Name:_____ Class_____ Index No____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

MATHEMATICS **4048/1**

Paper 1
Candidates answer on the question paper.
No additional materials are required.

Date: 28 Aug 2020
Time: 0800 – 1000 hours
Duration: 2h

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper has a total of 23 pages.

Setter: Mr KH Chiam **[Turn over]**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

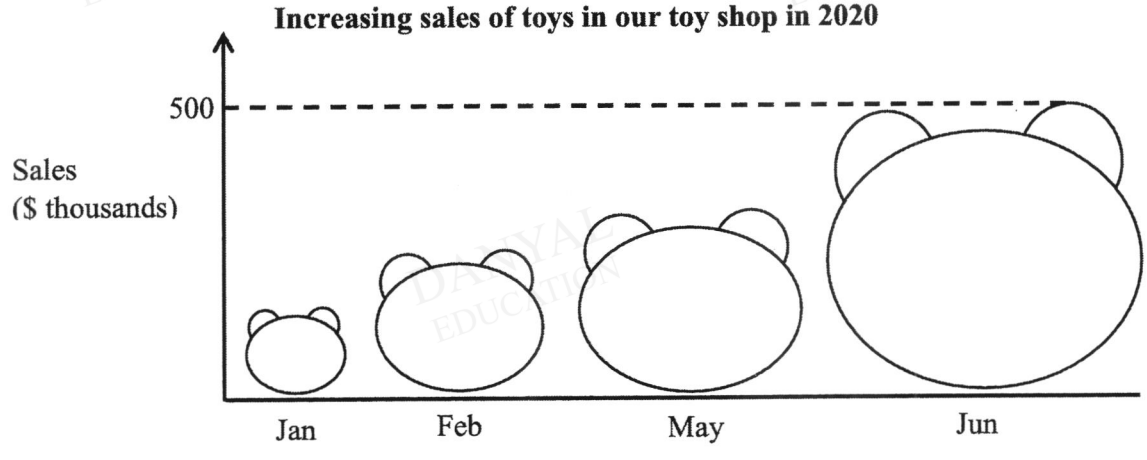
$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions

1 The number of beans in a bowl is given as 120, correct to the nearest ten.
Write down the minimum number of beans that could be in the bowl at this time.

Answer _____[1]

2 A toy shop put up this chart.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

_____

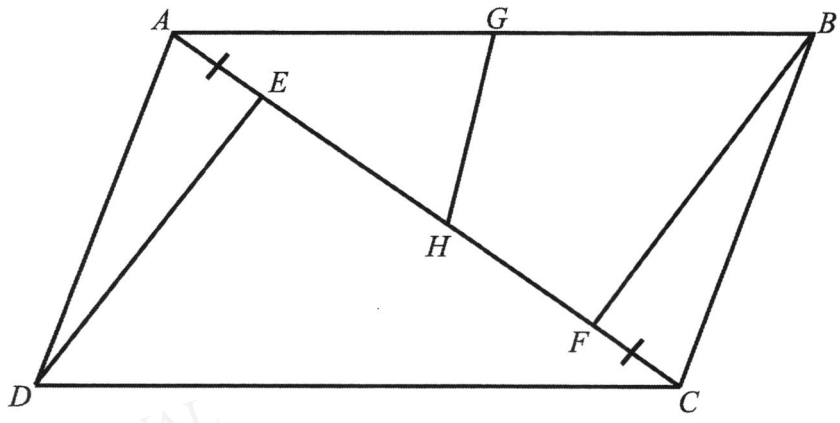
_____

_____

_____ [2]

3 For the parallelogram $ABCD$, $AE = CF$. DE is parallel to BF. G is the mid-point of AB.

a. Show that ΔDAE is congruent to ΔBCF.



[2]

b. Triangle AGH is similar to triangle ACB, $AB = 2$ m and $AC = 2.1$ m,
John claims that the length of AH is half of the length of AC. Using appropriate calculations, comment on the correctness of John's claim. [3]

4 (a) Express 540 as a product of its prime factors.

Answer _____ [1]

(b) The number $\frac{540p}{q}$ is a perfect square.

p and q are prime numbers.

Find the smallest value of p and the corresponding value of q.

Answer p = _____ [1]

q = _____ [1]

5 Express $8\frac{3}{4}\%$ as a

(a) decimal,

(b) fraction in its simplest form.

Answer *(a)* _____ [1]

(b) _____ [1]

6 A farm rears chickens and sheep.
Each chicken has 2 legs and each sheep has 4 legs.
The total number of legs is 100 while the total number of heads is 35.
Let the number of chickens be c and the number of sheep be s.

Solve for the values of c and s.

Answer c = _____

s = _____[3]

7 A bag contains 10 red marbles, 3 blue marbles and 2 white marbles.
Ethan picked two marbles at random.
Calculate the probability that he picked at least 1 white marble.

Answer _____[2]

8 (a) Solve the inequalities $2 \leq 3x + 2 < 17$.

Answer _____ [2]

(b) List all the prime numbers such that $2 \leq 3x + 2 < 17$.

Answer _____ [1]

9 I is inversely proportional to the square of r.

(a) When $r = \frac{1}{5}$, $I = 500$. Find I when $r = \frac{1}{10}$.

Answer _____[2]

(b) When the value of r is tripled, find the percentage decrease in the value of I.

Answer _____% [2]

10 Find **two pairs of numbers**, all numbers being smaller than 1400, that have a lowest common multiple of 5400 and a highest common factor of 90.

Answer _____ and _____ [3]

11 (a) Factorise $4a^2 + 12ab + 9b^2$

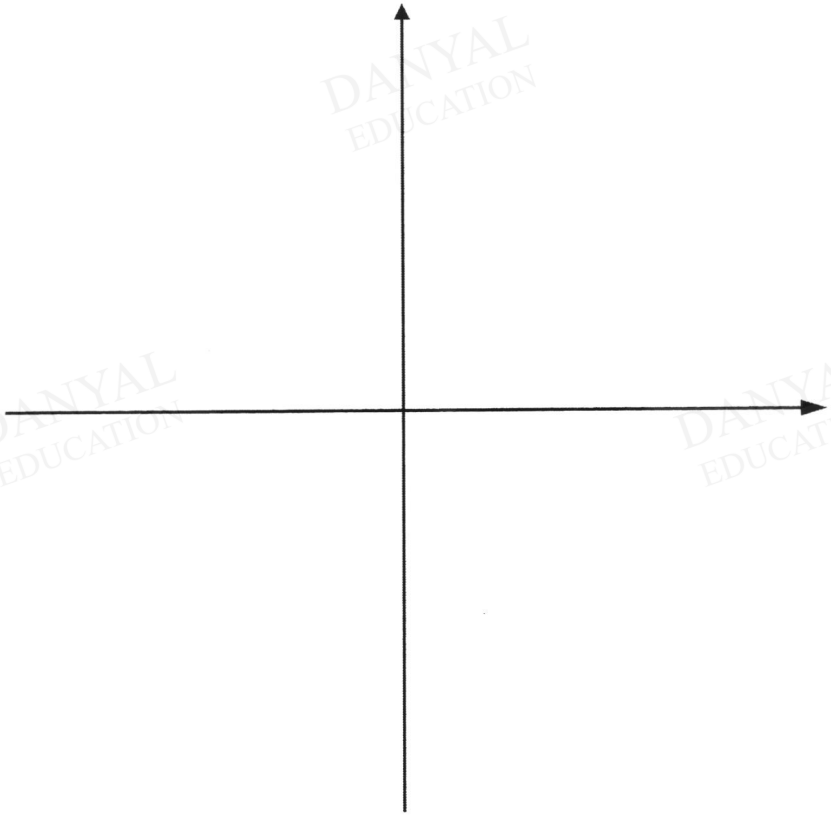
Answer _____ [1]

(b) Hence, completely factorise $4a^2 + 12ab + 9b^2 - 1$

Answer _____ [2]

(c) n is a positive integer.
Show that, for all n, $(3n+1)^2-(3n-1)^2$ is a multiple of 4. [2]

12 Sketch the graph of $y=-(x+3)^2+5$ on the axes below.
Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the curve. [3]

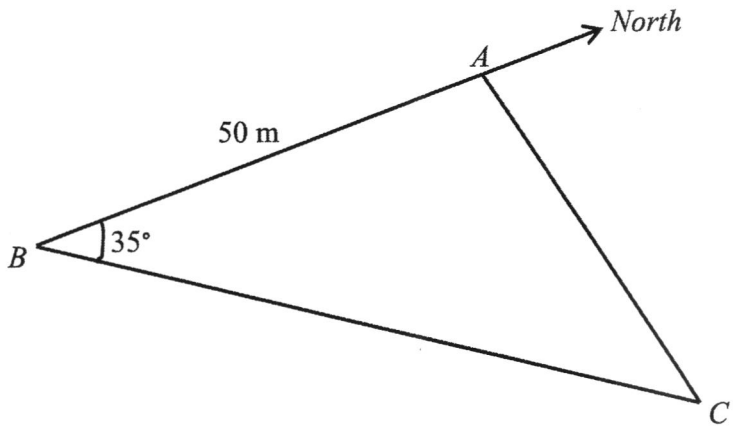


13. The sum of the first n terms of a sequence is given by $2n^2 - n$.

Find the 10^{th} term of this sequence.

Answer _____[2]

14



The diagram shows the positions of three points, A , B and C. Angle $ABC = 35°$. Point A is due north of point B. The bearing of point C from point A is 80°. AB is 50 m.

(a) Find the bearing of point B from point C.

Answer _____ [1]

(b) Calculate the length AC.

Answer _____ m [2]

15 Mary weighed 3 pet kittens.
The mean mass of the kittens was 200 grams.
The standard deviation of the masses of the kittens was 10 grams.

The scales used by Mary were found to be inaccurate.
The correct mass of each kitten was 20 grams less than Mary recorded.

(a) Write down the correct value for the mean and standard deviation (SD).

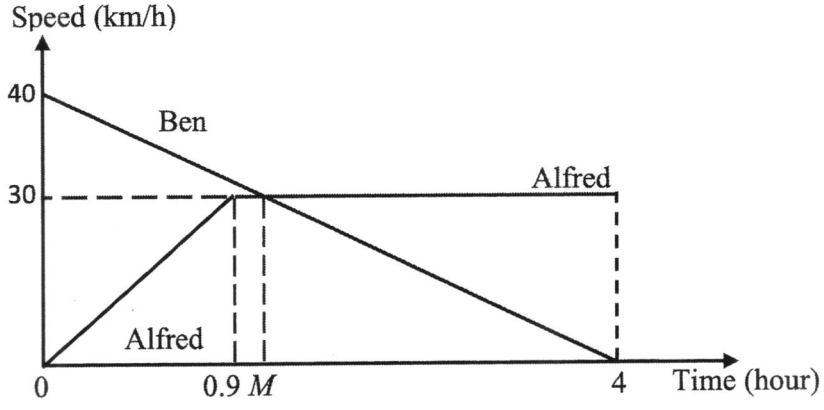
Answer Mean = _____ g[1]

SD = _____ g[1]

(b) After a number of years, the mass of each of the 3 kittens are tripled. Calculate the new standard deviation.

Answer SD = _____ g[1]

16 The speed-time graph shows the journeys for two drivers, Alfred and Ben. They travelled on the same road. Ben drove past Alfred at the time when Alfred started his journey.



(a) Charles commented that Alfred will meet Ben at time = M hour since the graphs intersect at time = M hours. Is Charles correct ? Explain.

_____

_____

_____

_____ [2]

(b) Calculate the value of M.

Answer _____ h [2]

17 $\xi = \{integers\ x :\ 2 \le x < 10\}$

$E = \{even\ numbers\}$

$M = \{multiples\ of\ 3\}$

$F = \{factors\ of\ 48\}$

(a) List the element(s) in

(i) $M \cap E$

Answer _____[1]

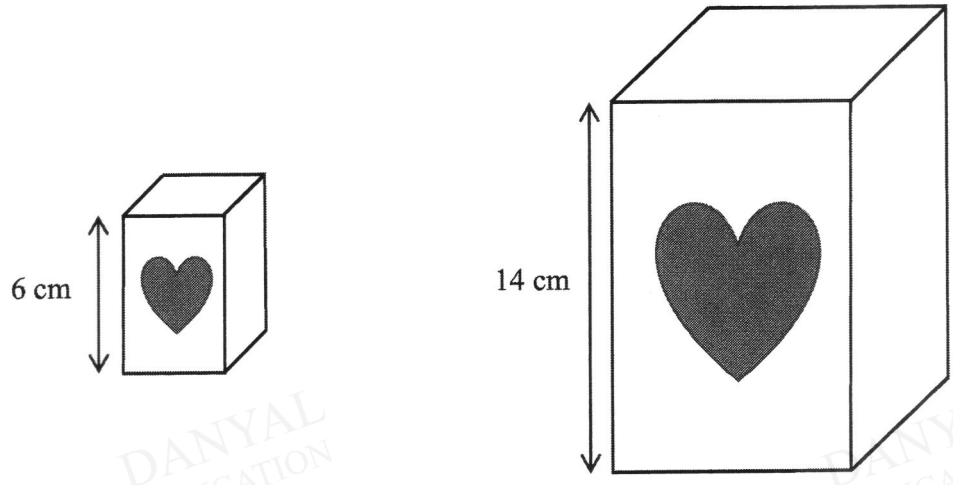
(ii) $(M \cup F)'$

Answer _____[2]

18 Cindy deposited $20000 in a bank account which offered an interest of 1.2% per annum compounded monthly. She withdrew her money after 1 year 6 months. Calculate the amount of interest earned.

Answer $ _____[2]

19. The diagram shows two geometrically similar figures made of the same material.
The length of the smaller figure is 6 cm.
The length of the larger figure is 14 cm.



(a) The area of the heart in the larger figure is 15 cm^2.
Calculate the area of the heart in the smaller figure.

Answer _____ cm^2 [2]

(b) If the mass of the smaller figure is 200 grams, calculate the mass of the larger figure.

Answer _____ g [2]

(c) The smaller figure is sold for \$2.50 while the larger figure is sold for \$ 25.50. Which figure will be a better buy? Explain.

Answer _____ [2]

20 Two cafe outlets, A and B sells coffee, tea and cakes.
Both outlets charge $ 2.50 for a cup of tea, $3.50 for a cup of coffee and $3.00 for a piece of cake.

(a) On a Saturday, Cafe A sells 100 cups of coffee, 70 cups of tea and 50 pieces of cake.
Cafe B sells 120 cups of coffee, 90 cups of tea, and 40 pieces of cake.

Represent this information in a 2×3 matrix $\boldsymbol{Q}$.

Answer Q = $\begin{pmatrix} & & \end{pmatrix}$ [1]

(b) By multiplying Q with another matrix, calculate the total amount collected by each of the two outlets respectively.

Answer: Outlet A: $ _____

Outlet B: $ _____ [2]

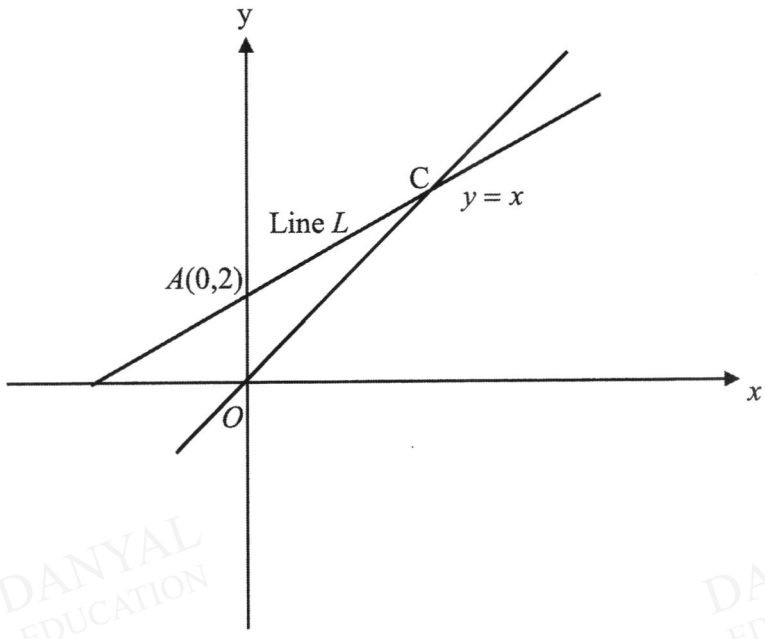
(c) Both outlets decided to give a special discount of 20% for coffee, 10% for tea and 10% for cakes.
By the multiplication of two matrices, calculate the new prices of a cup of coffee, of a cup of tea and of a piece of cake.

Answer: New price of a cup of coffee : $_____,

New price of a cup of tea : $_____,

New price of a piece of cake : $_____[2]

21



The line L passes through the point A (0,2) and meets the line $y = x$ at point C. O is the origin. The area of OAC is 3 units^2.

(a) What is the equation of line L ?

Answer _____[2]

(b) There is a point B such that OC is the line of symmetry of the figure $OACB$.

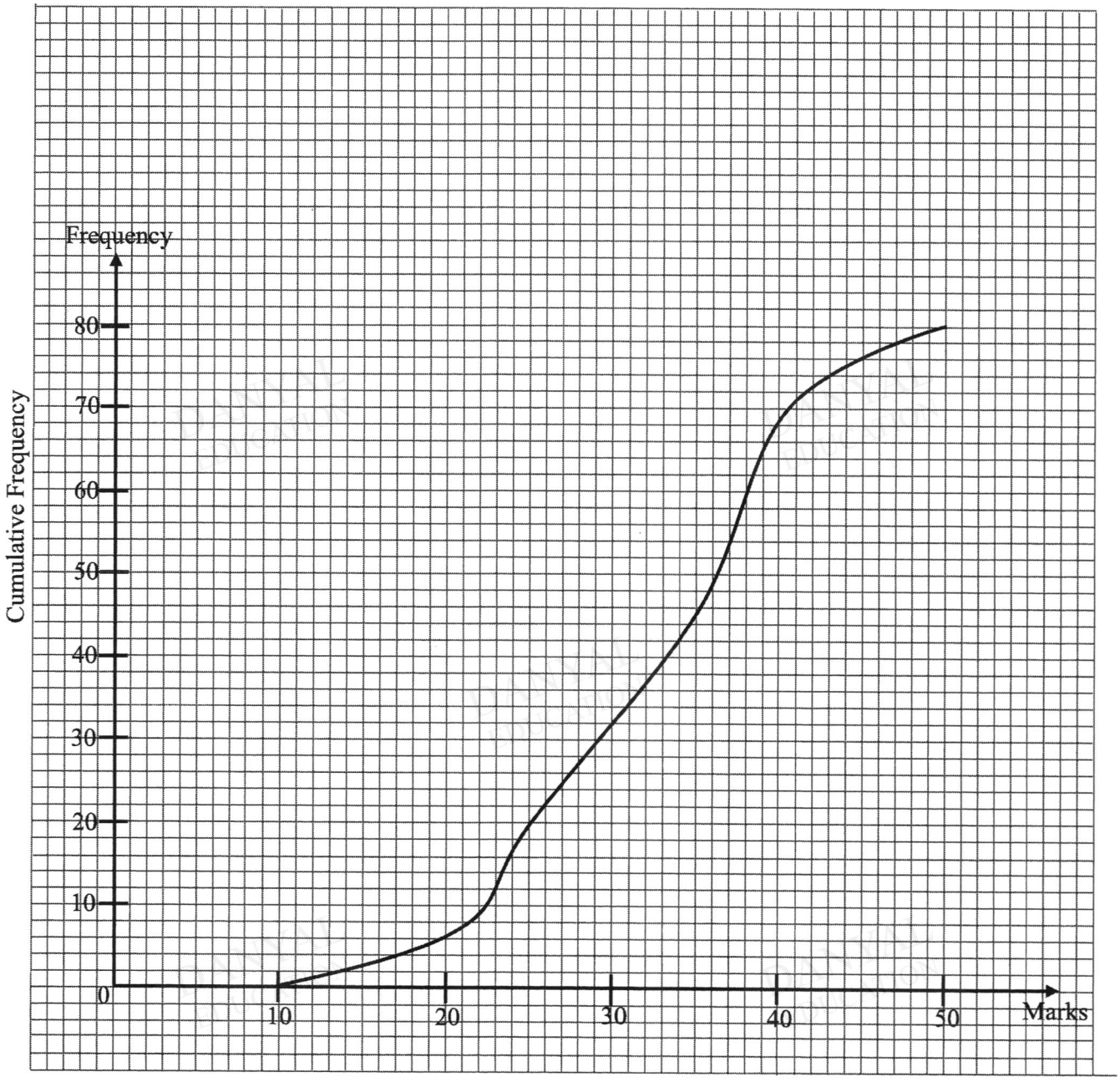
(i) What is the name of quadrilateral $OACB$?

Answer _____[1]

(ii) State the coordinates of the point B.

Answer B (,) [1]

22. The cumulative frequency represents the marks scored by a group of 80 students.



(a) Use the graph to find the number of students with a score of more than 26 marks.

Answer _____ [1]

(b) Calculate the range of the marks scored.

Answer _____[1]

(c) If another group of 80 students have the same median mark with a smaller interquartile range, how would the cumulative frequency graph of this second group of students differ from this cumulative frequency graph ?

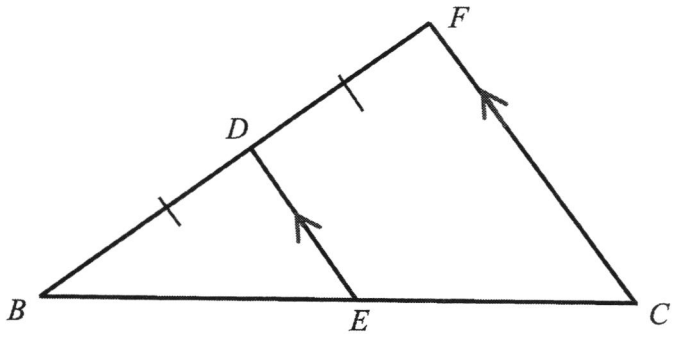
_____

_____ [1]

23 A building is drawn to a scale of 1: n.
The actual area of the building floor is 500 m^2 while the area of the building on the map is 20 cm^2.
Find the value of n.

Answer n = _____[2]

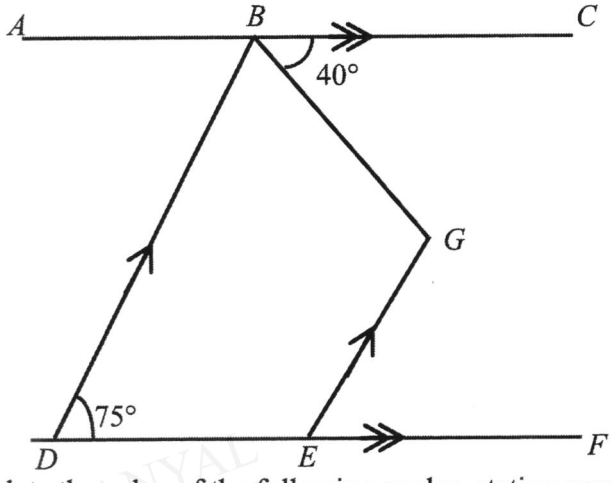
24 In the triangle BFC, D is the mid - point of BF. DE is parallel to FC.



Show that ΔBDE is similar to ΔBFC.

[2]

25. In the diagram, the straight line ABC is parallel to the line DEF.
The straight line DB is parallel to the line EG. $\angle BDE = 75°$ and $\angle CBG = 40°$.



Calculate the value of the following angles, stating your reasons clearly.

a. $\angle DBG$

[2]

b. $\angle BGE$

[1]

26. Explain whether a kite can be a rhombus.

[2]

----------END OF PAPER----------

Name:_____ Class_____ Index No____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

MATHEMATICS	**4048/2**
Paper 2	Date: 25 August 2020
Candidates answer on the question paper.	Time: 0800 – 1030
Additional materials: Graph paper	Duration: 2h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper has a total of 22 pages.

Setter: Chong Lin Lin **[Turn over]**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Simplify $\frac{2x}{x^0} \div \left(\frac{2y}{x}\right)^{-2}$, leaving your answer in positive indices.

Answer _____ [3]

(b) Express y in terms of p and q, given that $\frac{1}{q} = \frac{2}{y} + p$.

Answer _____ [3]

2 **(a)** Express $\frac{1}{x-3} - \frac{6}{x^2-9}$ as a single fraction in its simplest form.

Answer _____ [2]

(b) **(i)** Express $y = x^2 + 4x - 5$ in the form $y = (x+a)^2 + b$, where a and b are constants.

Answer _____ [2]

(ii) Write down the equation of the line of symmetry of the graph of $y = x^2 + 4x - 5$.

Answer _____ [1]

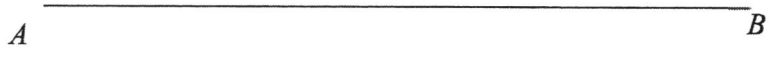
3 The line AB is shown below.

(a) Construct an equilateral triangle ABC. [1]

(b) Construct the perpendicular bisector of AB. [1]

(c) Construct the angle bisector of $\angle CAB$. [1]

(d) Mark clearly a possible point that is inside the triangle, equidistant from AC and AB, and is nearer to A than B. Label this point P. [1]



(e) The point Q is such that $\angle ACQ = 120°$ and $\frac{BQ}{BA} = 0.9$.

Find two possible positions of Q. Label these point Q_1 and Q_2.

Measure and write down the length AQ (where Q can be any of the two points).

Answer _____ cm [3]

4 The exterior angle of a regular n-sided polygon is 14° more than the exterior angle of a regular $(2n + 6)$-sided polygon.

(a) Write down and simplify an expression, in terms of n, each exterior angle of the $(2n + 6)$-sided polygon.

Answer _____ ° [1]

(b) Write down an equation in n and show that it reduces to $14n^2 - 138n - 1080 = 0$.

[3]

(c) Solve the equation $14n^2 - 138n - 1080 = 0$.

Answer _____ [2]

(d) Explain why one of the solutions above is rejected.

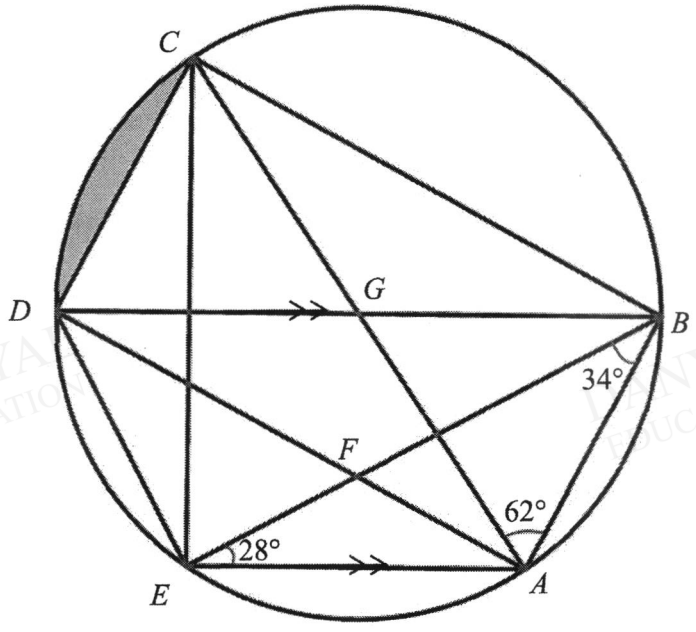
Answer

[1]

(e) Hence, find the value of each interior angle of the n-sided polygon.

Answer _____ ° [2]

5 The points A, B, C, D and E lie on the circle. $\angle AEB = 28^\circ$, $\angle CAB = 62^\circ$ and $\angle ABE = 34^\circ$. AE is parallel to BD.



(a) Find two other angles that are 62°, stating the angle property.

Answer _____ [2]

(b) Explain whether AC is the diameter of the circle.

Answer

[2]

(c) Find the following angles, stating all the angle properties.

(i) $\angle EBD$

(ii) $\angle DEA$

Answer **(i)** _____ °

(ii) _____ ° [3]

(d) Explain why G is the center of the circle.

Answer

[2]

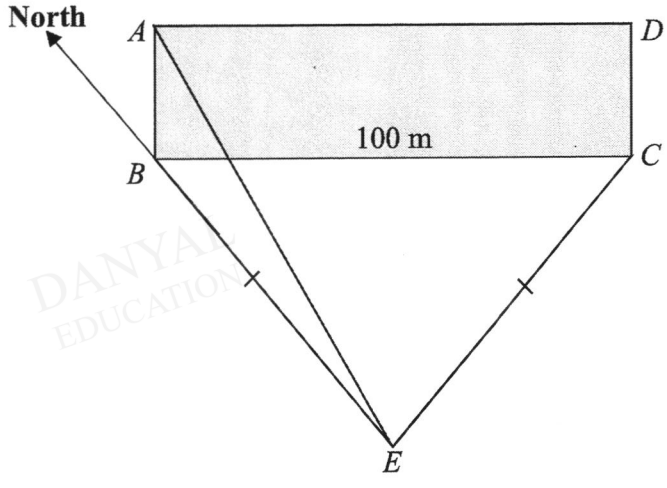
(e) If the radius of the circle is 7 cm, find the area of the shaded region.

Answer _____ cm^2 [3]

6 In the diagram, *ABCD* represents a vertical cliff face. The bottom of the cliff, *BC*, is 100 m and is at sea level. A boat is in the sea at *E*.

B is due north of *E* and the bearing of *C* from *E* is 070°.

E is known to be equidistant from *B* and *C*.



(a) Find the bearing of *C* from *B*.

Answer _____ ° [2]

(b) Show that *BE* = 87.17 m.

Answer

[2]

(c) Find the area of ΔBCE, leaving your answer to the nearest square metre.

Answer _____ m^2 [2]

(d) It is given that the angle of depression of E from A is 20.2°. Find the height of the cliff BA, leaving your answers in 3 significant figures.

Answer _____ m [2]

(e) P is a point on BC such that $\frac{Area\ of\ \Delta BEP}{Area\ of\ \Delta CEP} = \frac{3}{2}$.

Find the length of CP.

Answer _____ m [2]

7 Part of a pattern of numbers is shown in the table below.

		C_1	C_2	C_3	C_4	C_5	C_6
Row	1	2	3	4	5	6	7
Row	2	8	9	10	11	12	13
Row	3	14	15	16	17	18	19
Row	4	20	21	22	23	24	25
Row	5	26	27	28	29	30	31
*	*	*	*	*	*		
*	*	*	*	*	*		
Row	n		x	y			
Row	$n+1$		s	t			

(a) Express y, s and t in terms of x.

Answer $y =$ _____

$s =$ _____

$t =$ _____ [2]

(b) Show that the difference between xt and ys is always 6.

Answer

[2]

(c) Express x in terms of n.

Answer $x =$ _____ [1]

(d) Explain why xy would be in column number C_5, for any value of n.

Answer _____ [2]

(e) From the above, which column would ys lies in (C_1, C_2, C_3, C_4, C_5 or C_6)?

Answer _____ [1]

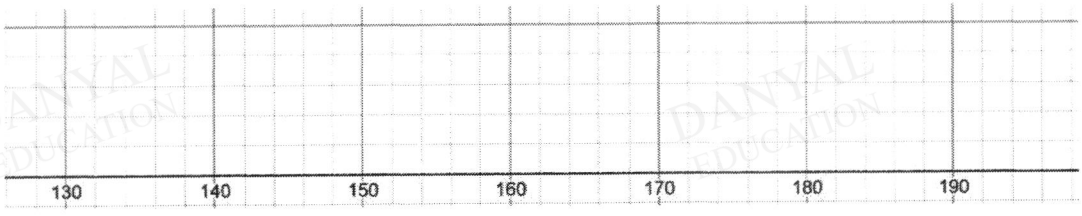
8 The ordered stem-and-leaf diagram records the height of 21 plants in Plantation *A*.

Stem	Leaf							
13	0	7	8	9	9			
14	1	1	2	6	7	8	9	9
15	0	2	3	5	6	8	8	8

Key: 14 | 8 means 148 cm

(a) Represent the above information in a box and whisker.

Answer



[3]

(b) State the modal height.

Answer _____ cm [1]

(c) Calculate the percentage of plants having a height of less than 150 cm.

Answer _____ % [1]

(d) Two plants are selected at random from Plantation A.

The students are asked to calculate the probability of selecting a plant that is less than 150 cm and another plant that is at least 150 cm.

(i) Alice says that the above probability is calculated using $\frac{8}{21}$ and $\frac{13}{21}$.

Explain why she is wrong.

Answer

[1]

(ii) Calculate the above-mentioned probability.

Answer _____ [2]

(e) 30 plants are planted in Plantation B on the same day. The range of height is 24 cm, the interquartile range is 10 cm and the median is 152 cm. Compare the height in Plantation A and B in two ways.

Answer

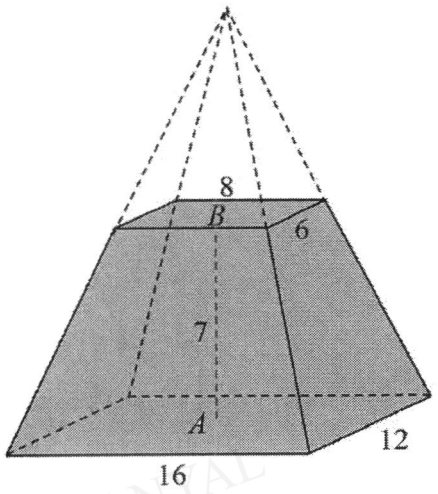
[2]

9 The frustum shown in the diagram is a metal solid obtained by cutting off the top part of a pyramid horizontally.

The dimension of the top rectangular plane is 8 cm by 6 cm.

The dimension of the bottom rectangular plane is 16 cm by 12 cm.

The height, AB, of the frustum is 7 cm where A is the center of the base.



(a) Explain why the height of the pyramid that is cut off is 7 cm.

Answer

[2]

(b) Show that the volume of the frustum is 784 cm^3.

Answer

[2]

(c) Assuming negligible loss in the process of melting, how many spherical balls (of radius 2 cm) can be formed if the frustrum is melted?

Answer _____ [2]

(d) 3 of the above frustums are to be packed into a rectangular box. To save cost, the seller should keep the amount of material of the rectangular box to the minimum.

Albert claims that the dimension should be 30 cm by 16 cm by 7cm (Box *A*).
Betty claims that the dimension should be 38 cm by 12 cm by 7cm (Box *B*).
Carol claims that the dimension should be 21cm by 16 cm by 12cm (Box *C*).

(i) Explain which box will be too small.

Answer

[1]

(ii) Which of the other two boxes would use the minimum amount of material?

Answer _____ [3]

10 A group of researchers brought 180 koalas to an island and tracked the number of koalas over a few years.

x (years)	0	0.5	1	2	3	4	5	6	6.5	7	7.5
y (no of animals)	180	177	175	172	p	172	175	180	183	187	191

(a) Using a scale of 2 cm on both axes to represent 1 year on the x-axis and 2 animals on the y-axis for $170 \leq y \leq 192$, plot the values given and join them with a smooth curve.

[3]

(b) Use your curve to estimate the value of p, corrected to nearest integer.

Answer _____ [1]

(c) By drawing a tangent, find the gradient at $x = 6$.

Answer _____ [2]

(d) The number of koalas (y) turns out to be related to an unknown quadratic function $y = (x + a)^2 + b$. Find the value of a and of b.

Answer $a =$ _____ $b =$ _____ [2]

(e) The researchers also brought 180 penguins to the same island and noticed that the number of penguins increased at a steady rate of 1 per year.

(i) Write down an equation connecting the number of penguins, y, and the number of years, x.

Answer _____ [1]

(ii) On the same graph in **(a)**, draw the graph of the equation in **(e)(i)** and state the point of intersection (x, y).

Answer _____ [2]

(iii) What would this point of intersection represent in this research?

Answer

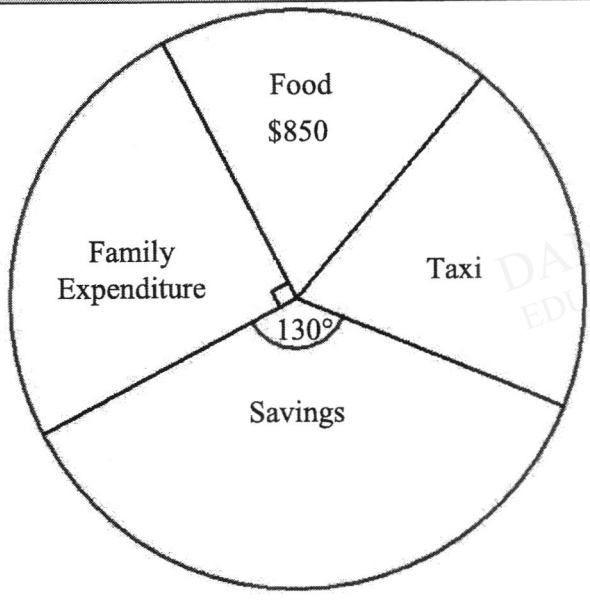
[1]

11 Charlotte takes the taxi every day. She is thinking of buying a car and wants to pay off the car loan as soon as possible. The tables below give the information Charlotte can use.

Charlotte's financial situation	
Money set aside to buy a car	\$50000
Monthly salary	\$4500

Information on the car and car loan	
Car Price	\$96000 Customer must take a minimum loan of 50% from the car dealer.
Loan Period (Number of years, x)	• $2 \leq x \leq 7$, x is an integer • Customer needs to decide on the loan period on the day he or she buys the car.
Interest rate of loan	2.45% per annum simple interest

Information on how Charlotte uses her monthly salary



(a) Determine the minimum loan amount.

Answer $ _____ [1]

(b) Charlotte calculated that if the loan period is 2 years and that she took a loan of $48000, her monthly car instalment will be $2098.00. Show that Charlotte is correct.

Answer

[2]

(c) Charlotte intends to use **no more than** her current taxi expenditure to pay for her monthly car instalment. Suggest the minimum loan period. Justify your answer with clear workings.

Answer _____ [7]

(d) Give a possible reason to explain why Charlotte's savings will be reduced even if her monthly instalment is the same as the current taxi expenditure.

Answer

[1]

End of paper

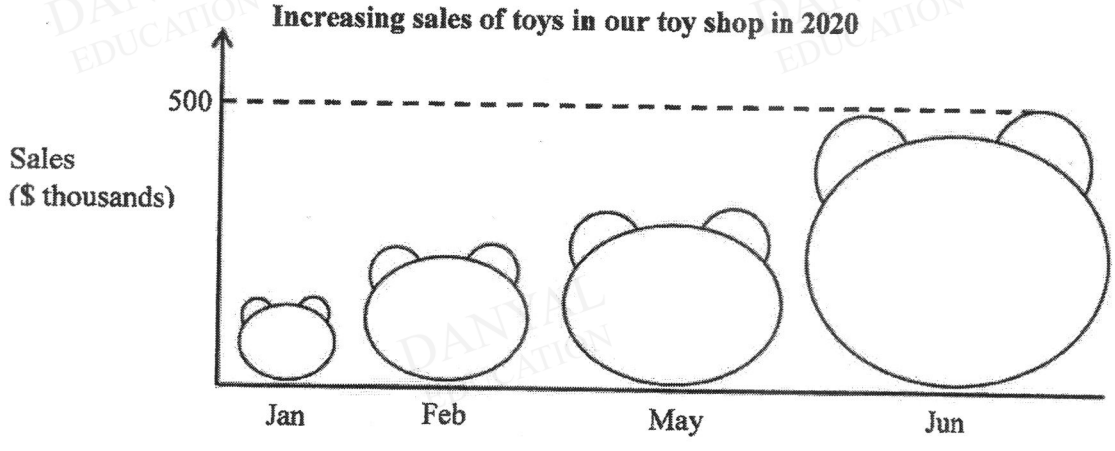
Answer **all** the questions

1 The number of beans in a bowl is given as 120, correct to the nearest ten.
Write down the minimum number of beans that could be in the bowl at this time.

115 – [1]

Answer _____ [1]

2 A toy shop put up this chart.



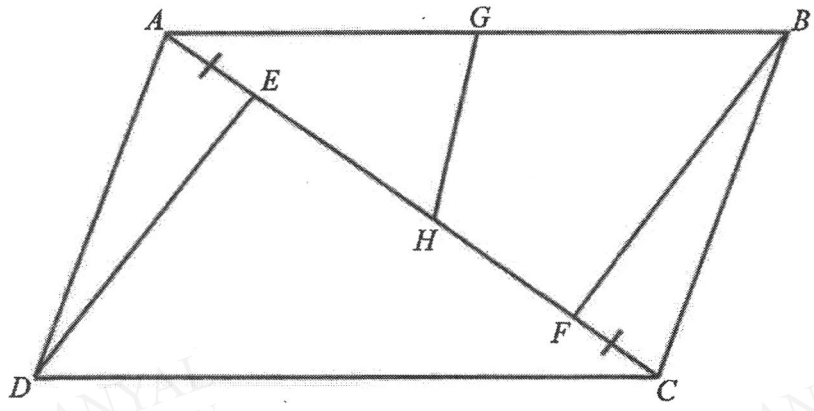
State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

There is no indication of sales figures for March, April and months after June [A1], there is incomplete data to support "Increasing sales ..." [A1] OR:

It is not known whether sales is proportional to height [2]
or area of shape for each month [A1],
readers may arrive at wrong sales figure [A1]

3 For the parallelogram $ABCD$, $AE = CF$. DE is parallel to BF. G is the mid-point of AB.

a. Show that ΔDAE is congruent to ΔBCF.



AD = CB (opp. sides of //gram)

$\angle DAE = \angle BCF$ (Alt. angles) — [1]

AE = CF (Given)

SAS Congruency test — [1]

[2]

b. Triangle AGH is similar to triangle ACB, $AB = 2$ m and $AC = 2.1$ m,
John claims that the length of AH is half of the length of AC. Using appropriate calculations,
Comment on John's claim as to whether he is right. [3]

$\angle CAB = \angle GAH$ (Common angle)

$\frac{AH}{AB} = \frac{AG}{AC}$ — [1]

$\frac{AH}{2} = \frac{1}{2.1}$

$AH = 0.95238$ — [1]

$\frac{1}{2} \times AC \neq \frac{1}{2} \times 2.1 = 1.05$m

$AH \neq 1.05$m

John's claim is not correct [1]

4 (a) Express 540 as a product of its prime factors.

$540 = 2^2 \times 3^3 \times 5$ —[1]

Answer $2^2 \times 3^3 \times 5$ [1]

(b) The number $\frac{540p}{q}$ is a perfect square.

p and q are prime numbers.

Find the smallest value of p and the value of q.

$$\frac{540p}{q} = \frac{2^2 \times 3^3 \times 5p}{q}$$

$$= \frac{2^2 \times 3^2 \times 3 \times 5p}{q}$$

$p = 5$, then $q = 3$

$p = 3$, then $q = 5$

Answer $p =$ 5 or 3 [1]

$q =$ 3 or 5 [1]

5 Express $8\frac{3}{4}\%$ as a

(a) decimal

(b) fraction in its simplest form

a) $\frac{8.75}{100} = 0.0875$ —[1]

b) $\frac{8.75}{100} = \frac{7}{80}$ —[1]

Answer *(a)* 0.0875 [1]

(b) 7/80 [1]

6 A farm rears chickens and sheep.
Each chicken has 2 legs and each sheep has 4 legs.
The total number of legs is 100 while the total number of heads is 35.
Let the number of chickens be c and the number of sheep be s.

Solve for the values of c and s.

$C + S = 35$ —(1)

$2C + 4S = 100$ —(2)

$C = 35 - S$ —(3)

$2(35 - S) + 4S = 100$

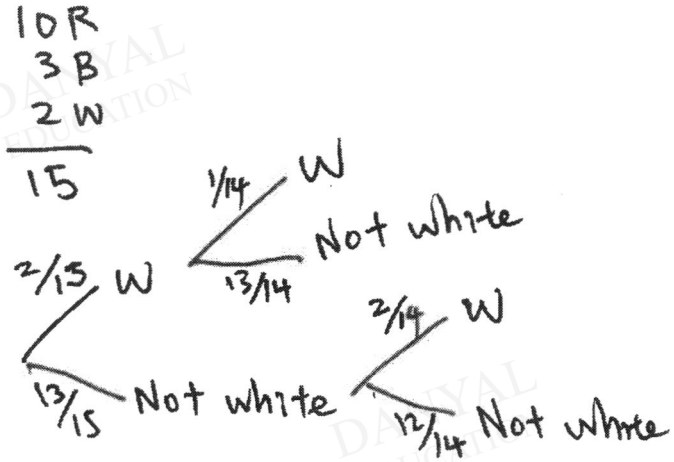
$S = 15$ —[1]

$C = 20$ —[1]

7 A bag contains 10 red marbles, 3 blue marbles and 2 white marbles.
Ethan picked two marbles at random.
Calculate the probability that there is at least 1 white marble.

$P(\text{atleast 1 white}) = 1 - \frac{13}{15} \times \frac{12}{14}$ — [1]

$= \frac{9}{35}$ — [1]



Answer 9/35 [2]

8 (a) Solve the inequalities $2 \leq 3x+2 < 17$.

$2 \leq 3x+2$ and $3x+2 < 17$

$0 \leq x$ and $x < 5$ —[1]

$0 \leq x < 5$ —[1]

Answer $0 \leq x < 5$ [2]

(b) List all the prime numbers such that $2 \leq 3x+2 < 17$.

2, 3 —[1]

Answer 2, 3 [1]

9 I is inversely proportional to the square of r.

(a) When $r = \frac{1}{5}$, $I = 500$. Find I when $r = \frac{1}{10}$.

$I = \frac{k}{r^2}$

$500 = \frac{k}{(\frac{1}{5})^2}$

$k = 20$ —[1]

$I = \frac{20}{(\frac{1}{10})^2}$

$I = 2000$ —[1]

Answer 2000 [2]

(b) When the value of r is tripled, find the percentage decrease in the value of I.

$I_0 = \frac{k}{r_0^2}$

$k = I_0 r_0^2$

$I_{new} = \frac{I_0 r_0^2}{(3r_0)^2}$

$= \frac{1}{9} I_0$ —[1]

% decrease $= 88\frac{8}{9}$ % —[1]

Answer $88\frac{8}{9}$ % [2]

10 Find **two pairs of numbers**, all numbers being smaller than 1400, that have a lowest common multiple of 5400 and a highest common factor of 90.

1st pair: $N_1 = 2 \times 3^3 \times 5^2 = 1350$

$N_2 = 2^3 \times 3^2 \times 5 = 360$

$Lcm = 2^3 \times 3^3 \times 5^2 = 5400$

$HCF = 2 \times 3^2 \times 5 = 90$

2nd pair: $n_1 = 2 \times 3^2 \times 5^2 = 450$

$n_2 = 2^3 \times 3^3 \times 5 = 1080$

Working [1m]

1st pair = 360, 1350 — [1]

2nd pair 450, 1080 — [1]

Answer 360, 1350 and 450, 1080 [3]

11 (a) Factorise $4a^2 + 12ab + 9b^2$

$(2a)^2 + 2(2a)(3b) + (3b)^2$

$= (2a + 3b)^2$ — [1]

Answer $(2a+3b)^2$ [1]

(b) Hence, completely factorise $4a^2 + 12ab + 9b^2 - 1$

$(2a+3b)^2 - 1^2$ — [1]

$= (2a+3b+1)(2a+3b-1)$ — [1]

Answer $(2a+3b+1)(2a+3b-1)$ [2]

(c) n is a positive integer.
Show that, for all n, $(3n+1)^2-(3n-1)^2$ is a multiple of 4. [2]

$(3n+1)^2-(3n-1)^2$

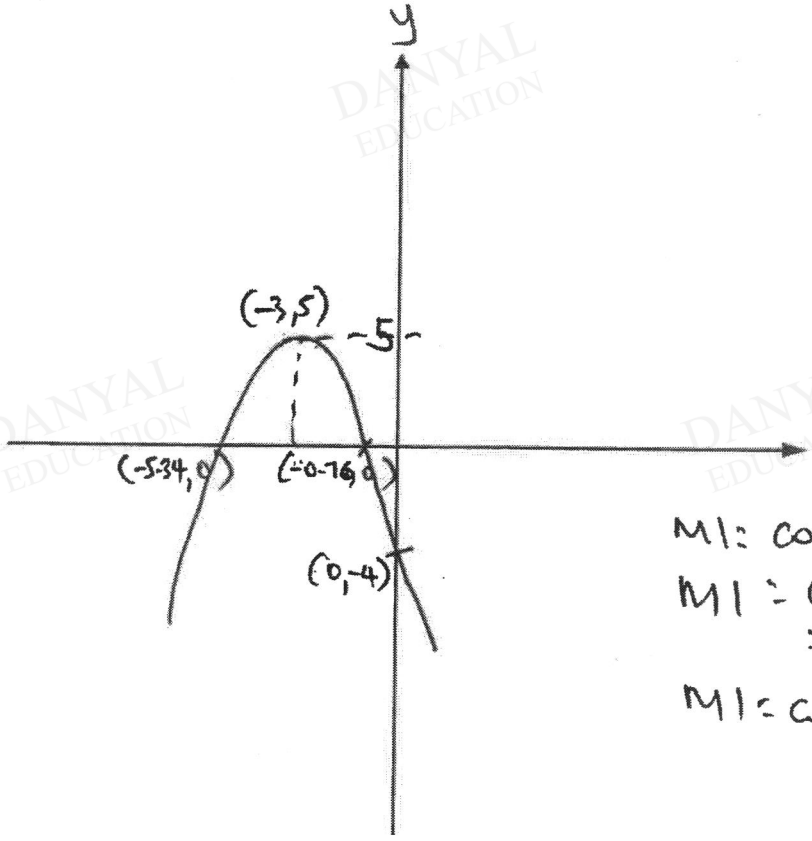
$=(3n+1+3n-1)[3n+1-(3n-1)]$

$=(6n)(2)$

$=12n$ — [1]

$=4(3n)$ is a multiple of 4. — [1]

12 Sketch the graph of $y=-(x+3)^2+5$ on the axes below.
Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the curve. [3]



M1: correct shape
M1: Correct x and y intercept
M1: Correct turning pt.

13. The sum of the first n terms of a sequence is given by $2n^2 - n$.

Find the 10th term of this sequence.

Sum of Term 1 $= 2(1)^2 - 1 = 1$

Sum of Term 2 $= 2(2)^2 - 2 = 6$

Sum of Term 3 $= 2(3)^2 - 3 = 15$

Sum of Term 4 $= 2(4)^2 - 4 = 28$

Term 1 $= 1$ — [1]

Term 2 $= 6 - 1 = 5$

Term 3 $= 15 - 6 = 9$

Term 4 $= 28 - 15 = 13$

Common difference is 4 — [1]

$T_n = 1 + 4(n-1)$

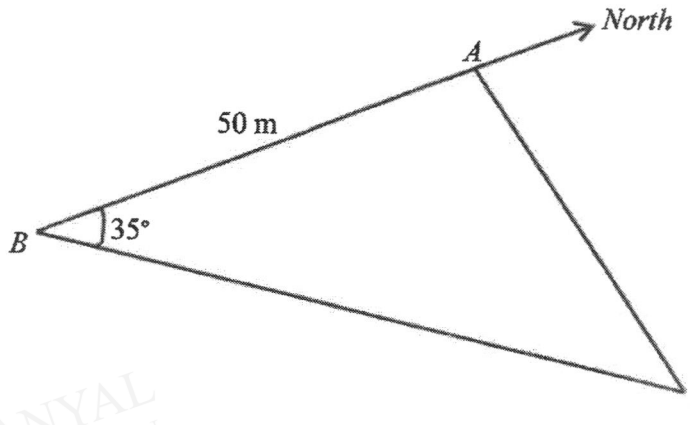
$T_n = 4n - 3$

$T_{10} = 4(10) - 3$

$= 37$ — [1]

Answer 37 [3]

14



The diagram shows the positions of three points, A , B and C. Angle $ABC = 35°$. Point A is due north of point B. The bearing of point C from point A is 80°. AB is 50 m.

(a) Find the bearing of point B from point C.

$\angle BCN_2 = 180° - 35° = 145°$

Bearing of point B from point C
$= 360° - 145°$
$= 215°$ — [1]

Answer 215° [1]

(b) Calculate the length AC.

$\angle BAC = 180° - 80° = 100°$

$\angle ACB = 180° - 100° - 35° = 45°$ — [0]

$\frac{50}{\sin 45°} = \frac{AC}{\sin 35°}$ — [1]

$AC = \frac{50}{\sin 45°} \times \sin 35°$

$= 40.6\text{m}$ — [1]

Answer 40.6 m [2]

15 Mary weighed 3 pet kittens.
The mean mass of the kittens was 200 grams.
The standard deviation of the masses of the kittens was 10 grams.

The scales used by Mary were found to be inaccurate.
The correct mass of each kitten was 20 grams less than Mary recorded.

(a) Write down the correct value for the mean and standard deviation (SD).

Correct mean = 200 - 20
= 180 g. — [1]

S.D. = 10 g. — [1]

Answer Mean = 180 g[1]

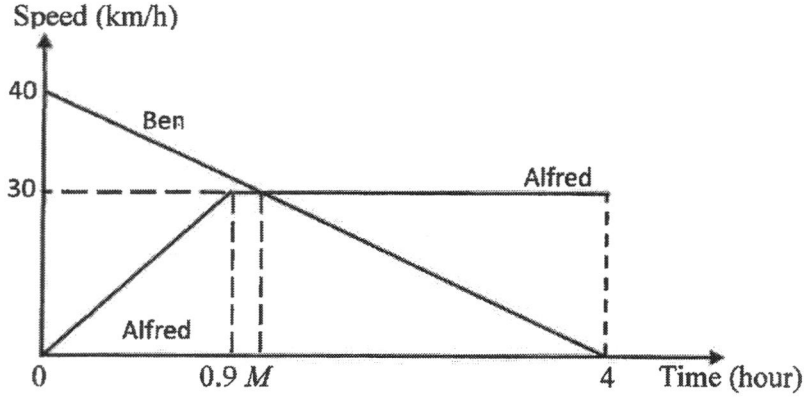
SD = 10 g[1]

(b) After a number of years, the mass of each of the 3 kittens are tripled. Calculate the new standard deviation.

New SD = 3 × 10
= 30g

Answer SD = 30 g[1]

16 The speed-time graph shows the journeys for two drivers, Alfred and Ben. They travelled on the same road. Ben drove past the point at the same time that Alfred started his journey.



(a) Charles commented that Alfred will meet Ben at time = M hour since the graphs intersect at time = M hours. Is Charles correct ? Explain.

No, [1] at time M, both vehicles have the same speed, but the distance covered need not be the same [1]

[2]

(b) Calculate the value of M.

$a_{Ben} = \frac{0-40}{4} = -10 km/h^2$ — [1]

$V_{Ben} = 40 - 10M$

$40 - 10M = 30$

$M = 1$ hour — [1]

Or $\frac{40}{4} = \frac{30}{4-M}$ — [1]

$M = 1$ hour — [1]

Answer 1 h [2]

17 $\xi = \{integers\ x : 2 \leq x < 10\}$

$E = \{even\ numbers\}$

$M = \{multiples\ of\ 3\}$

$F = \{factors\ of\ 48\}$

(a) List the elements in

(i) $M \cap E$

$M = \{3, 6, 9\}$

$E = \{2, 4, 6, 8\}$

$M \cap E = \{6\}$

Answer $\{6\}$ [1]

(ii) $(M \cup F)'$

$F = \{2, 3, 4, 6, 8\}$

$M \cup F = \{2, 3, 4, 6, 8, 9\}$

$(M \cup F)' = \{5, 7\}$

Answer $\{5, 7\}$ [2]

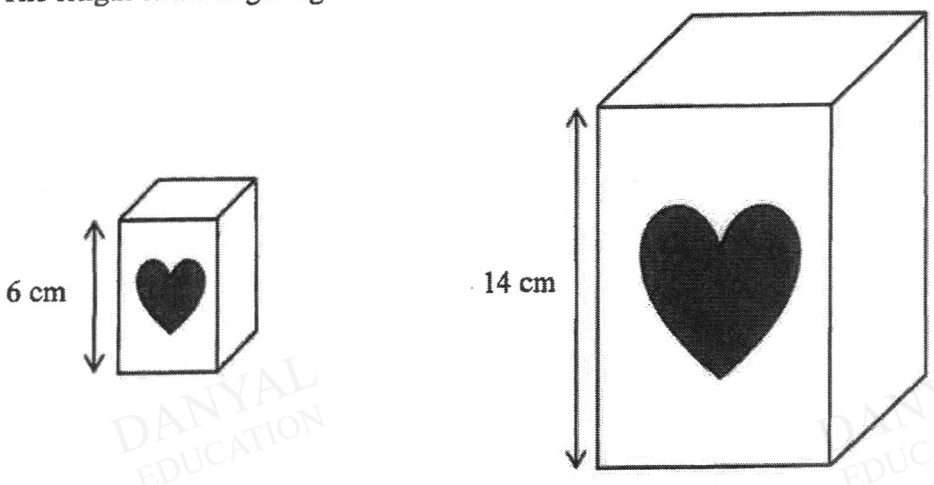
18 Cindy deposited \$20000 in a bank account which offered an interest of 1.2% per annum compounded monthly. She withdrew her money after 1 year 6 months. Calculate the amount of interest earned.

$I = 20000\left(1 + \frac{0.1}{100}\right)^{18} - 20000$ [1]

$= \$363.08$ [1]

Answer \$ 363.08 [2]

19. The diagram shows two geometrically similar figures made of the same material.
The length of the smaller figure is 6 cm.
The length of the larger figure is 14 cm.



(a) The area of the heart in the larger figure is 15 cm^2.
Calculate the area of the heart in the smaller figure.

$\frac{Area_{small}}{15} = \left(\frac{6}{14}\right)^2$ – [1]

$Area_{small} = 2.76 cm^2$

Answer 2.76 cm^2 [2]

(b) If the mass of the smaller figure is 200 grams, calculate the mass of the larger figure.

$\frac{200}{Mass_L} = \left(\frac{6}{14}\right)^3$ – [1]

$Mass_L = 2540g$ – [1]

Answer 2540 g [2]

(c) The smaller figure is sold for \$2.50 while the larger figure is sold for \$ 25.50. Which figure will be a better buy? Explain.

Small figure

200g → \$2.50

1g costs \$0.0125

Large figure

2540.7408g → \$25.50

1g costs \$ 0.010036] – [1]

Answer Larger figure [2]

17 Larger figure is a better buy – [1]

20 Two cafe outlets, *A* and *B* sells coffee, tea and cakes.
Both outlets charge $ 2.50 for a cup of tea, $3.50 for a cup of coffee and $3.00 for a piece of cake.

(a) On a Saturday, Cafe *A* sells 100 cups of coffee, 70 cups of tea and 50 pieces of cake.
Cafe *B* sells 120 cups of coffee, 90 cups of tea, and 40 pieces of cake.

Represent this information in a 2×3 matrix $\boldsymbol{Q}$.

Answer $Q = \begin{pmatrix} 100 & 70 & 50 \\ 120 & 90 & 40 \end{pmatrix}$ [1]

(b) By multiplying Q with another matrix, calculate the total amount collected by each of the two outlets respectively.

$$\begin{pmatrix} 100 & 70 & 50 \\ 120 & 90 & 40 \end{pmatrix} \begin{pmatrix} 3.50 \\ 2.50 \\ 3.00 \end{pmatrix} = \begin{pmatrix} 675 \\ 765 \end{pmatrix}$$

[1] [1]

Answer: Outlet *A*: $ 675

Outlet *B*: $ 765 [2]

(c) Both outlets decided to give a special discount of 20% for coffee, 10% for tea and 10% for cakes.
By the multiplication of two matrices, calculate the new prices of a cup of coffee, of a cup of tea and of a piece of cake.

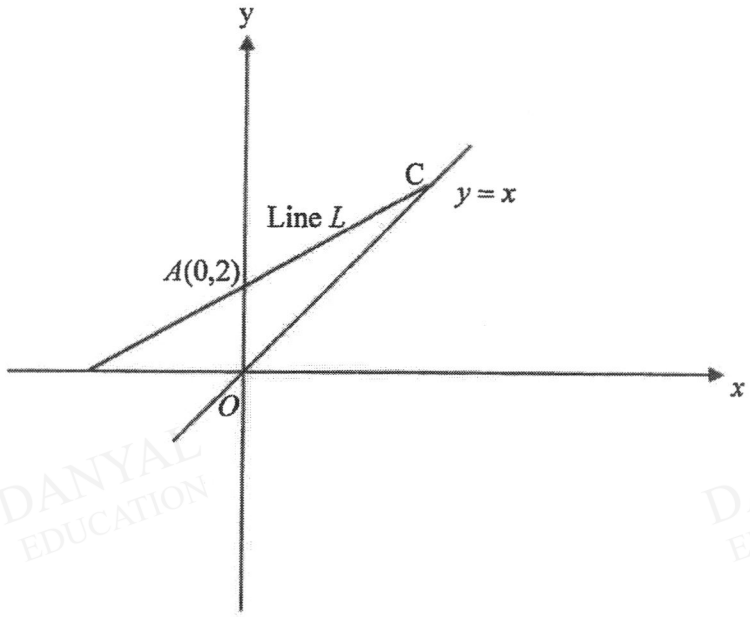
$$\begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix} \begin{bmatrix} 3.50 \\ 2.50 \\ 3.00 \end{bmatrix} = \begin{bmatrix} 2.80 \\ 2.25 \\ 2.70 \end{bmatrix}$$

Answer: New price of a cup of coffee : $ 2.80,

New price of a cup of tea : $ 2.25,

New price of a piece of cake : $ 2.70 [2]

21



The line L passes through the point A (0,2) and meets the line $y = x$ at point C. O is the origin. The area of OAC is 3 units^2.

(a) What is the equation of line L ?

$\frac{1}{2} \times 2 \times \text{height} = 3$

height $= 3$

C $(3,3)$

gradient $= \frac{3-2}{3-0} = \frac{1}{3}$ ——[1]

Equation : $y = \frac{1}{3}x + 2$ —[1]

Answer $y = \frac{1}{3}x + 2$ [2]

(b) There is a point B such that OC is the line of symmetry of the figure $OACB$.

(i) What is the name of quadrilateral $OACB$?

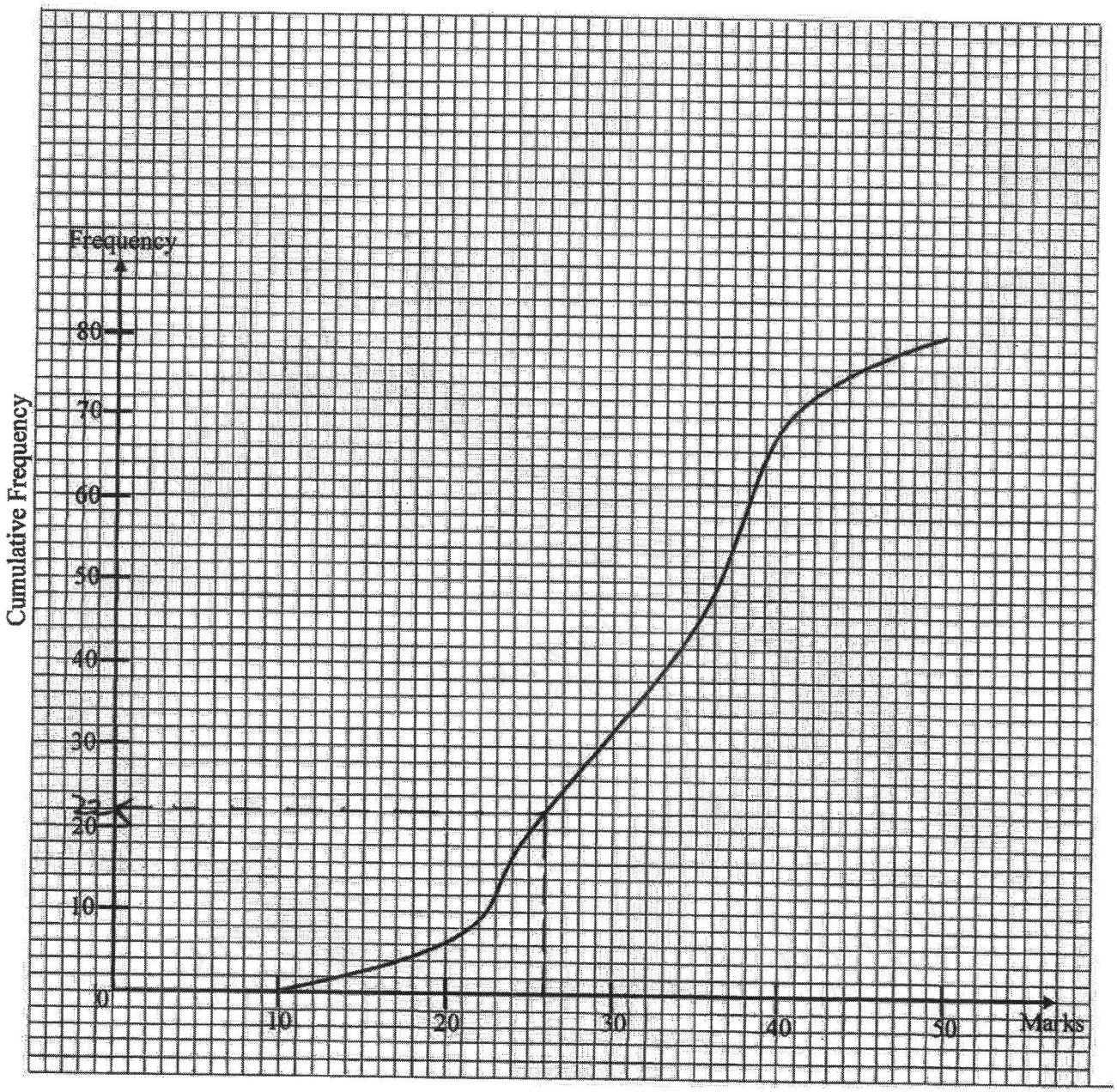
Answer kite [1]

(ii) State the coordinates of the point B.

Answer B (2 , 0) [1]

[1]

22. The cumulative frequency represents the marks scored by a group of 80 students.



(a) Use the graph to find the number of students with a score of more than 26 marks.

No of students
With a score $>26 = 80 - 22 = 58$ —[1]

Answer 58 [1]

(b) Calculate the range of the marks scored.

range = 50 − 10
= 40 marks

Answer 40 marks [1]

(c) If another group of 80 students have the same median mark with a smaller interquartile range, how would the cumulative frequency graph of this second group of students differ from this cumulative frequency graph ?

The cumulative frequency graph will be steeper −[1]

[1]

23 A building is drawn to a scale of 1: n.
The actual area of the building floor is 500 m^2 while the area of the building on the map is 20 cm^2.
Find the value of n.

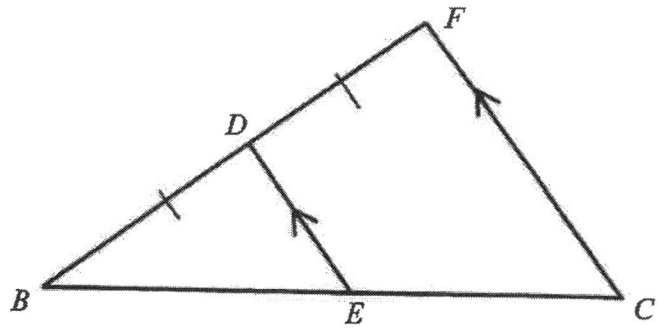
$\left(\frac{1}{n}\right)^2 = \frac{20}{500 \times 100 \times 100}$ −[1]

$1 : n = 1 : 500$

$n = 500$ −[1]

Answer n = 500 [2]

24 In the triangle BFC, D is the mid - point of BF. DE is parallel to FC.



Show that ΔBDE is similar to ΔBFC.

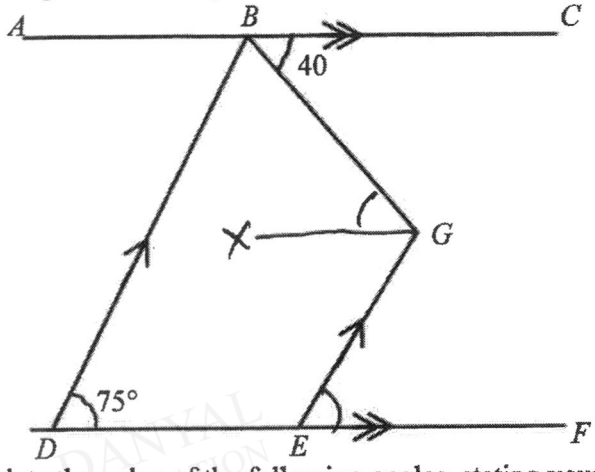
$\angle BDE = \angle BFC$ (Corresponding angles are equal)] – [1]

$\angle DBE = \angle FBC$ (Common angle)

AA Similarity test – [1]

[2]

25. In the diagram, the straight line ABC is parallel to the line DEF.
The straight line DB is parallel to the line EG. $\angle BDE = 75°$ and $\angle CBG = 40°$.



Calculate the value of the following angles, stating your reasons clearly.

a. $\angle DBG$

$\angle DBC = 180° - 75° = 105°$ (Sum of int. $\angle$s is 180°) [1]

$\angle DBG = 105° - 40°$ "10" level write in full"

$= 65°$ [1]

[2]

b. $\angle BGE$

$\angle BGX = 40°$ (alt. $\angle$s) "10" level write in full.

$\angle GEF$ 75° (corr. $\angle$s)

$\angle XGE = 75°$ (alt $\angle$s)

$\angle BGE = 40 + 75°$ 115° — [1] [1]

26. Explain whether a kite can be a rhombus.

Kite has 2 pairs of 1 adjacent sides [1]

If the 2 pairs of sides have equal length,

then kite can be a rhombus [1]

(rhombus is a kite) [2]

----------END OF PAPER----------

Name:_____ Class_____ Index No____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

MATHEMATICS	**4048/2**
Paper 2	Date: 28 August 2020
Candidates answer on the question paper.	Time: 0800-1030
Additional materials: Graph paper	Duration: 2h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper has a total of 22 pages.

Setter: Chong Lin Lin **[Turn over]**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Simplify $\frac{2x}{x^0} \div \left(\frac{2y}{x}\right)^{-2}$, leaving your answers in positive indices.

$$\frac{2x}{x^0} \div \left(\frac{2y}{x}\right)^{-2} = \frac{2x}{1} \div \left(\frac{x}{2y}\right)^{2} \quad \text{[M1]}$$

$$= \frac{2x}{1} \div \frac{x^2}{4y^2}$$

$$= \frac{2x}{1} \times \frac{4y^2}{x^2} \quad \text{[M1]}$$

$$= \frac{8y^2}{x} \quad \text{[A1]}$$

Answer [3]

(b) Express y in terms of p and q and s, given that $\frac{1}{q} = \frac{2}{y} + p$.

$$\frac{1}{q} = \frac{2}{y} + p$$

$$\frac{2}{y} = \frac{1}{q} - p \quad \text{[M1]}$$

$$\frac{2}{y} = \frac{1}{q} - \frac{pq}{q}$$

$$\frac{2}{y} = \frac{1-pq}{q} \quad \text{[M1]}$$

$$\frac{y}{2} = \frac{q}{1-pq}$$

$$y = \frac{2q}{1-pq} \quad \text{[M1]}$$

Answer [3]

2 **(a)** Express $\frac{1}{x-3}-\frac{6}{x^2-9}$ as a single fraction in its simplest form.

$$\frac{1}{x-3}-\frac{6}{x^2-9}$$
$$=\frac{1}{x-3}-\frac{6}{(x-3)(x+3)}$$
$$=\frac{(x+3)-6}{(x-3)(x+3)} \quad \text{[M1]}$$
$$=\frac{(x-3)}{(x-3)(x+3)}$$
$$=\frac{1}{(x+3)} \quad \text{[A1]}$$

Answer [2]

(b) **(i)** Express $y=x^2+4x-5$ in the form $y=a-(x+b)^2$, where a and b are constants.

$$y=x^2+4x-5$$
$$=(x+2)^2-4-5$$
$$=(x+2)^2-9$$

[M1,A1] method mark for complete square

Or [B1, B1] for getting a and b right

Answer [2]

(ii) Write down the equation of the line of symmetry

x = -2

Answer [1]

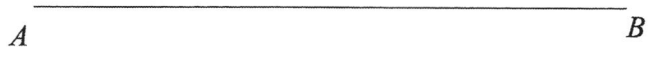
3 The line AB is shown below.

(a) Construct an equilateral triangle ABC. [1]

(b) Construct the perpendicular bisector of AB. [1]

(c) Construct the angle bisector of $\angle CAB$. [1]

(d) Mark clearly a possible point that is inside the triangle, equidistant from AC and AB, and is nearer to A than B. Label this point P. [1]



AB = 9.5

AQ=8.55 or 14.4 Allow ±0.1

(e) The point Q is such that $\angle ACQ = 120°$ and $\frac{BQ}{BA} = 0.9$.

Find two possible positions of Q. Label these point Q_1 and Q_2.

Measure and write down the length AQ (where Q can be any of the two points).

Draw a line such that $\angle ACQ = 120°$ [M1]

Draw a circle center B, radius (BQ) = 8.55 cm [M1]

Mark both Q_1 and Q_2 [A1]

Answer cm [3]

4 The exterior angle of a regular n-sided polygon is 14° more than the exterior angle of a regular $(2n + 6)$-sided polygon.

(a) Write down and simplify an expression, in terms of n, each exterior angle of the $(2n + 6)$-sided polygon.

Each ext angle of $(2n + 6)$ sides polygon

$= \frac{360}{2n+6}$ or $\frac{180}{n+3}$ [A1] Accept both

Accept $\frac{360}{n} - 14$

Answer _____ ° [1]

(b) Write down an equation in n and show that it reduces to $14n^2 - 138n - 1080 = 0$.

$\frac{360}{n} - \frac{180}{n+3} = 14$ [M1]

$\frac{360(n+3) - 180n}{n(n+3)} = 14$ [M1]

$360n + 1080 - 180n = 14n(n+3)$

$1080 + 180n = 14n^2 + 42n$ [M1]

$14n^2 + 42n - 180n - 1080 = 0$

$14n^2 - 138n - 1080 = 0$

[3]

(c) Solve the equation $14n^2 - 138n - 1080 = 0$.

$14n^2 - 138n - 1080 = 0$

$n = \dfrac{-(-138) \pm \sqrt{(-138)^2 - 4(14)(-1080)}}{2(14)}$ [M1]

$n = 15$ or $\dfrac{-36}{7}$ [A1]

Answer [2]

(d) Explain why one of the solutions above is rejected.

Answer

n is a positive integer since it is the number of sides of a polygon

[1]

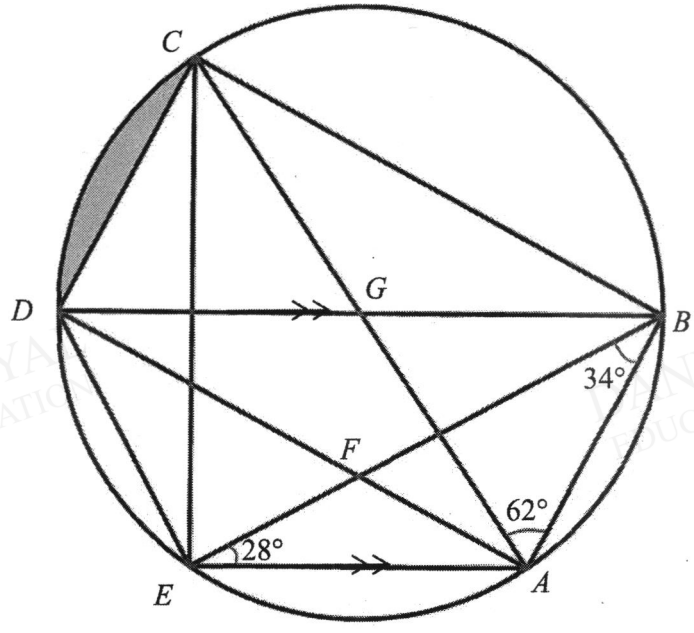
(e) Hence, find the value of each interior angle of the n-sided polygon.

Each exterior angle $= \dfrac{360}{15} = 24°$ [M1]

Each interior angle
$= 180 - 24 = 156°$[A1]

Answer ° [2]

5 The points A, B, C, D and E lie on the circle. $\angle AEB = 28^\circ$, $\angle CAB = 62^\circ$ and $\angle ABE = 34^\circ$. AE is parallel to BD.



(a) Find two other angles that is 62°, stating the angle property.

$\angle BEC$ and $\angle BDC$ [A1]

(angles in the same segment) [A1]

Answer [2]

(b) Explain whether AC is the diameter of the circle.

Answer

$\angle BEC + \angle BEA = 62 + 28 = 90^\circ$

or

$\angle BDA = 28^\circ$ (angles in the same segment)

$\angle BDC + \angle BDA = 62 + 28 = 90^\circ$

[M1] show how they get right angle

∠AEC or ADC = 90°, AC is the diameter. (right angle in semicircle) [M1]

[2]

(c) Find the following angles, stating all the angle properties.

(i) $\angle EBD$

(ii) $\angle DEA$

$\angle EBD = 28°$ (alt $\angle$s, AE//BD) [A1]

$\angle DEA$

$= (180\text{-}28\text{-}34)°$ (angles in opposite segments/ opp angles in cyclic quad)

$= 118°$ [A1]

[M1] all necessary angle properties written for (i) (ii)

Answer **(i)** _____ °

(ii) _____ ° [3]

(d) Explain why G is the center of the circle.

Answer

Many possible explanations

Example

$\angle GBA = 34 + 28 = 62°$. [M1]

Since $\angle GBA = \angle GAB$, GA=GB = radius of circle. [M1]

[2]

(e) If the radius of the circle is 7 cm, find the area of the shaded region.

Area of the shaded region

$$= \frac{180-62-62}{360} \times \pi(7)^2 - \frac{1}{2} \times (7)^2 \times \sin(180-62-62)$$

[M1 – area of sector]

[M1 – area of triangle]

$= 3.63449$

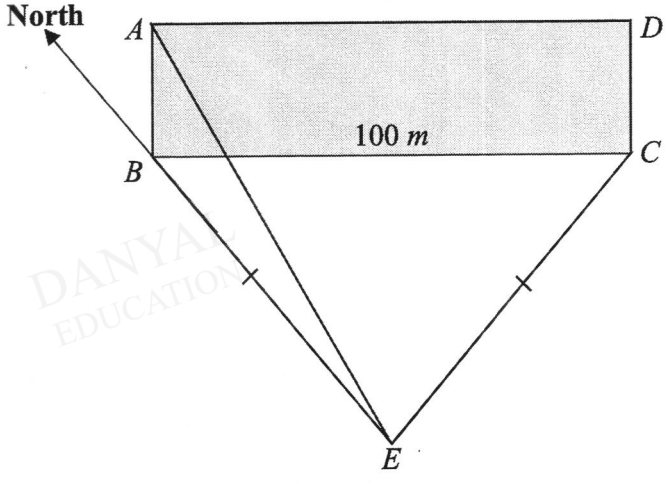
$\approx 3.63 \text{ cm}^2$ [A1]

Answer _____ cm^2 [3]

6 In the diagram, *ABCD* represent a vertical cliff face. The bottom of the cliff, *BC*, is 100 m and is at sea level. A boat is in the sea at *E*.

B is due north of *E* and the bearing of *C* from *E* is 070°.

E is known to be equidistant from *B* and *C*.



(a) Find the bearing of *C* from *B*.

$$\angle EBC = \frac{180^\circ - 70^\circ}{2}$$

$$= 55^\circ \quad [M1]$$

The bearing of C from B = 125° [A1]

Answer _____ ° [2]

(b) Show that $BE = 87.17\ m$.

Answer

$$\frac{\sin 70^\circ}{100} = \frac{\sin 55^\circ}{BE} \quad [M1]$$

$$BE = \frac{100 \sin 55^\circ}{\sin 70^\circ} \quad [M1]$$

$$BE = 87.17 \text{ m}$$

[2]

(c) Find the area of ΔBCE, leaving your answer to the nearest square metre.

Area

$= 0.5\,(87.17)^2 \sin 70°$ (accept using more accurate BE) [M1]

$= 3570.178356$

$= 3570\ m^2$ [A1]

Answer _____ m^2 [2]

(d) It is given that the angle of depression of E from A is 20.2°. Find the height of the cliff BA, leaving your answers in 3 significant figures.

$$\tan 20.2 = \frac{AB}{87.17}$$

$AB = 87.17 \tan 20.2$ [M1] make height the subject

AB = 32.1 m (3 significant figures) [A1]

Answer _____ m [2]

(e) P is a point on BC such that $\dfrac{Area\ of\ \Delta BEP}{Area\ of\ \Delta CEP} = \dfrac{3}{2}$.

Find the length of CP.

$$\frac{Area\ of\ \Delta BEP}{Area\ of\ \Delta CEP} = \frac{3}{2}.$$

So $\dfrac{BP}{CP} = \dfrac{3}{2}$ [M1]

CP = 40 m [A1 or B2]

Answer _____ m [2]

7 Part of a pattern of numbers is shown in the table below.

		C_1	C_2	C_3	C_4	C_5	C_6
Row	1	2	3	4	5	6	7
Row	2	8	9	10	11	12	13
Row	3	14	15	16	17	18	19
Row	4	20	21	22	23	24	25
Row	5	26	27	28	29	30	31
*	*	*	*	*	*		
*	*	*	*	*	*		
Row	n		x	y			
Row	$n+1$		s	t			

(a) Express y, s and t in terms of x.

Row	n		x	$y = x + 1$		
Row	$n+1$		$s = x + 6$	$t = x + 7$		

Answer

$y =$

$s =$

$t =$ [2]

(b) Explain why the difference between xt and ys is always 6.

Answer

$xt - ys$

$= x(x+7) - (x+1)(x+6)$

$= x^2 + 7x - (x^2 + 7x + 6)$ [M1]

$= -6$

Difference $= 6$ [M1]

[2]

(c) Express x in terms of n.

$x = (6n - 3)$

Answer $x =$ [1]

(d) Explain why xy would be in column number C_5, for any value of n.

xy

$= (6n-3)(6n-2)$

$= 36n^2 - 30n + 6$

$= 6(6n^2 - 5n + 1)$ [M1] Take 6 as common factor

Since 6 is a factor, xy is multiple of 6. Therefore it is in C_5. [M1]

Answer [2]

(e) From the above, where would ys lies in (C_1, C_2, C_3, C_4, C_5 or C_6)?

C_5

Answer [1]

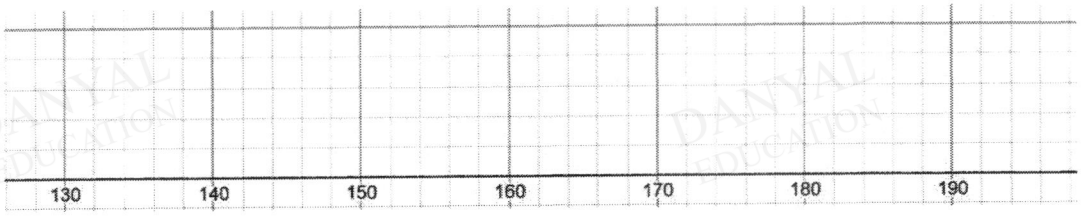
8 The ordered stem-and-leaf diagram records the height of 21 plants in Plantation *A*.

Stem	Leaf							
13	0	7	8	9	9			
14	1	1	2	6	7	8	9	9
15	0	2	3	5	6	8	8	8

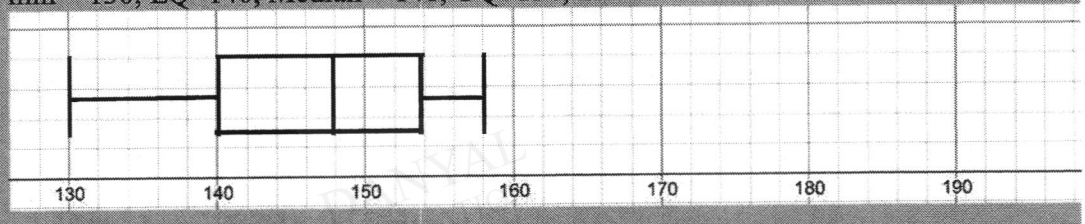
Key: 14 | 8 means 148 cm

(a) Represent the above information in a box and whisker.

Answer



min = 130, LQ=140, Median = 148, UQ=154, Max = 158



[3]

(b) State the modal height.

158cm

Answer _____ cm [1]

(c) Calculate the percentage of students having a height of less than 150 cm.

Percentage (<160)

$$=\frac{13}{21}\times 100\%$$

$$=61.905\%$$

$$=61.9\%$$---[B1]

Answer _____ % [1]

(d) Two trees are selected at random from Plantation *A*.

The students are asked to calculate the probability of selecting a plant that is less than 150 and another plant that is more than or equal to 150.

(i) Alice says that the above probability is calculated using $\frac{8}{21}$ and $\frac{13}{21}$.

Explain why she is wrong.

Answer

Possible answer

(1) After selecting the first tree, there are only 20 trees left.

(2) P(select one < 150 and one at least 150)

= P(first plant at least 150, second less than 150) + P(first plant less than 150, second at least 150)

[1]

(ii) Calculate the above-mentioned probability.

P(select one < 150 and one at least 150) should be

$= \frac{8}{21} \times \frac{13}{20} \times 2$ [M1]

$= \frac{52}{105}$ [A1]

Answer [2]

(e) 30 plants are planted in Plantation *B* on the same day. The range of height is 24 cm, the interquartile range is 10 cm and the median is 152 cm. Compare the height in Plantation *A* and *B* in two ways.

Answer

One mark each

(1) $\text{Median}_A < \text{Median}_B$: The plants in Plantation B are taller. (or grow faster)

(2) Calculate $IQR_A = 14$. $IQR_A > IQR_B$: The spread in height of plantation for Plantation A is wider.

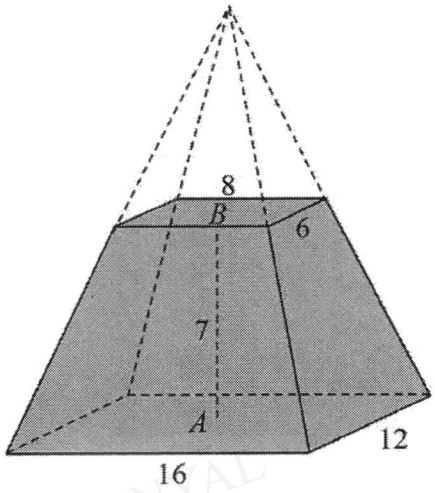
[2]

9 The frustum shown in the diagram is a metal solid obtained by cutting off the pyramid.

The dimension of the top rectangular plane is 8 cm by 6 cm.

The dimension of the bottom rectangular plane is 16 cm by 12 cm.

The height, AB, of the frustum is 7 cm where A is the center of the base.



(a) Explain why the height of the pyramid that is cut off is 7 cm.

Answer

The cone that is cut off is similar to the bigger cone. [M1]

$\frac{l_1}{l_2} = \frac{b_1}{b_2} = \frac{\text{height of small cone (cut off)}}{\text{height of larger cone}}$ [M1]

$\frac{8}{16} = \frac{6}{12} = \frac{h}{h+7}$

$\frac{1}{2} = \frac{h}{h+7}$

$h + 7 = 2h$

$h = 7$

[2]

(b) Show that the volume of the frustum is 784 cm^3.

Answer

Volume of frustum

= Volume of big cone - Volume of small cone

$= \frac{1}{3}(12 \times 16 \times 14) - \frac{1}{3}(6 \times 8 \times 7)$ [M1, M1 - Volume of big and small cone]

$= 784 \text{ cm}^3$

[2]

(c) Assuming negligible loss in the process of melting, how many spherical balls (of radius 2 cm) can be formed if the frustrum is melted?

Approximate no of spheres

$= 784 \div \left(\frac{4}{3}\pi \times 2^3\right)$ [M1]

$= 23.39577$

No of spherical balls that can be formed = 23 [A1]

Answer [2]

(d) 3 of the above frustums are to be packed into a rectangular box. To save cost, the seller should keep the amount of material of the rectangular box to the minimum.

Albert claim that the dimension should be 30 cm by 16 cm by 7cm (Box *A*).
Betty claim that the dimension should be 38 cm by 12 cm by 7cm (Box *B*).
Carol claim that the dimension should be 21cm by 16 cm by 12cm (Box *C*).

(i) Explain which box will be too small.

Answer

Box B. The minimum length should be (16+8+16) = 40cm. not 38 cm

[1]

(ii) Which of the other two boxes would use the minimum amount of material?

Calculate surface area

Surface Area (Box A) = $2\times[(30\times16)+(30\times7)+(16\times7)]=1604$ cm^2

Surface Area (Box C) = $2\times[(12\times16)+(12\times21)+(16\times21)]=1560$ cm^2

Answer: Box C

[M1, M1, A1]

Answer [3]

10 A group of researchers brought 180 koalas to an island and tracked the number of koalas over a few years.

x (years)	0	0.5	1	2	3	4	5	6	6.5	7	7.5
y (no of animals)	180	177	175	172	p	172	175	180	183	187	191

(a) Using a scale of 2 cm on both axes to represent 1 year on the x-axis and 2 animals on the y-axis for $170 \leq y \leq 192$, plot the values given and join them with a smooth curve.

[3]

(b) Use your curve to estimate the value of p, corrected to nearest integer.

p = 171

Answer [1]

(c) By drawing a tangent, find the gradient $x = 6$.

Draw a suitable tangent. [M1]

Gradient = 6 ± 1 [A1]

Answer [2]

(d) The number of koalas (y) turns out to be related to an unknown quadratic function $y = (x + a)^2 + b$. Find the value of a and of b.

Answer $a =$ -3 $b = 171$ [2]

(e) The researchers also brought 180 penguins to the same island and noticed that the number of penguins increased at a steady rate of 1 per year.

(i) Write down an equation connecting the number of penguins, y, and the number of years, x.

$y = x + 180$

Answer _____ [1]

(ii) On the same axes, draw the graph of the equation in **(i)** and state the point of intersection (x, y).

Draw line [M1]

Accept (0,180) and (7,187) [A1]

Answer _____ [2]

(iv) What would this point of intersection represent in this research?

Answer

On the 7th year, the number of penguins = the number of koalas = 187.

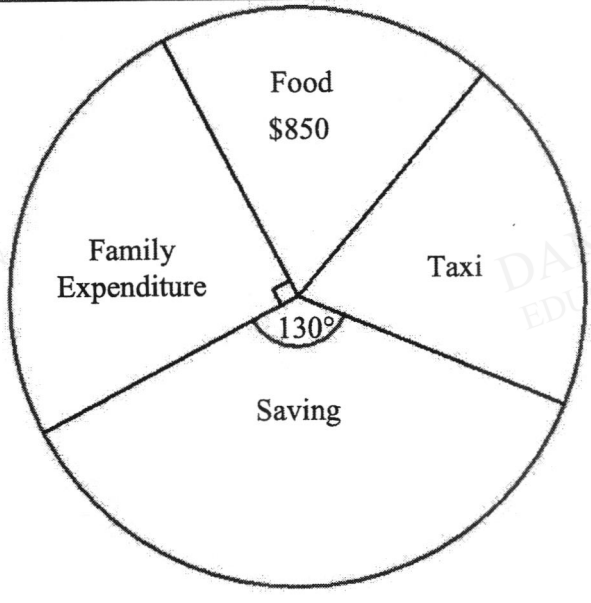
[1]

11 Charlotte takes the taxi every day. She is thinking of buying a car and wants to pay off the car as soon as possible. The tables below give the information Charlotte can use.

Charlotte's financial situation	
Money set aside to buy a car	\$50000
Monthly salary	\$4500

Information on the car and car loan	
Car Price	\$96000 Customer must take a minimum loan of 50% from the car dealer.
Loan Period (Number of years, x)	• $2 \leq x \leq 7$, x is an integer • Customer needs to decide on the loan period on the day he or she buys the car.
Car agent's interest rate	2.45% per annum simple interest

Information on how Charlotte uses her monthly salary



(a) Determine the minimum loan amount.

Loan amount =

$$= \frac{50}{100} \times 96000$$

$$= 48000$$

[A1]

Answer $ _____ [1]

(b) Charlotte calculated that if the loan period is 2 years and that she took a loan of \$48000, her monthly car instalment will be \$2098.00. Show that Charlotte is correct.

Answer

Monthly instalment

$$= \frac{\text{Loan} + \text{Simple interest}}{\text{Number of months}}$$

$$= \frac{48000 + 48000 \times \frac{2.45}{100} \times 2}{24}$$ [M1 calculate simple interest]

$= 2098$ [M1 show correct steps to calculate instalement]

[2]

(c) Charlotte intends to use **no more than** her current taxi expenditure to pay for her monthly car instalment. Suggest the minimum loan period. Justify your answer with clear working.

First part of calculation (3 marks)

Angle representing Food

$$= \frac{850}{4500} \times 360^\circ \quad \text{[M1 - convert info from pie chart]}$$

$$= 68^\circ$$

Current monthly expenditure

$$= \frac{360 - 90 - 130 - 68}{360} \times 4500 \quad \text{[M1 - calculate angle, form ratio]}$$

$$= 900 \quad \text{[M1]}$$

Method 1 (3 marks) : Forming an algebraic inequality to solve

Method 2 (3 marks) : Calculating monthly instalment for appropriate number of years 4 years and 5 years. Explain why it will not be more than 5 years.

Minimum number of years = 5 years [A1]

Answer [7]

(d) Give a possible real-life scenario to explain why Charlotte's savings will be reduced even if her monthly instalment is the same as the current taxi expenditure.

Answer

Any logical real-context reasons such as carpark fee, petrol, car maintenance.

[1]

End of paper

12

a