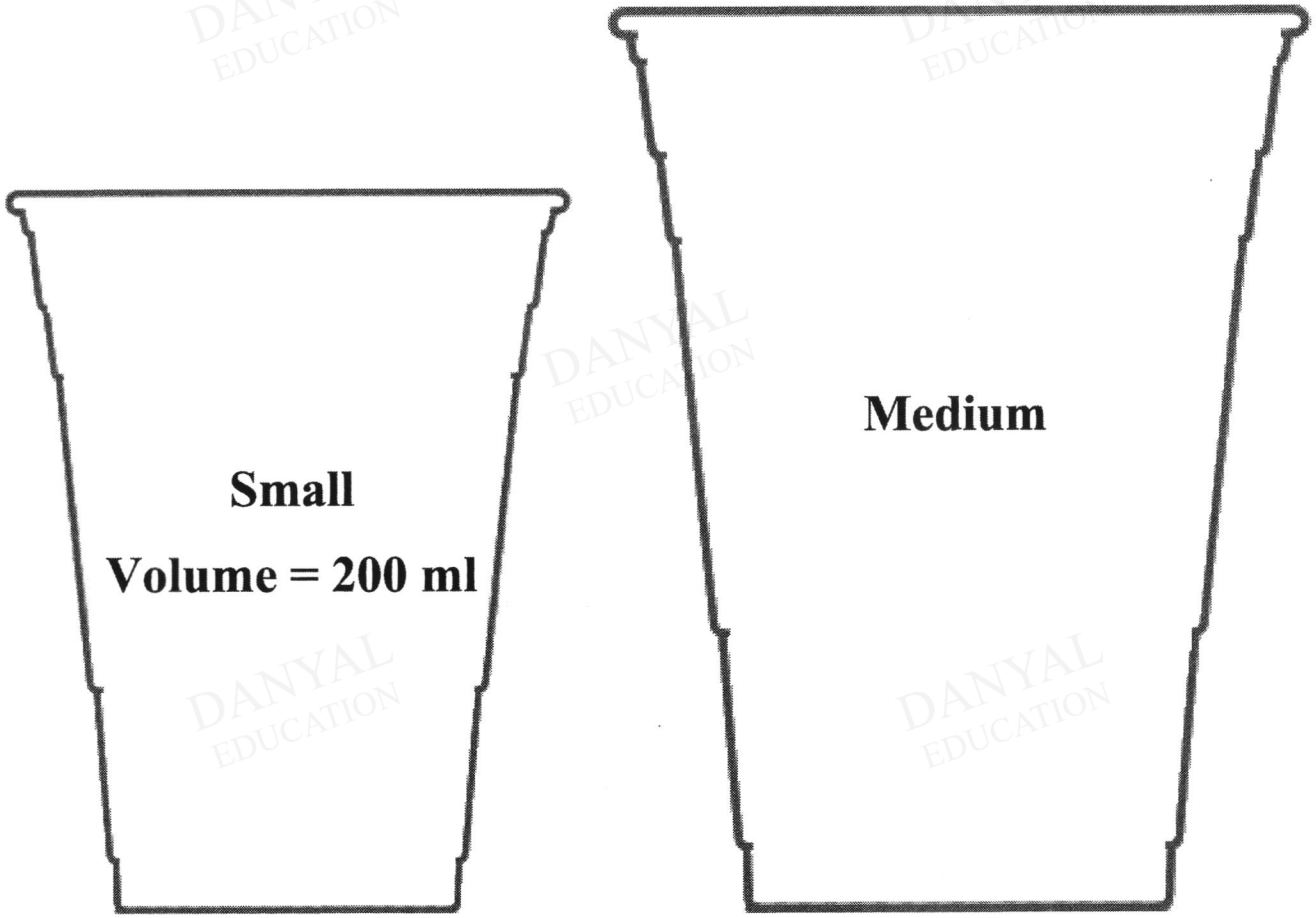
[1]

10 DeBest Bubble Tea Shop purchases their plastic cups from Bubble Planet Supplies.
Below is a pamphlet on the dimensions of the plastic cups.
Due to printing problems, some information are missing.

BUBBLE PLANET SUPPLIES

Picture shows actual size of cup

Item type	Plastic Cup
Sizes Available	Small, Medium
Uses	Drinks (Hot/Cold)



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(a) Estimate the heights of the small and medium cup.

Answer Height of small cup : ..... cm

Height of medium cup : ..... cm [1]

(b) Given that the two cups are geometrically similar, find the volume of the medium cup.

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Answer .....ml [2]

(c) Allison is planning to order a drink from DeBest Bubble Tea Shop.
She has read the following health advice:

> To avoid developing diabetes, the Health Promotion Board (HPB) recommends a daily calorie intake from sugary food (e.g. a cup of bubble tea) of no more than 10% of one's daily energy intake.

She then finds the following information on the website of DeBest Bubble Tea Shop.

Table 1

Sugar Level
Quarter Sugar (25%)
Half Sugar (50%)
Less Sugar (75%)
Full Sugar (100%) *

* A medium-sized cup of drink with full sugar (100%) contains 95 ml of sugar syrup.

Table 2

	Approximate amount of calories
Green tea	85 calories per 500 ml
Milk tea	110 calories per 500 ml
Black tea	70 calories per 500 ml
Sugar syrup	50 calories per 15 ml
Honey	60 calories per 15 ml

Note : A cup of flavoured tea is made by adding sugar syrup according to customers' preferred sugar level and topping up the remaining amount with flavoured tea.

For customers with daily energy intake between 2400 and 2900 calories

Healthier Choice Option

Medium-sized milk tea with 75% sugar level

Allison's daily energy intake is about 2800 calories.
She decides to select the healthier choice option drink and thinks that she will meet the daily calorie intake as recommended by HPB.

Is she correct?
Justify your decision with calculations.

Writing space for Q10 (c)

Answer:

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[7]

Answer Key

1. $\sqrt[3]{-64}$, -0.7^2 , $\frac{23}{45}$, $(-0.86)^2$

2. The **3D presentation** of the pie-chart **makes it seem like** T. Flour holds the largest proportion of the pie chart, and hence the largest percentage of sales, **when in actual fact** is equal to Flour King at 33%

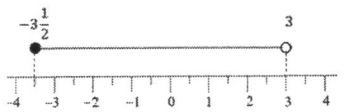
3(a) $\frac{q^2}{p^4}$ (b) 11 4(a) 2917.2 (b) 2900 5. $(5q-2)(r-3s)$

6(a) 20 000 (b) 2.6 km (c) 50% 7. $\frac{2x-3}{(1+x)(1-x)}$ 9(a) $x=4, y=3$

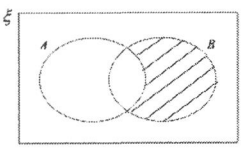
9(b) Since the powers of the prime factors of 3024 are not even (multiples of 2), 3024 is not a square number.

10. 19.6 cm 11. $x=11\frac{4}{7}$ 12. 32π 13. \$1239.32 14. 15 men 15. $n=7$

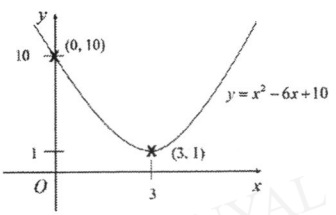
16(a) $x=7$ (b) $x=10$ (c) $x=19$ 17(a) $-3\frac{1}{2} \le x < 3$ (b)



18(a) $\frac{2}{5}$ (b) $x=\frac{6}{25y^2-z^2}$ 19(a)(i) 5, 7, 9, 10 (a)(ii) 2, 3 (b)

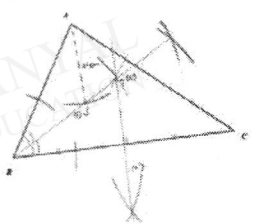


20(i) $(x-3)^2+1$ (ii) (3, 1) (iii)

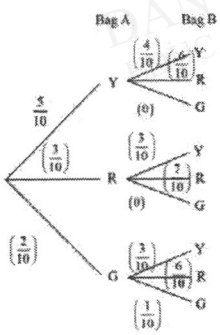


20(iv) Since the graph will not intersect the line $y = k$, when $k < 1$, there are no solutions when $k < 1$.

21(a) & (b)



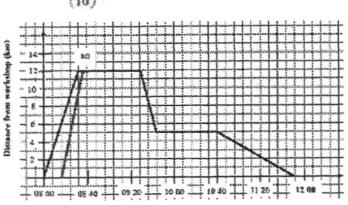
21(c) 220 m



22(b)(i) $SR = 4$ cm (b)(ii) $-\frac{4}{5}$ 23(i)

23 (ii)(a) $\frac{43}{100}$ (ii)(b) $\frac{57}{100}$ (c) $\frac{7}{20}$

24(a) 112 mins (b) 3 or 3.5 km (c) $26\frac{1}{4}$ km (d)



Answer Key

1(a) $y=\frac{3}{4}x+4$ (c) $y=-\frac{4}{3}x-10$

(d) Since gradient of line h = gradient of line l, the lines are parallel and do not intersect.

2(a)(i) $a=-3, b=5, c=9$ (a)(ii) $4n-7$

2(b)(i) 4th line: $\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$; 5th line: $\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$ (b)(ii) nth line: $\frac{1}{n}-\frac{1}{n+1}=\frac{1}{n(n+1)}$

(b)(iii) Since n is not an integer, he is not correct. (b)(iv) $1=\frac{1}{2}+\frac{1}{6}+\frac{1}{7}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$

3(a) $\mathbf{A}=\begin{pmatrix}390 & 300 & 350\\ 150 & 200 & 180\end{pmatrix}$ (b) $\mathbf{H}=\begin{pmatrix}0.15 & 0.45\end{pmatrix}$ (c)(i) $\mathbf{R}=\begin{pmatrix}610 & 550 & 550\\ 410 & 430 & 350\end{pmatrix}$

(c)(ii) $\mathbf{M}=\begin{pmatrix}1710\\ 1190\end{pmatrix}$

(c)(iii) The elements represent the **total weight of** newspapers and clothes collected **respectively** by the 3 classes, from the **2 collections / weeks**.

(c)(iv) \$792

4(a) $\frac{500}{x}$ (b) $\$(x+1)\left(\frac{500}{x}-3\right)$ (d) $x=9.63$ or -17.30 (e) 49

5(a) $p=15$ (c) 34 (d) -0.15 (e)(ii) Any integer value of $17\le x\le 73$

6(c) \$1391 (d) 806 m^3

7(b) 1520 m (c) 076.2° (d) 1420 m (e) 164 m

8(a)(i) 15° (ii) 75° (iii) 105° (iv) 15° (b) On the circle

(c) False.

9(a)(i)

Test	Lower Quartile	Median	Upper Quartile	Interquartile range
1	**24**	30	**37**	**13**
2	20	**35**	45	25

(a)(ii) 45 (iii) Disagree. (b)(i) 56.1 km/h (b)(ii) 9.98

10(a) Height of small cup = 9.9 cm, height of medium cup = 12.4 cm

(b) 393 ml (c) She is not correct.

Mathematics Paper 1 Solutions (Prelim 2020)

Answer **all** the questions.

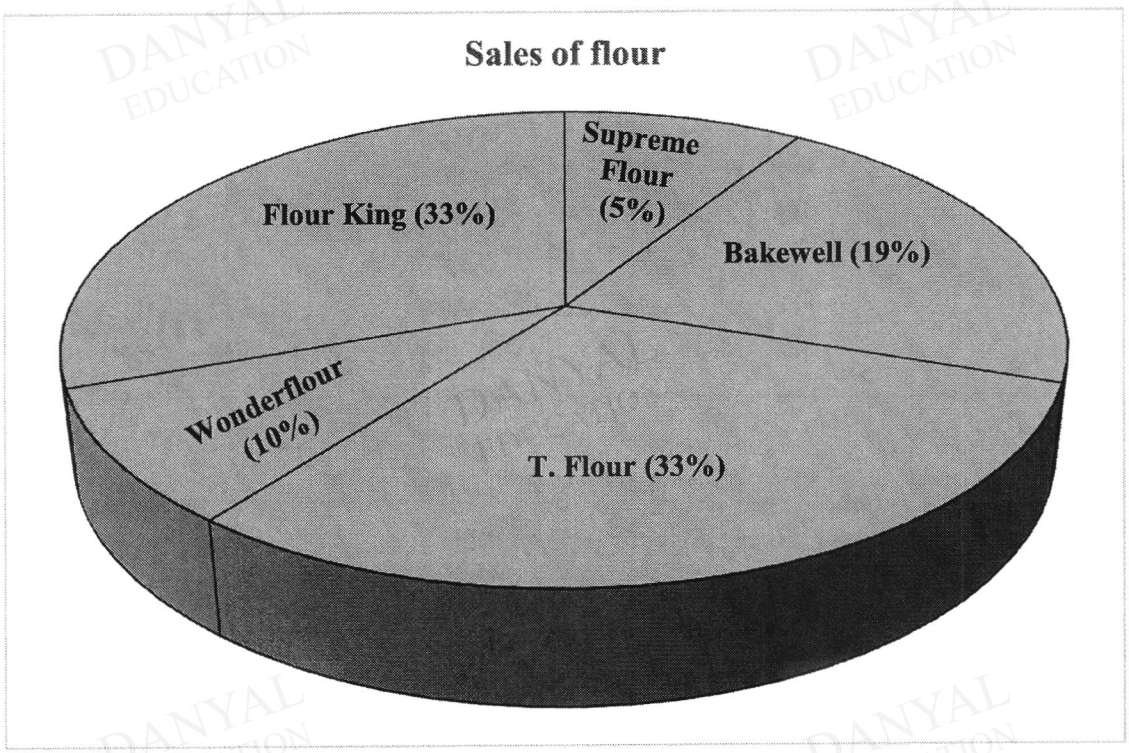
1 Write the following in order of the size, starting with the **smallest**.

$$\frac{23}{45} \quad \sqrt[3]{-64} \quad (-0.86)^2 \quad -0.7^2$$

Answer $\sqrt[3]{-64}$, -0.7^2 , $\frac{23}{45}$, $(-0.86)^2$ [1]

---

2 The following pie chart represents the percentage of the sales of five different brands of flour in a supermarket.



Explain how the chart above may be misleading.

The **3D presentation** of the pie-chart **makes it seem like** T. Flour holds the largest proportion of the pie chart, and hence the largest percentage of sales, **when in actual fact** is equal to Flour King at 33%. [1]

---

3 **(a)** Simplify $\left(\frac{p^6}{q^3}\right)^{-\frac{2}{3}}$, leaving your answer in positive indices.

$$\left(\frac{p^6}{q^3}\right)^{-\frac{2}{3}} = \frac{p^{-4}}{q^{-2}}$$
$$= \frac{q^2}{p^4}$$

Answer $\frac{q^2}{p^4}$ [2]

(b) Given that $8^{1+b} = 16^{b-2}$, find the value of b.

$$8^{1+b} = 16^{b-2}$$
$$2^{3(1+b)} = 2^{4(b-2)}$$

Comparing powers,

$$3(1+b) = 4(b-2)$$
$$3+3b = 4b-8$$
$$-b = -11$$
$$b = 11$$

Answer $b =$ 11 [2]

---

4 **(a)** Calculate $\frac{18.6^3}{\sqrt{23}-2.59}$.
Write down the first five digits of your answer.

Answer 2917.2 [1]

(b) Write your answer to **part (a)** correct to 2 significant figures.

Answer 2900 [1]

---

5 Factorise $5qr+6s-2r-15qs$ completely.

$$5qr+6s-2r-15qs = 5qr-15qs+6s-2r$$
$$= 5q(r-3s)-2(r-3s)$$
$$= (5q-2)(r-3s)$$

Answer $(5q-2)(r-3s)$ [2]

---

6 The area of an 8 km^2 park is represented on a map by an area of 200 cm^2.

(a) If the map has a scale of $1:n$, find the value of n.

Area scale

Map : Actual

200 cm^2 : 8 km^2

1 cm^2 : 0.04 km^2

Length scale

Map : Actual

$\sqrt{1\ cm^2} : \sqrt{0.04\ km^2}$

1 cm : 0.2 km

1 cm : 0.2×1000×100 cm

1 : 20 000

Answer $n =$ 20 000 [2]

(b) If the perimeter of the park on the map is 13 cm, calculate the actual perimeter in kilometres.

Actual perimeter = 13×0.2

= 2.6 km

Answer 2.6 km [1]

(c) A renovation was done on the park and its area on the map is now represented by the following parallelogram $ABCD$, with $AB = 25$ cm, $AD = 13$ cm and $DO = \frac{1}{5}DC$. Find the percentage change in the area of the park.

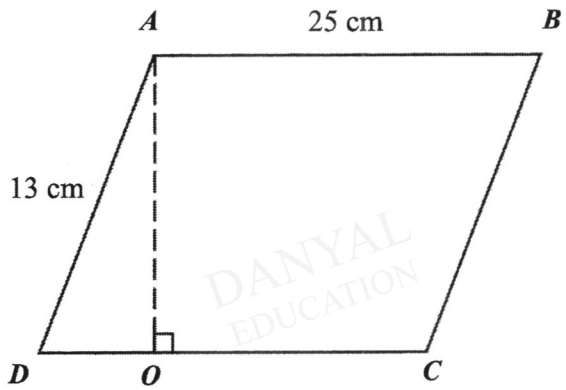
$DO = \frac{1}{5} \times 25$

$= 5$ cm

By Pythagoras' Theorem,

$AO = \sqrt{13^2 - 5^2}$

$= 12$ cm



Area of parallelogram = 12×25

= 300 cm^2

Percentage change $= \frac{300 - 200}{200} \times 100\%$

$= 50\%$

Answer 50% [3]

7 Write as a single fraction in its simplest form $\frac{5x}{1-x^2}+\frac{3}{x-1}$.

$$\begin{aligned}\frac{5x}{1-x^2}+\frac{3}{x-1} &= \frac{5x}{(1+x)(1-x)}+\frac{3}{x-1}\\ &= \frac{5x}{(1+x)(1-x)}-\frac{3}{1-x}\\ &= \frac{5x-3(1+x)}{(1+x)(1-x)}\\ &= \frac{5x-3-3x}{(1+x)(1-x)}\\ &= \frac{2x-3}{(1+x)(1-x)}\end{aligned}$$

Answer $\frac{2x-3}{(1+x)(1-x)}$ [3]

---

8 Without using a calculator, show that $5^{2018}-5^{2017}$ is an even number.

Answer

$$\begin{aligned}5^{2018}-5^{2017} &= 5^{2017}(5-1)\\ &= 5^{2017}\times 4\end{aligned}$$

Option 1: Since 4 is an **even factor** of $5^{2018}-5^{2017}$, $5^{2018}-5^{2017}$ is an even number.

Option 2: Since $5^{2018}-5^{2017}$ is a **multiple of 4, which is an even number**, $5^{2018}-5^{2017}$ is an even number.

Option 3: Since **4 is an even number**, and an even number **multiplied by any number is even**, $5^{2018}-5^{2017}$ is an even number.

[2]

---

9 Written as a product of its prime factors, $3024 = 2^x \times 3^y \times 7$.

(a) Find the values of x and y.

Answer $x =$ 4

$y =$ 3 [2]

(b) Explain if 3024 is a square number.

Answer **Since the powers of the prime factors of 3024 are not even (multiples of 2), 3024 is not a square number.** OR **3024 is not a square number as it cannot be expressed as a product of 2 identical integers**. [1]

---

10 A cylindrical container has a radius of 5.4 cm and a capacity of 1.8 litres.
Calculate the height of the container.

1.8 litres = 1800 ml
= 1800 cm^3

Height $= 1800 \div \pi(5.4)^2$
$= 19.648...$ cm
$= 19.6$ cm (3sf)

Answer 19.6 cm [2]

---

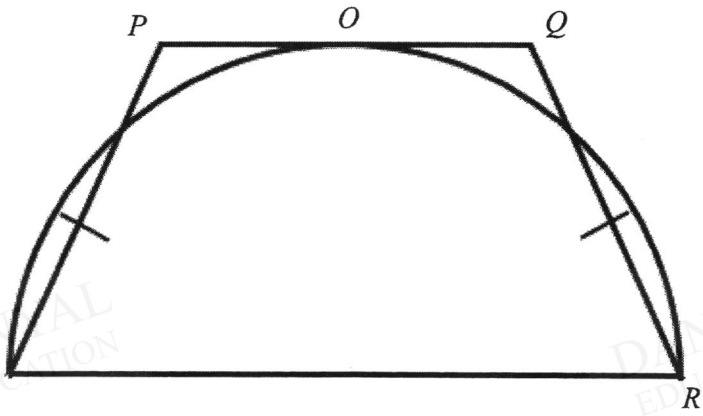
11 Solve the equation $\frac{2x-3}{3} - \frac{x+2}{5} = 4$.

$$\frac{2x-3}{3} - \frac{x+2}{5} = 4$$
$$\frac{5(2x-3)-3(x+2)}{15} = 4$$
$$10x - 15 - 3x - 6 = 60$$
$$7x - 21 = 60$$
$$7x = 81$$
$$x = 11\frac{4}{7}$$

Answer $x =$ $11\frac{4}{7}$ [2]

---

12 The following diagram shows a semicircle ROS and an isosceles trapezium $PQRS$, where $PS = (2x+2y)$ cm, $QR = (3y-2x+5)$ cm, $RS = (5y+x-1)$ cm. Given that $PS = QR$ and that the height of the trapezium is $(x+2y)$ cm, find the exact area of the semicircle ROS, giving your answer in terms of π.



$2x+2y = 3y-2x+5$

$4x-y = 5$ ----- (1)

$2(x+2y) = 5y+x-1$

$2x+4y = 5y+x-1$

$x-y = -1$ ----- (2)

(1) − (2):

$3x = 6$

$x = 2$

Sub. $x = 2$ into (2).

$2-y = -1$

$-y = -3$

$y = 3$

Radius of semicircle $= 2+2(3)$

$= 8$ cm

Area of semicircle $= \frac{1}{2}\pi(8)^2$

$= 32\pi$ cm^2

Answer 32π cm^2 [4]

13 Mrs Lai would like to deposit \$1080 in a bank which pays 7 % interest per annum compounded half-yearly. Calculate the amount of money in the account at the end of 2 years.

$$\text{Total} = \text{P}\left(1+\frac{r}{100}\right)^n$$

$$=1080\left(1+\frac{\frac{7}{2}}{100}\right)^4$$

$$= \$1239.32 \text{ (nearest cents)}$$

Answer \$ ..... 1239.32 ..... [2]

---

14 5 men are hired to paint a house. If an additional man is hired, the painting can be completed 4 days earlier. Calculate the number of additional men to be hired if the painting is to be completed 18 days earlier.

Let the number of men be m and the number of days be d.

$m \propto \frac{1}{d}$

$m = \frac{k}{d}$, where k is a non-zero constant

When $m = 5$,

$5 = \frac{k}{d}$

$k = 5d \quad ---(1)$

When $m = 6$,

$6 = \frac{k}{d-4} \quad ---(2)$

Sub (1) into (2).

$6 = \frac{5d}{d-4}$

$6d - 24 = 5d$

$d = 24$

When $d = 24 - 18 = 6$,

$m = \frac{k}{6}$

$k = 5d$

$= 5(24)$

$= 120$

$m = \frac{120}{6}$

$= 20$

Additional number of men $= 20 - 5$

$= 15$

Answer ..... 15 ..... [3]

---

15 A polygon has n sides. Two of its exterior angles are $24°$ and $86°$, while the other $(n-2)$ exterior angles are $50°$ each. Calculate the value of n.

$$24° + 86° + 50°(n-2) = 360°$$
$$110 + 50n - 100 = 360$$
$$50n = 350$$
$$n = 7$$

Answer $n =$ 7 [2]

---

16 The table below shows the number of enrichment lessons attended by students.

Number of lessons	0	1	2	3	4	5
Number of students	1	3	x	8	5	2

(a) Write down the largest possible value of x given that 3 is the only mode.

Answer $x =$ 7 [1]

(b) Write down the largest possible value of x given that the median is 3.

Answer $x =$ 10 [1]

(c) Calculate the value of x given that the mean is 2.5.

$$\frac{0(1)+1(3)+2x+3(8)+4(5)+5(12)}{19+x} = 2.5$$
$$\frac{57+2x}{19+x} = 2.5$$
$$57 + 2x = 47.5 + 2.5x$$
$$-0.5x = -9.5$$
$$x = 19$$

Answer $x =$ 19 [2]

---

17 (a) Solve the inequalities $x-9 \le 3x-2 < \frac{2}{3}x+5$.

$x-9 \le 3x-2$ and $3x-2 < \frac{2}{3}x+5$

$x-9 \le 3x-2$

$-2x \le 7$

$x \ge -3\frac{1}{2}$

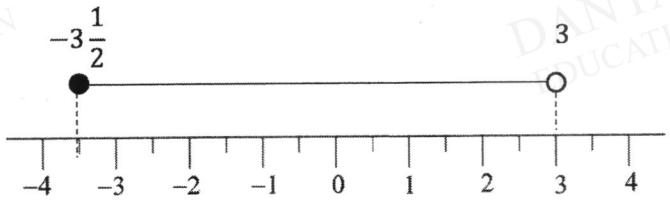
$3x-2 < \frac{2}{3}x+5$

$2\frac{1}{3}x < 7$

$x < 3$

Answer $-3\frac{1}{2} \le x < 3$ [2]

(b) Represent your answer to part **(a)** on the number line below.



[1]

---

18 Given that $5y = \sqrt{\frac{6+xz^2}{x}}$,

(a) find the value of y when $x = 2$ and $z = -1$.

$5y = \sqrt{\frac{6+2(-1)^2}{2}}$

$5y = 2$

$y = \frac{2}{5}$

Answer $y = \frac{2}{5}$ [2]

(b) express x in terms of y and z.

$25y^2 = \frac{6+xz^2}{x}$

$25xy^2 = 6+xz^2$

$25xy^2 - xz^2 = 6$

$x(25y^2 - z^2) = 6$

$x = \frac{6}{25y^2 - z^2}$

Answer $x = \frac{6}{25y^2 - z^2}$ [2]

---

19 $\xi = \{x \text{ is an integer: } 1 \le x \le 10\}$

$A = \{\text{factors of } 24\}$

$B = \{\text{prime numbers}\}$

(a) List the elements in

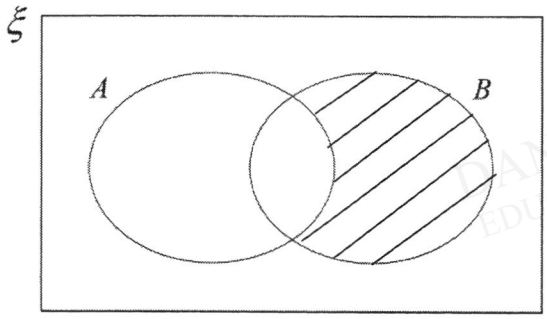
(i) A',

Answer 5,7,9,10 [1]

(ii) $A \cap B$.

Answer 2,3 [1]

(b) On the Venn diagram, shade the region which represents $(A \cup B')'$.



[1]

---

20 **(i)** Express $x^2 - 6x + 10$ in the form $(x-p)^2 + q$.

$$x^2 - 6x + 10 = x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 10$$

$$= (x-3)^2 - (-3)^2 + 10$$

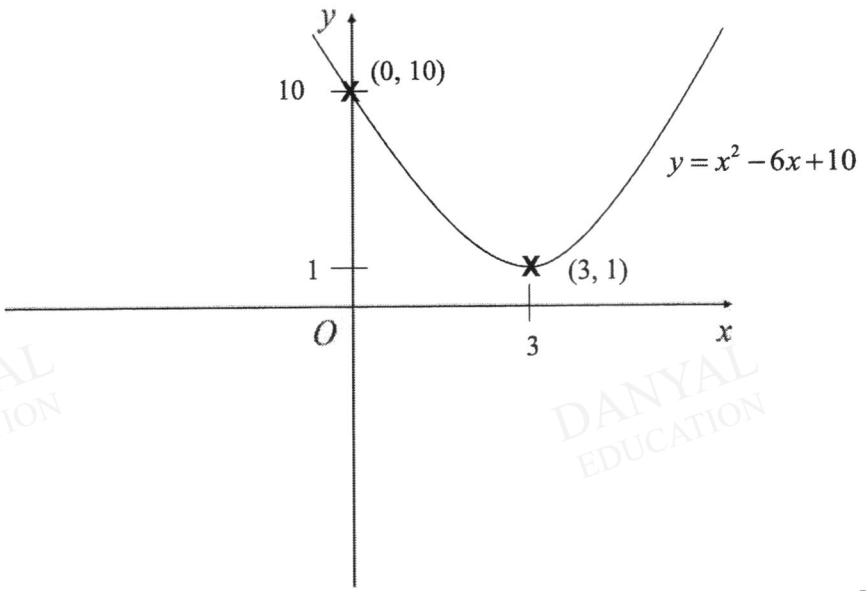
$$= (x-3)^2 + 1$$

Answer $(x-3)^2 + 1$ [2]

(ii) Write down the coordinates of the minimum point of the graph of $y = x^2 - 6x + 10$.

Answer (.... 3 ..... , ... 1 .....) [1]

(iii) Sketch the graph of $y = x^2 - 6x + 10$ on the axes below.
Indicate clearly the value where the graph crosses the y-axis.



[2]

(iv) Explain why the equation $x^2 - 6x + 10 = k$ does not have solutions for some values of k.

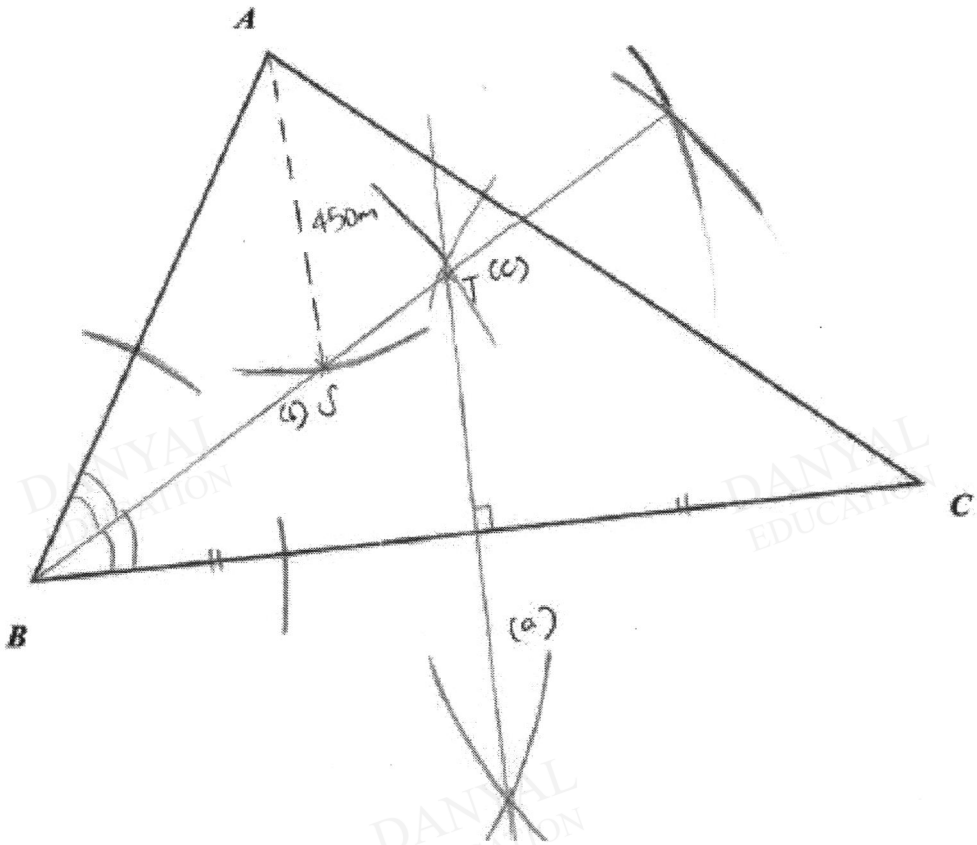
Answer

Since the graph will not intersect the line $y = k$ when $k < 1$, there are no solutions when $k < 1$.

OR

As $x^2 - 6x + 10$ will not go below its minimum value of 1, it will not have solutions for $k < 1$. [1]

21 The diagram is a plan of a triangular field *ABC*, drawn to a scale of 1 cm to 100 m.



(a) Draw the perpendicular bisector of *BC*. [1]

(b) A structure, *S*, in the field is 450 m from *A* and is equidistant from *BA* and *BC*.

By making appropriate constructions on the diagram, indicate clearly the position of *S*. [2]

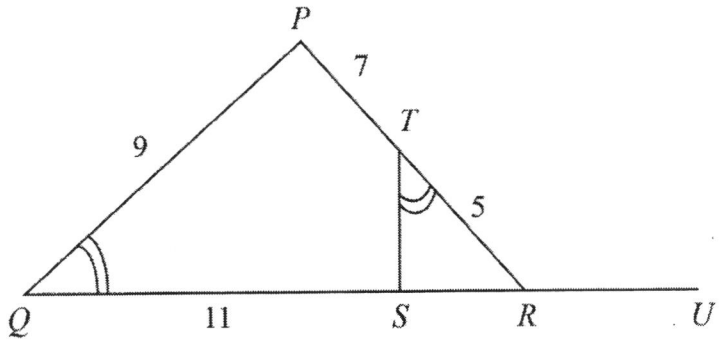
(c) Another structure, *T*, is to be built equidistant from *B* and *C* and from *BA* and *BC*. By locating *T*, find the actual distance, in metres from *S* to *T*.

Distance = 2.2×100

= 220 m

Answer 220 m [1]

22 In the diagram, $QSRU$ is a straight line. $PQ = 9$ cm, $PT = 7$ cm, $TR = 5$ cm and $QS = 11$ cm. Angle PQR = Angle STR.



(a) Show that triangles PQR and STR are similar.

Answer

$\angle PQR = \angle STR$ (given)

$\angle PRQ = \angle SRT$ (common angle)

Hence triangles PQR and STR are similar.

[2]

(b) Find

(i) the length of SR,

$\frac{PR}{SR} = \frac{PQ}{ST} = \frac{QR}{TR}$ (ratio of corresponding sides are equal)

$\frac{12}{SR} = \frac{9}{ST} = \frac{11+SR}{5}$

$60 = 11SR + SR^2$

$SR^2 + 11SR - 60 = 0$

$(SR - 4)(SR + 15) = 0$

$SR = 4$ or $SR = -15$ (reject, $SR > 0$)

Answer $SR =$ 4 cm [3]

(ii) $\cos \angle PRU$, given that angle TSR is a right angle.

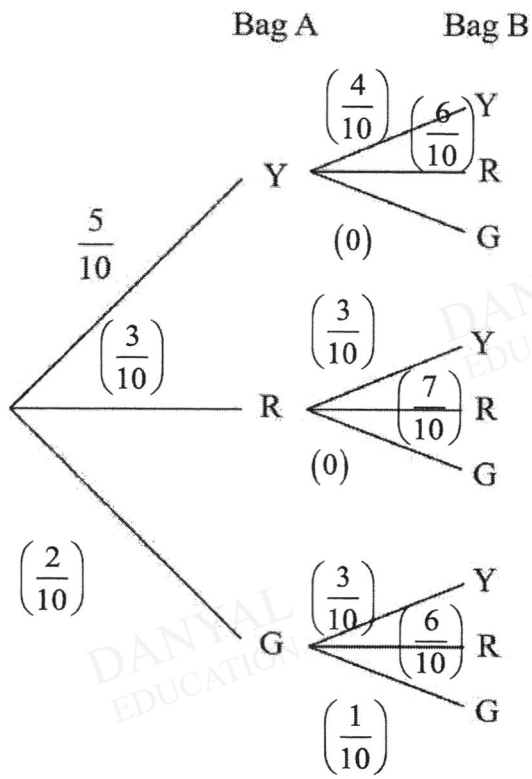
$\cos \angle PRU = -\cos \angle TRS$

$= -\frac{4}{5}$

Answer $-\frac{4}{5}$ [1]

23 Bag A contains 10 coloured balls of which 5 are yellow, 3 are red and the remaining balls are green. Bag B contains 3 yellow balls and 6 red balls.
A ball is drawn at random from Bag A and placed in Bag B. A second ball is then drawn from Bag B.

(i) Complete the tree diagram to show this information.



[2]

(ii) Find the probability that

(a) both balls drawn are of the same colour,

$$\text{P(same colour)} = \left(\frac{5}{10}\times\frac{4}{10}\right)+\left(\frac{3}{10}\times\frac{7}{10}\right)+\left(\frac{2}{10}\times\frac{1}{10}\right)$$
$$= \frac{43}{100}$$

Answer $\frac{43}{100}$ [2]

(b) both balls drawn are of different colours,

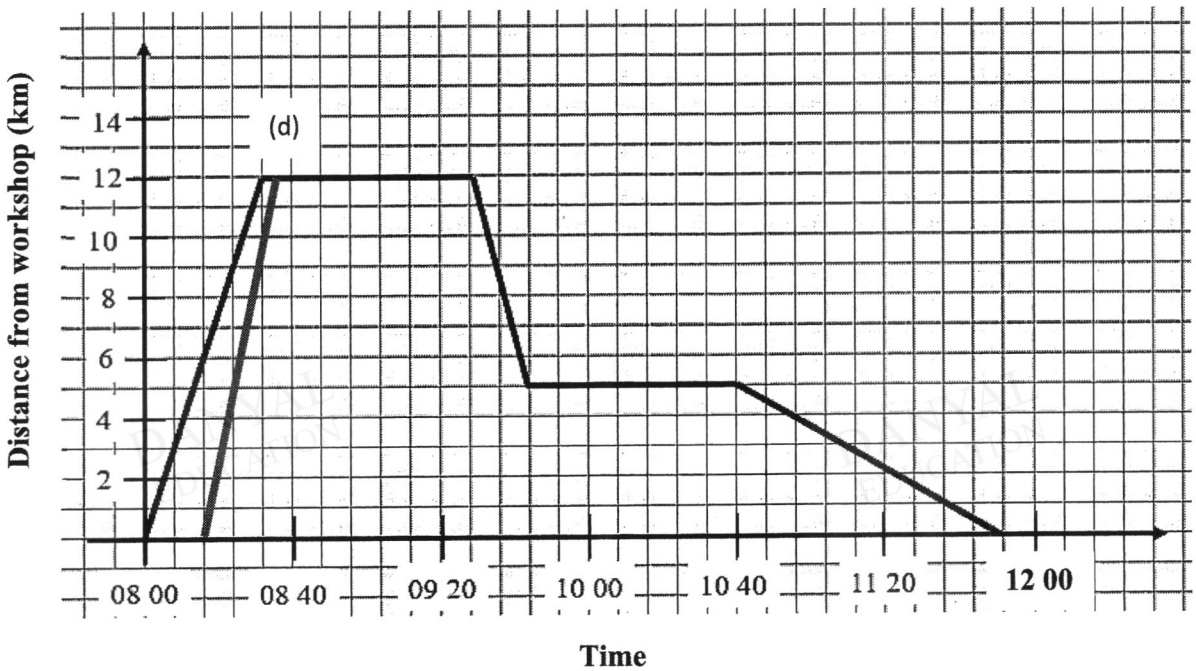
Answer $\frac{57}{100}$ [1]

(c) a yellow ball is drawn from Bag B.

$$\text{P(YY, RY, GY)} = \left(\frac{5}{10}\times\frac{4}{10}\right)+\left(\frac{3}{10}\times\frac{3}{10}\right)+\left(\frac{2}{10}\times\frac{3}{10}\right)$$
$$= \frac{7}{20}$$

Answer $\frac{7}{20}$ [2]

24 David drove from his workshop to repair customer A's computer. On his way back, he stopped to repair customer B's computer. The graph shows his entire journey.



(a) How long did he take to repair both computers in total?

Answer ..... 112 ..... minutes [1]

(b) How far was he from customer A at 08 24?

Answer ..... 3 or 3.5 ..... km [1]

(c) Find the speed of his travel from Customer A to Customer B.

$$\text{Speed} = \frac{7}{16/60}$$

$$= 26\frac{1}{4} \text{ km/h}$$

Answer ..... $26\frac{1}{4}$ ..... km/h [1]

(d) David realised he had forgotten one of his repair tools. His sister, Sarah, left his workshop at 08 16 to bring him his tool.
She drove towards customer A's place at a constant speed of 36km/h.
Show her journey on the above graph.

$$\text{Time taken} = \frac{12}{36} h$$

$$= 20 \text{ min} \rightarrow \text{2.5 sqaures after 08 16}$$

[1]

1 A is the point $(-20, -11)$ and B is the point $(8, 10)$.

(a) Find the equation of the line AB.

$$\text{gradient } AB = \frac{10-(-11)}{8-(-20)}$$
$$= \frac{3}{4}$$

Equation of AB:

$$y-10 = \frac{3}{4}(x-8)$$
$$y = \frac{3}{4}x - 6 + 10$$
$$y = \frac{3}{4}x + 4$$

Answer $y = \frac{3}{4}x + 4$ [2]

(b) Show that the line AB does not pass through the point $(1, 5)$.

Answer

substitute $x = 1$ into $y = \frac{3}{4}x + 4$,

$$y = 4\frac{3}{4} \neq 5$$

Since (1,5) does not satisfy the equation of AB, AB does not pass through (1,5). [2]

(c) A line l, perpendicular to the line AB, passes through the point $(-15, 10)$.
The product (gradient of l) × (gradient of AB) equals –1.

Use this information to find the equation of the line l.

$$m_l \times \frac{3}{4} = -1$$
$$m_l = -\frac{4}{3}$$

Equation of l:

$$y-10 = -\frac{4}{3}(x+15)$$
$$y = -\frac{4}{3}x - 10$$

Answer $y = -\frac{4}{3}x - 10$ [2]

(d) The equation of another line h is $4x + 3y - 15 = 0$.

Without solving for x and y, explain whether line l intersects line h.

Answer

For $4x + 3y - 15 = 0$

$$y = -\frac{4}{3}x + 5$$

Since gradient of line h = gradient of line l, the 2 lines are parallel and hence do not intersect.

[2]

2 (a) Each term in this sequence is found by adding the same number to the previous term.

$$a,\ 1,\ b,\ c,\ 13,\ .....$$

(i) Find the values of a, b and c.

let the number added be x

$13-3x=1$

$3x=12$

$x=4$

$\therefore a=-3,\ b=5,\ c=9$

Answer $a=$...-3....., $b=$ ...5..., $c=$...9 [2]

(ii) Write down, in terms of n, a formula for the n^{th} term.

$4n-7$

Answer $4n-7$ ..... [1]

(ii) Explain why the terms of the sequence are all odd numbers.

Answer (1) $4n$ is an even number and 7 is odd. An even number subtract an odd number will always result in an odd number.

(2) $4n-7=2\left(2n-\frac{7}{2}\right)$. As $\left(2n-\frac{7}{2}\right)$ is not a positive integer, 4n-7 cannot be expressed as a multiple of 2, it is an odd number. [1]

(b) Observe the following difference of unit fractions.
(Unit fractions : fractions with numerator equal to 1)

1st line : $1-\frac{1}{2}=\frac{1}{2}$

2nd line : $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

3rd line : $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$

.
.
.

10th line : $\frac{1}{10}-\frac{1}{11}=\frac{1}{110}$

.
.
.

(i) Write down the 4th line and the 5th line.

Answer 4th line : $\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$

5th line : $\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$ [2]

(ii) Write down the n^{th} line.

Answer nth line : $\frac{1}{n}-\frac{1}{n+1}=\frac{1}{n(n+1)}$

[1]

(iii) A student claims that $\frac{1}{421}$ exists in row n.
Is he correct? Justify your answers with working.

Answer

$n(n+1)=421$

$n^2+n-421=0$

$n=\frac{-1\pm\sqrt{(-1)^2-4(1)(-421)}}{2}$

$n=20.02..$ or $n=-21.0..$

Since n is not a positive integer, the student is not correct.

[2]

(iv) By adding the first 3 lines, we obtain $1-\frac{1}{4}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}$. Thus, $1=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$.

Using this information, express 1 as a sum of seven different unit fractions.

Answer

Adding 1st 6 lines,

$1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$

$1-\frac{1}{7}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$

$1=\frac{1}{2}+\frac{1}{6}+\frac{1}{7}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$

[1]

3 As part of a Values in Action project, three Secondary 4 classes from Brightgrove Secondary School collected old newspapers and clothes to raise funds for charity.
The collection was done over two weeks.

The following table shows the weight of the collections made in kilograms (kg), by the three classes, 4E, 4F and 4G in Week 1.

	Week 1		
	4E	4F	4G
Newspapers (kg)	390	300	350
Clothes (kg)	150	200	180

(a) Represent the weights of the newspapers and clothes collected in Week 1 in a 2×3 matrix **A**.

Answer $\mathbf{A} = \begin{pmatrix} 390 & 300 & 350 \\ 150 & 200 & 180 \end{pmatrix}$ [1]

(b) 1 kg of newspapers is sold at \$0.15 and 1 kg of clothes is sold at \$0.45. Represent this information in a 1×2 row matrix **H**.

Answer $\mathbf{H} = \begin{pmatrix} 0.15 & 0.45 \end{pmatrix}$ [1]

(c) The collection done by the same classes in Week 2 is given by the matrix **B.**

$$\mathbf{B} = \begin{pmatrix} 220 & 250 & 200 \\ 260 & 230 & 170 \end{pmatrix}$$

(i) Evaluate the matrix $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

$$\mathbf{R} = \begin{pmatrix} 390 & 300 & 350 \\ 150 & 200 & 180 \end{pmatrix} + \begin{pmatrix} 220 & 250 & 200 \\ 260 & 230 & 170 \end{pmatrix}$$
$$= \begin{pmatrix} 610 & 550 & 550 \\ 410 & 430 & 350 \end{pmatrix}$$

Answer $\begin{pmatrix} 610 & 550 & 550 \\ 410 & 430 & 350 \end{pmatrix}$ [1]

(ii) Given that $\mathbf{L} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, evaluate the matrix $\mathbf{M} = \mathbf{RL}$.

$$\mathbf{M} = \begin{pmatrix} 610 & 550 & 550 \\ 410 & 430 & 350 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1710 \\ 1190 \end{pmatrix}$$

Answer $\begin{pmatrix} 1710 \\ 1190 \end{pmatrix}$ [2]

(iii) State what the elements of **M** represent.

Answer
The elements represent the **total weight of** newspapers and clothes collected **respectively** by the 3 classes, from the **2 collections / weeks**.

[1]

(iv) Evaluate the matrix **HM.** Hence, state the total amount of money raised by the 3 classes.

$$\begin{pmatrix} 0.15 & 0.45 \end{pmatrix} \begin{pmatrix} 1710 \\ 1190 \end{pmatrix}$$
$$= (256.5 + 535.5)$$
$$= (792)$$

Hence, amount of money raised = \$ 792

Answer \$ .....792..... [2]

4 A shop owner bought some essential oil for \$500.
She paid \$$x$ for each litre of essential oil.

(a) Find, in terms of x, an expression for the number of litres she bought.

$\frac{500}{x}$

Answer $\frac{500}{x}$ [1]

(b) Due to a leak, she lost 3 litres of essential oil.
She sold the remainder of the essential oil for \$1 per litre more than she paid for it.

Write down an expression, in terms of x, for the sum of money she received.

no. of litres left $= \frac{500}{x} - 3$

sum of money received $= (x+1)\left(\frac{500}{x} - 3\right)$

Answer \$ $(x+1)\left(\frac{500}{x} - 3\right)$ [2]

(c) She made a profit of \$20.

Write down an equation to represent this information and show that it reduces to

$$3x^2 + 23x - 500 = 0 \ .$$

Answer

$(x+1)\left(\frac{500}{x} - 3\right) - 500 = 20$

$(x+1)\left(\frac{500}{x} - 3\right) = 520$

$500 - 3x + \frac{500}{x} - 3 = 520$

$-3x - 23 + \frac{500}{x} = 0$

$\times -x]$ $3x^2 + 23x - 500 = 0$ (shown)

[3]

(d) Solve the equation $3x^2 + 23x - 500 = 0$, giving your solutions correct to 2 decimal places.

Answer

$$x = \frac{-23 \pm \sqrt{(23)^2 - 4(3)(-500)}}{2(3)}$$

$$= \frac{-23 \pm \sqrt{6529}}{6}$$

$x = 9.633...$ or $x = -17.300...$

$= 9.63$(2 dec.pl) $= -17.30$ (2 dec.pl)

Answer $x =$ .....9.63..... or $x =$ .....-17.30..... [4]

(e) Find, correct to the nearest whole number, how many litres of essential oil she sold.

$$\text{no. of litres sold} = \frac{500}{9.633...} - 3$$

$$= 48.90..$$

$$= 49 \text{ (nearest whole number)}$$

Answer .....49..... [2]

5 When x number of books are printed by a book store, the printing cost, \$$y$, of each book can be modelled by the equation

$$y = \frac{240}{x} + 12$$

The table below gives some values of x and the corresponding values of y.

x	10	20	30	40	60	80	120
y	36	24	20	18	16	p	14

(a) Calculate the value of p.

Answer $p =$ .....15..... [1]

(b) On the grid opposite,

use a scale of 2 cm to represent 20 units, draw a horizontal x-axis for $0 \le x \le 120$,
use a scale of 2 cm to represent 5 units, draw a vertical y-axis for $0 \le y \le 40$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate the number of books to be printed if the book store wishes to achieve the printing cost of \$19 for each book.

Answer .....34..... [1]

(d) By drawing a tangent, find the gradient of the curve at (40, 18).

$$\text{Gradient} = -\frac{24-15}{60-0} = -0.15$$

[actual gradient by differentiation $= -0.15$]

Answer .....-0.15..... [2]

(e) The selling price of each book is given to be \$$\left(30 - \frac{x}{5}\right)$.

(i) On the same grid used in part **(b)**, draw the graph of $y = \left(30 - \frac{x}{5}\right)$ for $0 \le x \le 120$. [2]

(ii) Assuming that all printed books are sold, write down a possible value of x such that the book store will make a profit

[1]

Answer Any integer value of $17 \le x \le 73$

6

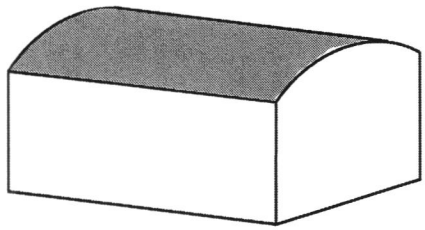


Diagram 1

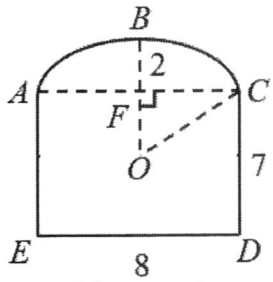


Diagram 2

Diagram 1 shows a garage and Diagram 2 shows the cross section of its end.
The owner needs to order a new roof, represented by the shaded area, for his garage.
The roof is represented by arc ABC, of a circle with centre O and radius r m. $ACDE$ is a rectangle. AC intersects OB at F and $\angle OFC = 90°$.
The owner has these measurements : $ED = 8$ m, $BF = 2$ m, $CD = 7$ m, and the length of the garage is 12 m.

(a) By expressing OF in terms of r, show that $r = 5$ m.
Answer

$OF = r - 2$

$OC^2 = OF^2 + FC^2$

$r^2 = (r-2)^2 + 4^2$

$r^2 = r^2 - 4r + 4 + 16$

$4r = 20$

$r = 5$ (shown)

[3]

(b) Show that angle AOC is approximately 1.855 radians.

Answer

Consider triangle FOC,

$\frac{CF}{OC} = \sin \angle FOC$

$\sin \angle FOC = \frac{4}{5}$

$\angle FOC = \sin^{-1}\left(\frac{4}{5}\right)$

$= 0.92729...$

$\angle AOC = 2(0.92729...)$

$= 1.8545...$

$= 1.855$ (4 sig.fig) (shown)

[2]

(c) The material for the roof costs \$12.50 per m^2.

Find the cost of the new roof. Give your answer to the nearest dollar.

$$\text{Arc length ABC} = 5(1.8545..)$$
$$= 9.2729...$$
$$\text{surface area} = 9.2729... \times 12$$
$$= 111.2754...$$
$$\text{Cost of new roof} = 111.2754... \times \$12.50$$
$$= \$1390.94...$$
$$= \$1391 \text{ (nearest dollar)}$$

Answer \$ .....1391..... [3]

(d) Calculate the volume of the garage.

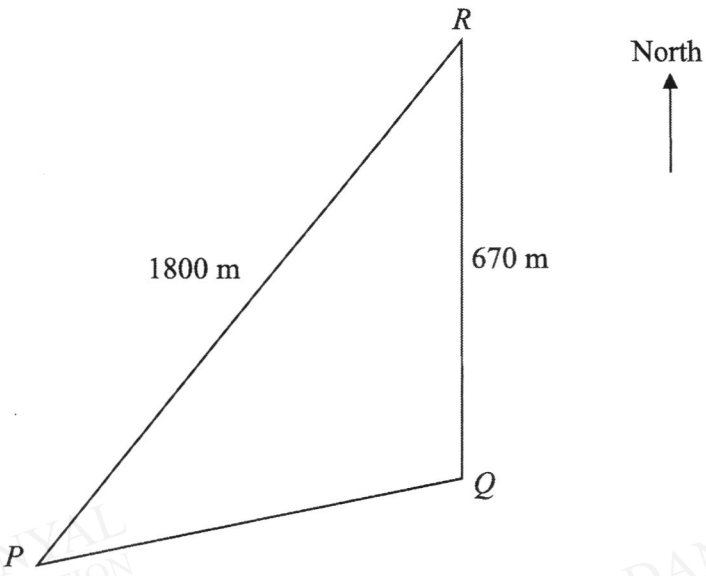
$$\text{Area of sector} = \frac{1}{2}(5)^2(1.8545...)$$
$$= 23.1812...$$
$$\text{Area of triangle } AOC = \frac{1}{2}(5)^2 \sin(1.8545...)$$
$$= 12.000...$$
$$\text{Area of segment} = 23.1812... - 12.000...$$
$$= 11.18088...$$
$$\therefore \text{Volume of garage} = (11.18088... + 7(8)) \times 12$$
$$= 806.17...$$
$$= 806 \text{ m}^3 \text{ (3 sig.fig)}$$

Alternative method

$$\text{Area of sector} = \frac{1}{2}(5)^2(1.8545...)$$
$$= 23.1812...$$
$$\text{Area of AOCDE (2 trapeziums)} = 2 \times \frac{1}{2}(4+7)(4)$$
$$= 44 \text{ cm}^2$$
$$\text{Volume of garage} = \text{area of cross section} \times 12$$
$$= (23.1812... + 44) \times 12$$
$$= 67.1812... \times 12$$
$$= 806.1744...$$
$$= 806 \text{ cm}^3$$

Answer .....806.....m^3 [4]

7



P, *Q* and *R* represent three points on an island.
Q is 670 m from *R* and *Q* is due south of *R*.
P is 1800 m from *R*.

(a) Given that the bearing of *P* from *R* is 235°, show that angle $PRQ = 55^\circ$.

Answer

$\angle PRQ = 235^\circ - 180^\circ$
$= 55^\circ$ (shown)

[1]

(b) Calculate *PQ*.
Using Cosine Rule,

$PQ = \sqrt{(1800)^2 + (670)^2 - 2(1800)(670)\cos 55^\circ}$
= 1518.365...
= 1520 m (3 s.f.)

Answer .....1520..... [2]

(c) Find the bearing of *Q* from *P*.

$\frac{\sin \angle QPR}{670} = \frac{\sin 55}{PQ}$ [Sine Rule]

$\sin \angle QPR = \frac{670 \times \sin 55^\circ}{(1518.365...)}$

Will be 21.166 (if use 1520)

$\angle QPR = 21.1900...^\circ$

$\angle N_1PQ = 21.1900...^\circ + 55^\circ$

$= 76.19...^\circ$

∴ Bearing of *Q* from *P* $= 076.2^\circ$ (1d.p)

Answer 076.2° [3]

(d) A hiker walks in a straight line from P to R and stops at a rest point X, where X is closest to Q. Calculate PX.

$QX \perp PR$

$\cos 55^\circ = \dfrac{XR}{670}$

$XR = 670 \cos 55^\circ$

$\therefore PX$

$= 1800 - 670\cos 55^\circ$

$= 1415.703$ m

$= 1420$ m (3sf)

OR $\cos 21.1900^\circ = \dfrac{PX}{1518.365...}$

$PX = 1420$ m (3sf)

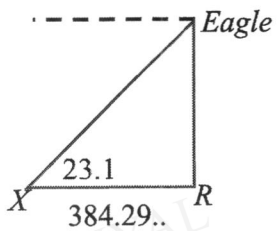
DANYAL EDUCATION

Answer ..... 1420 ..... m [2]

(e) At X, the hiker spots an eagle hovering vertically above R.

The angle of elevation of the eagle from X is 23.1°.

Calculate the height of the eagle from the ground.



$$\tan 23.1^\circ = \frac{\text{Height of eagle}}{1800 - 1415.703...}$$

Height of eagle

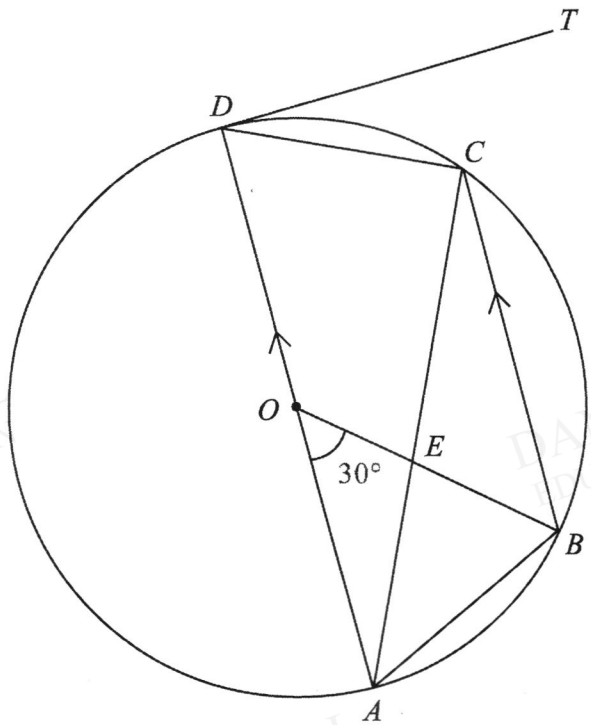
$\tan 23.1^\circ \times 384.297..$

$= 163.9165...$

$= 164$m (3 sig.fig)

Answer ..... 164 ..... m [2]

8 In the diagram, A, B, C and D are points on the circumference of a circle with centre O. TD is a tangent to the circle. AD is a diameter of the circle and TD is a tangent to the circle. OB intersect AC at E, angle $AOB = 30°$ and AD is parallel to BC.



(a) Find, giving reasons to your workings,

(i) angle BCA,

$\angle BCA = 30^\circ \div 2$ ($\angle$ at centre = 2$\angle$s at circumference)

$= 15^\circ$

Answer 15° [1]

(ii) angle CDA,

$\angle ACD = 90$ (right angle in semicircle)

$\angle CAO = \angle BCA = 15$ (alternate angles, $BC // AD$)

$\therefore \angle CDA = 180 - 15 - 90$ ($\angle$ sum of triangle)

$= 75^\circ$

Answer 75° [2]

(iii) angle ABC,

$\angle ABC = 180^\circ - \angle CDA$ ($\angle$s in opposite segment)

$= 180^\circ - 75^\circ$

$= 105^\circ$

Answer 105° [2]

(iv) angle TDC.

$\angle ODT = 90^\circ$ (tangent $\perp$ radius)

$\angle TDC = 90° - 75°$

$= 15°$

Answer 15° [2]

(b) It is given that angle BXA = angle BCA.

Explain whether point X should lie inside, on or outside the circle.

Answer

Since $\angle BXA = \angle BCA$, the two angles must be in the **same segment**.

Hence X should lie **on** the circle.

OR

X should lie outside the circle, such that

$BC = BX$ and CAX lie on a straight line

(or triangle BCX is isosceles). [2]

(c) "A circle with diameter AC can be drawn through the points A, B and C."

Determine, with clear explanation, if the above statement is true or false.

Answer

$\angle ABC = 105° \neq 90°$

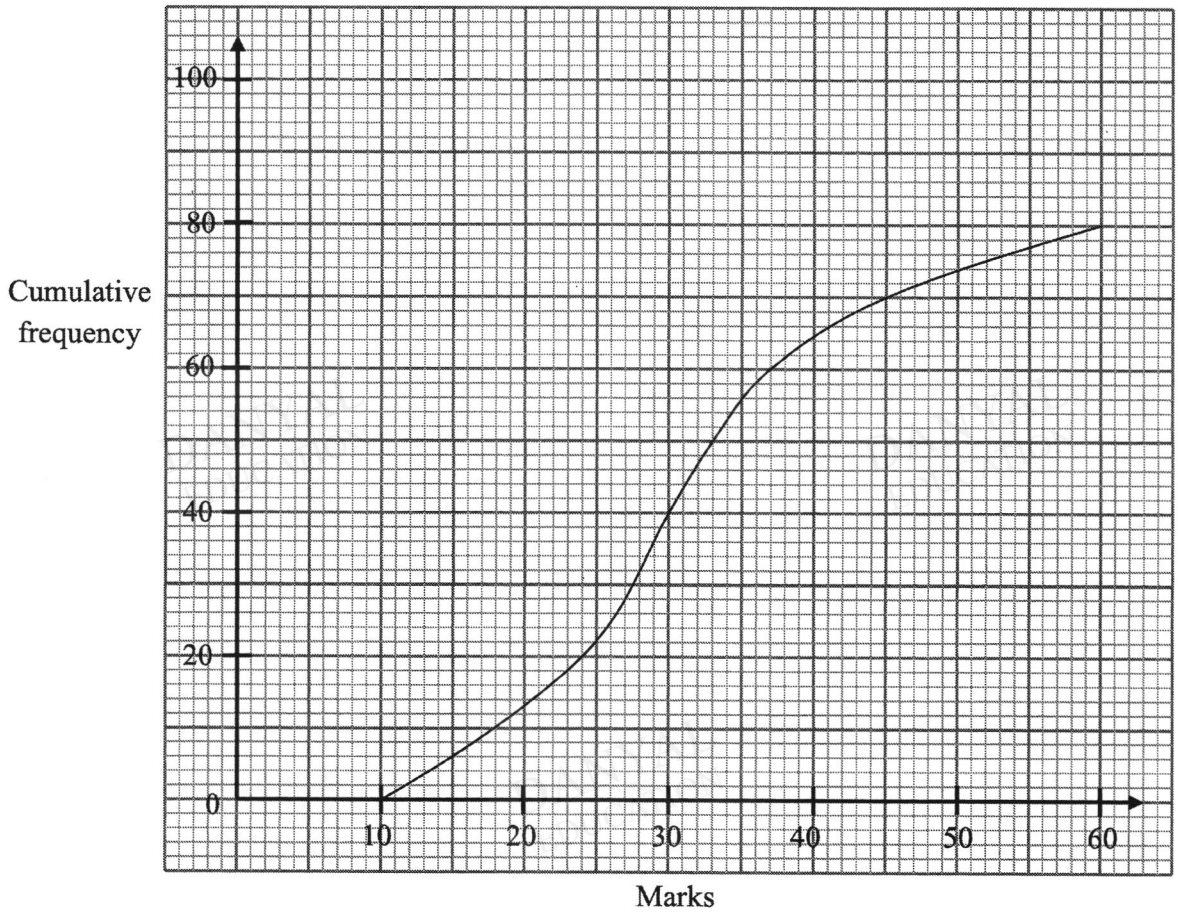
Since angle property right angle in a semicircle is not fulfilled,

AC cannot be the diameter of the circle.

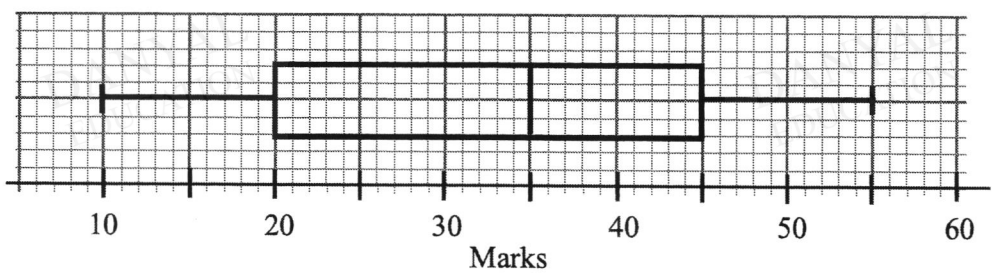
The statement is false.

[2]

9 (a) The cumulative frequency graph shows the distribution of the scores of a Geography test (Test 1) taken by 80 students. Test 1 was marked out of 60.



This box-and-whisker plot represents the distribution of the scores of the same group of students for another Geography Test (Test 2). Test 2 was marked out of 60.



(i) Use the two diagrams to complete this table for the two tests.

Test	Lower Quartile	Median	Upper Quartile	Interquartile range
1	**24**	30	**37**	**13**
2	20	**35**	45	25

[3]

To obtain a distinction for Test 1 and Test 2, a student needs to score at least x marks.

(ii) If 12.5% of the students scored distinction for Test 1, find x.

87.5% scored $< x$ marks
87.5% of 80 = 70 students
$x = 45$

Answer 45 [2]

(iii) "There is a higher proportion of students who scored distinction for Test 1."

Do you agree with this statement? Give a reason for your answer.

Answer
I disagree with the statement.

The upper quartile for Test 2 is 45 marks, which means that 25% of students scored distinction for Test 2, which is more than that of Test 1 (12.5%). A higher proportion of student scored distinction for Test 2 instead of Test 1. [2]

(b) The table below summarises the speeds of 95 cars on a stretch of road.

Speed (x km/h)	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$	$65 < x \leq 75$	$75 < x \leq 85$
Number of cars	13	29	40	8	5

Calculate an estimate of

(i) the mean speed,

Answer 56.1 km/h [1]

(ii) the standard deviation.

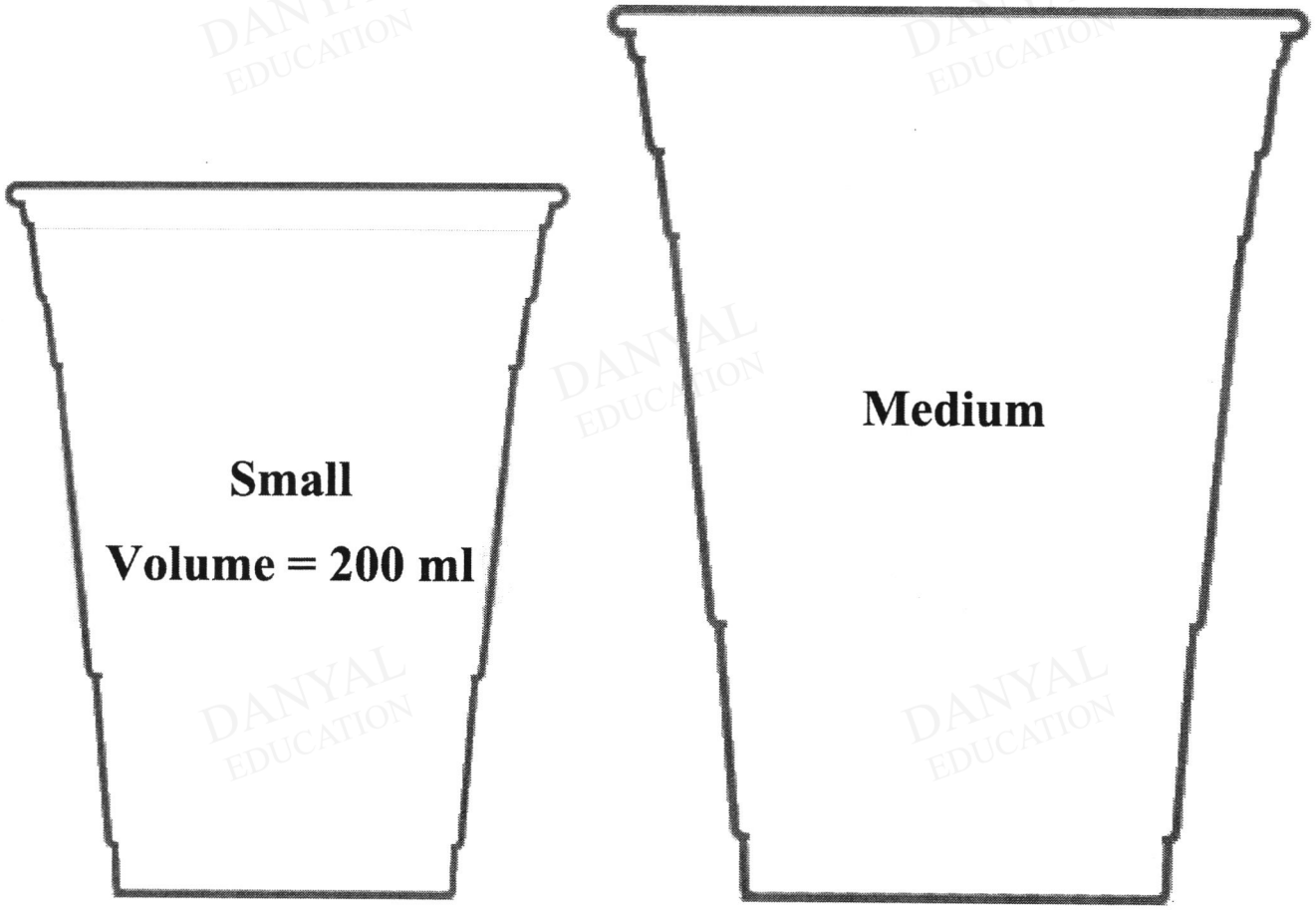
Answer 9.98 [1]

10 DeBest Bubble Tea Shop purchases their plastic cups from Bubble Planet Supplies.
Below is a pamphlet on the dimensions of the plastic cups.
Due to printing problems, some information are missing.

BUBBLE PLANET SUPPLIES

Picture shows actual size of cup

Item type	Plastic Cup
Sizes Available	Small, Medium
Uses	Drinks (Hot/Cold)



(a) Estimate the heights of the small and medium cup.

Answer Height of small cup : ..... 9.9 ..... cm

Height of medium cup : ..... 12.4 ..... cm [1]

(b) Given that the two cups are geometrically similar, find the volume of the medium cup.

$$\left(\frac{12.4}{9.9}\right)^3 = \frac{V_M}{200}$$

$$V_M = \left(\frac{12.4}{9.9}\right)^3 \times 200$$

$$= 392.99...$$

$$= 393 \text{ ml (3 sig.fig)}$$

Answer ..... 393 .....ml [2]

DANYAL EDUCATION

(c) Allison is planning to order a drink from DeBest Bubble Tea Shop.
She has read the following health advice:

> To avoid developing diabetes, the Health Promotion Board (HPB) recommends a daily calorie intake from sugary food (e.g. a cup of bubble tea) of no more than 10% of one's daily energy intake.

She then finds the following information on the website of DeBest Bubble Tea Shop.

Table 1

Sugar Level
Quarter Sugar (25%)
Half Sugar (50%)
Less Sugar (75%)
Full Sugar (100%) *

* A medium-sized cup of drink with full sugar (100%) contains 95 ml of sugar syrup.

Table 2

	Approximate amount of calories
Green tea	85 calories per 500 ml
Milk tea	110 calories per 500 ml
Black tea	70 calories per 500 ml
Sugar syrup	50 calories per 15 ml
Honey	60 calories per 15 ml

Note : A cup of flavoured tea is made by adding sugar syrup according to customers' preferred sugar level and topping up the remaining amount with flavoured tea.

For customers with daily energy intake between 2400 and 2900 calories

Healthier Choice Option

Medium-sized milk tea with 75% sugar level

Allison's daily energy intake is about 2800 calories.
She decides to select the healthier choice option drink and thinks that she will meet the daily calorie intake as recommended by HPB.

Is she correct?
Justify your decision with calculations.

Writing space for Q10 (c)

Answer:

volume of medium cup = 393 ml

Amount of sugar syrup in 75%

$= 0.75 \times 95$

$= 71.25$ ml

Amount of milk tea

$= 393 - 71.25$

$= 321.75$ ml

Amount of calories in :

$$\text{Sugar syrup} = \frac{71.25}{15} \times 50$$
$$= 237.5$$

$$\text{milk tea} = \frac{321.75}{500} \times 110$$
$$= 70.785$$

total number of calories from 1 cup of bubble tea

$= 237.5 + 70.785$

$= 308.285$

$= 308$ calories (3 sig.fig)

Allison's recommended daily calorie intake from sugary food

$$= \frac{10}{100} \times 2800$$

$= 280$

since $308 > 280$, Allison will not meet the recommendation by HPB.

She is not correct.

[7]