NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

NAME

SUBJECT CLASS 2ma2 REGISTRATION NUMBER

MATHEMATICS

9758/01

Paper 1

15 September 2020

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)
Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

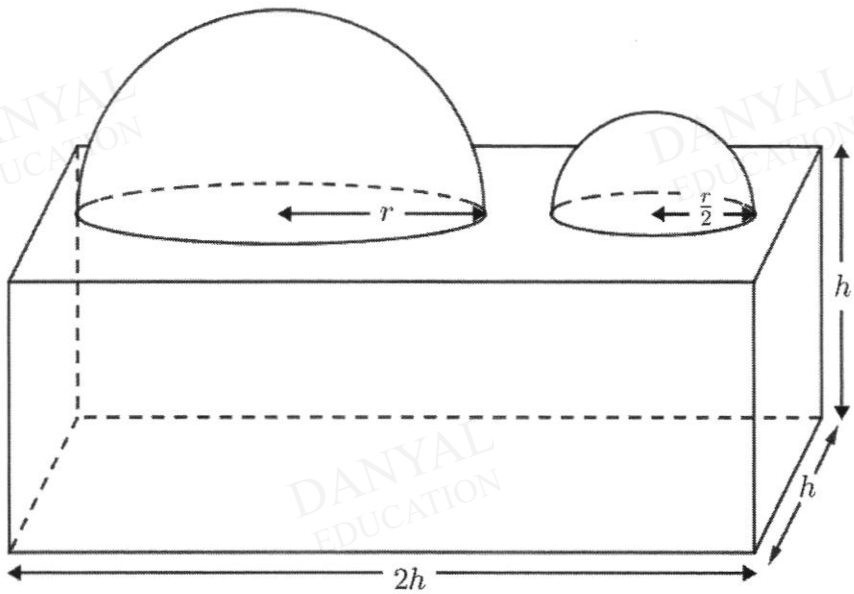
The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	4	
2	5	
3	7	
4	8	
5	8	
6	8	
7	8	
8	8	
9	10	
10	10	
11	10	
12	14	
Presentation Deduction		– 1 / – 2
TOTAL	100	

1 An arithmetic series has first term a and common difference d, where a and d are non-zero. It is given that the sixth, fourth and third terms of the arithmetic series are equal to the first three terms respectively of a convergent geometric series. Find, in terms of a, the sum of the terms of the geometric series after, and including, the 15th term. [4]

2 By rationalising $u_r = \dfrac{1}{\sqrt{r}+\sqrt{r-1}}$, find an expression for $\sum_{r=1}^{n} u_r$.
Deduce the sum from the seventeenth to the hundredth terms. [5]

3 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\dfrac{4}{3}\pi r^3$.]



A model of an observatory is made up of two parts.

- The main building is modelled by a closed rectangular box with two circular discs of radii r cm and $\dfrac{r}{2}$ cm removed from the top. The box is of length $2h$ cm, width h cm and height h cm.
- The telescope domes are modelled by the curved surfaces of two hemispheres of radii r cm and $\dfrac{r}{2}$ cm.

Both parts are joined together as shown in the diagram and the model is made of material of negligible thickness. It is given that the volume of the model is a fixed value 500 cm^3 and the model has external surface area $S \text{ cm}^2$.
Use differentiation to find the value of r that gives a stationary value of S.
Hence, determine whether the value of r found gives a minimum or maximum value of S. [7]

4 It is given that $f(x) = 10 - kx - \dfrac{9}{x-1}$, where k is a constant.

(a) In the case where $1 < k < 9$, sketch the graph of $y = f'(x)$, labelling the x-intercepts and the equations of any asymptotes. Leave your answers in terms of k. [3]

For the remainder of the question, assume that $k = 1$.

(b) Sketch the graph of $y = f(x)$, labelling the equations of any asymptotes.
Hence solve the inequality

$$f(x) \geq \frac{9}{|x-1|}.$$ [5]

5 The curve C has equation $y = a(cx - b)^2$, where a, b and c are positive constants.

(i) Describe fully a sequence of (no more than three) transformations which would transform the graph of $y = x^2$ onto the graph of C. [3]

(ii) Given that C passes through the points with coordinates $(-1, 147), (2, 3)$ and $(4, 27)$, find a and c in terms of b. [3]

(iii) It is given further that the point with coordinates $(-5, 25)$ on $y = x^2$ corresponds to the y-intercept on $y = a(cx - b)^2$. State the integer values of a, b and c. [2]

6 The integral test for convergence, known as the Maclaurin-Cauchy test, is a method used to test for convergence of an infinite series of non-negative terms. The Maclaurin-Cauchy test states that

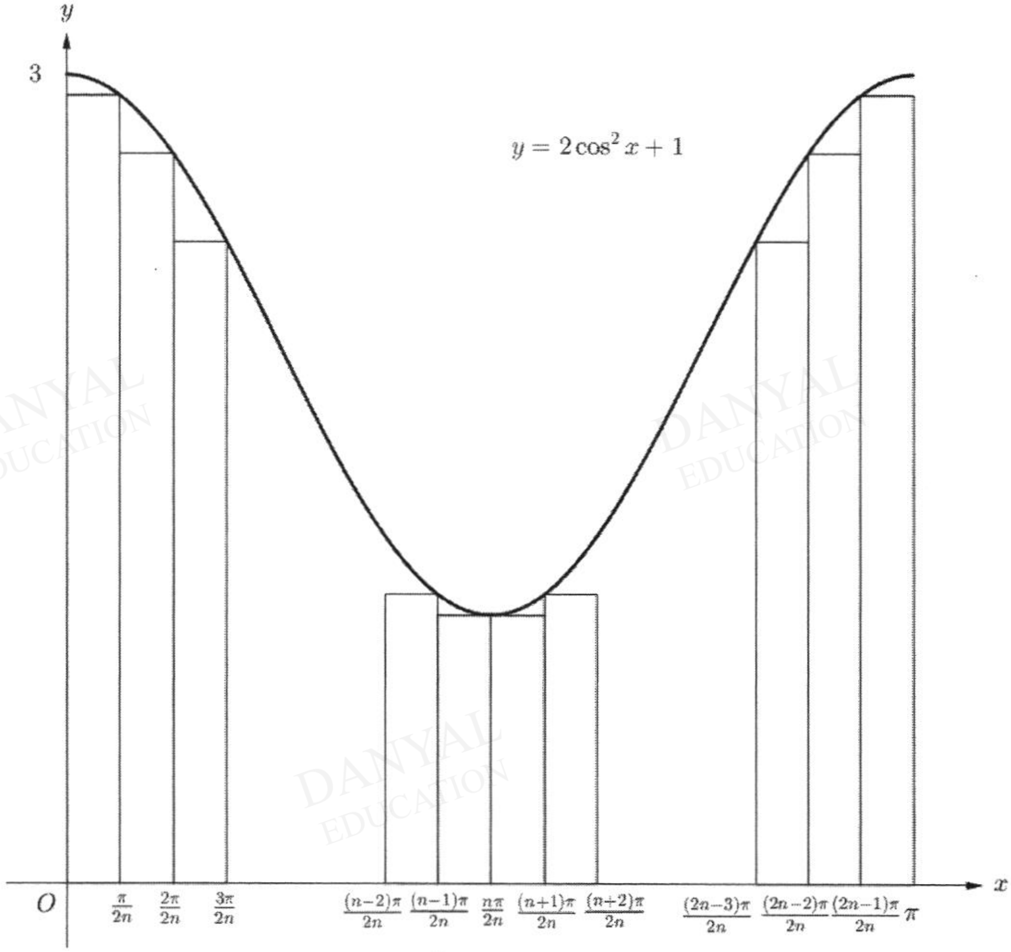
"For a non-negative and non-increasing function f defined on the interval $[1, \infty)$, a series of the form $\sum_{r=1}^{\infty} f(r)$ converges if $\int_1^{\infty} f(x)\, dx$ is finite."

(i) Show that $\dfrac{\tan^{-1} x}{1 + x^2}$ is positive and decreasing on the interval $[1, \infty)$. [3]

(ii) Use the Maclaurin-Cauchy test to determine if $\displaystyle\sum_{r=1}^{\infty} \frac{\tan^{-1} r}{1 + r^2}$ converges. [5]

7 **(a)** Evaluate exactly $\int_0^{\pi} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right)+\pi \, dx$. [2]

(b) **(i)**



The curve with equation $y = 2\cos^2 x + 1$, for $0 \le x \le \pi$, is shown in the diagram. Rectangles, each of width $\dfrac{\pi}{2n}$, are drawn under the curve. Show that the total area S_n of all $2n$ rectangles is given by

$$\frac{2\pi}{n}\left\{\cos^2\left(\frac{\pi}{2n}\right)+\cos^2\left(\frac{2\pi}{2n}\right)+\cdots+\cos^2\left(\frac{(n-1)\pi}{2n}\right)\right\}+\pi .$$ [3]

(ii) Find the exact sum to infinity S_{∞}. [3]

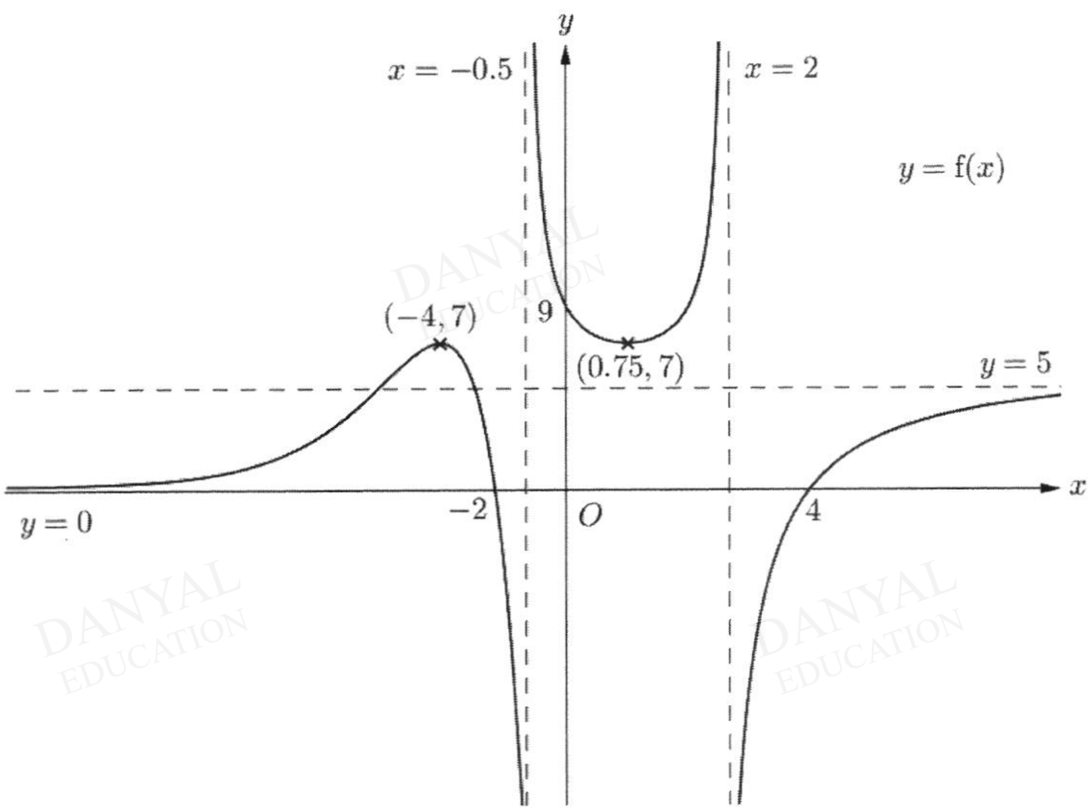
8 A blogger reviews a data plan with unlimited data. The download speed is x megabytes per second, t seconds after the download has started. This download speed decreases at a rate proportional to the difference between x and the eventual limit of 40 megabytes per second. The download speed falls from a value of 60 megabytes per second to 50 megabytes per second in the first 30 minutes.

(i) Write down a differential equation for this situation. Solve this differential equation to get x as an exact function of t. [5]

The total amount of data downloaded daily by time t seconds is y gigabytes. The fair usage policy enforces that the download speed will be reduced to a speed which cannot be used to stream videos if $y > 500$. You may assume that 1 gigabyte is 1000 megabytes.

(ii) The blogger starts using the data plan on the morning of a particular day to stream videos. How long would he be able to stream videos continuously? Give your answer in hours to 1 decimal place. [3]

9



The diagram shows the curve with equation $y = \mathrm{f}(x)$, for $x \in \mathbb{R}, x \neq -\frac{1}{2}, x \neq 2$. The curve crosses the axes at $x = -2, x = 4$ and $y = 9$, and has turning points with coordinates $(-4, 7)$ and $(0.75, 7)$. The curve has asymptotes with equations $x = -0.5, x = 2, y = 0$ and $y = 5$.

(i) Sketch the curve $y = \dfrac{1}{\mathrm{f}(x)}$, labelling any axial intercepts and turning points, and the equations of any asymptotes. [4]

For the remainder of the question, the domain of f is $[k,2)\cup(2,\infty)$, where k is a constant.

(ii) State the least value of k for which the function f^{-1} exists. [1]

The function g is defined by

$$g : x \mapsto \frac{2x+a}{x-b} \text{ for } x \in \mathbb{R},\ x > b,$$

where $a, b \in \mathbb{R}^+$.

(iii) Find the range of g. [2]

(iv) Find $(fg)^{-1}(0)$. Leave your answer in terms of a and b. [3]

10 A curve C has equation $x^2 = \dfrac{\ln(y+1)}{(y+1)^2}$. The points P and Q on C each have y-coordinate $e-1$.

(i) State the equation of the line of symmetry of C. [1]

(ii) **(a)** A vase is formed by rotating the region bounded by the curve C and the line PQ through π radians about the y-axis. Find the exact volume of the vase obtained. [4]

(b) The normals to C at P and Q meet at the point N. Find the exact coordinates of N. [5]

11 **Do not use a calculator in answering this question.**

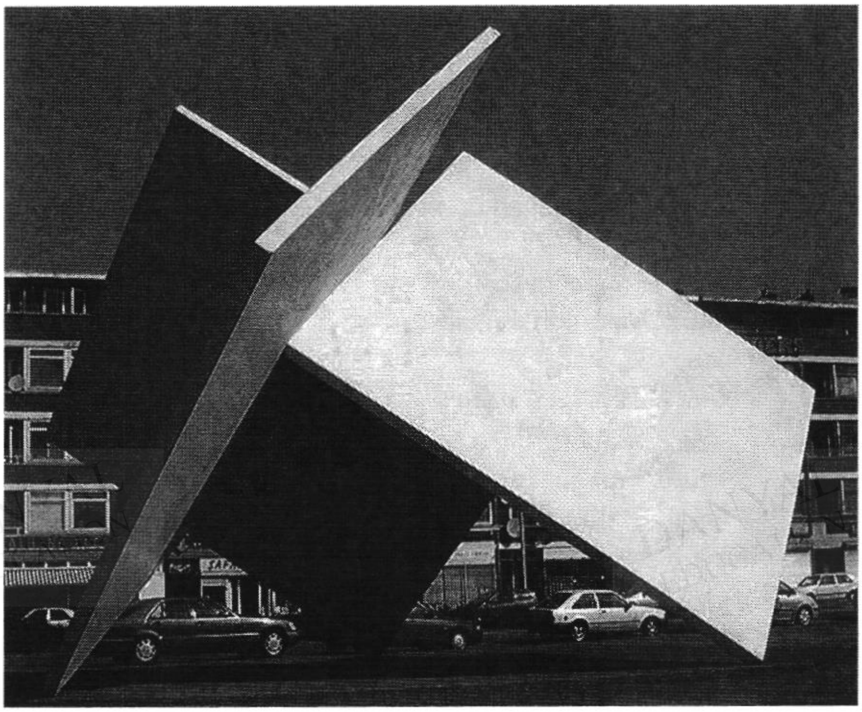
(a) The complex numbers z and w are such that $w - 6\alpha z = -5\alpha i + 3$ and $\alpha z^2 + iw = 10\alpha + 3i$, where α is a non-zero real constant.
Given that $|z| > 1$, find z and w in terms of α (where applicable). [3]

(b) The complex numbers z_1 and z_2 are given by $3 - i\sqrt{3}$ and $2i$ respectively.

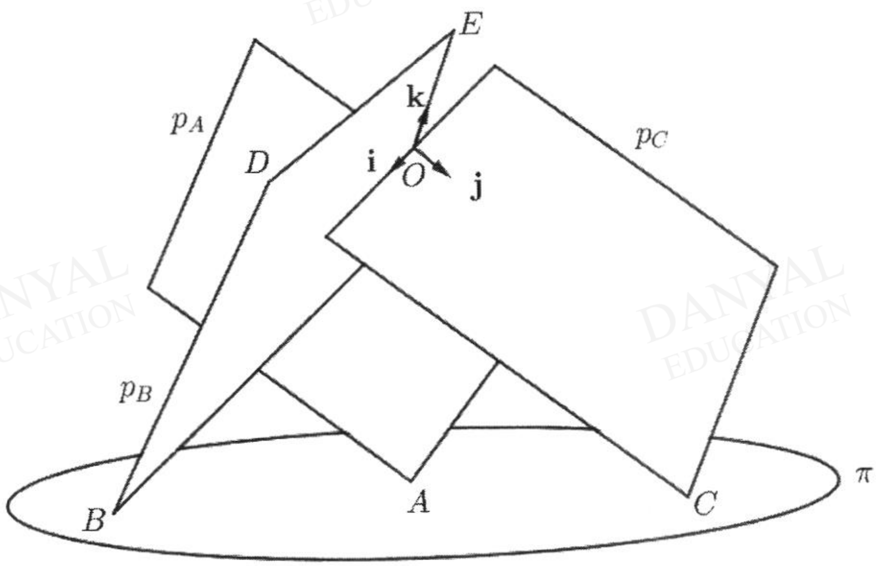
(i) Find the possible integer values of n such that $\dfrac{{z_1}^{3n}}{{z_2}^{2n}}$ is purely imaginary with a positive imaginary part. [3]

(ii) Given that z_1 is a root of the equation $px^3 - 9x^2 + 6x + 36 = 0$, find the value of the real number p. Using the value of p found, express $px^3 - 9x^2 + 6x + 36$ as a product of three linear factors. [4]

12



The picture shows a concrete sculpture in Rotterdam, Netherlands, made of three identical square slabs of side 8 m. The three slabs are assembled perpendicular to each other such that they meet at one common point O. Each slab has one vertex fixed to the flat horizontal ground, represented by the plane π, and one vertex fixed to the centre of another slab.



- The slab p_A is part of the plane $x=0$. It has one vertex, point A, on π and another vertex on the centre of p_C.
- The slab p_B is part of the plane $y=0$. It has one vertex, point B, on π and another vertex on the centre of p_A. The remaining two vertices are denoted as D and E.
- The slab p_C is part of the plane $z=0$. It has one vertex, point C, on π and another vertex on the centre of p_B.

The point O is taken as the origin and perpendicular unit vectors **i**, **j** and **k** are defined, as shown in the diagram, with **i**, **j**, **k** along one edge of p_C, p_A, p_B respectively. For example, point D has position vector $8\mathbf{i}+4\mathbf{k}$. It is assumed that the slabs are made of materials with negligible thickness.

(i) Show that a Cartesian equation of π is $x+y-z=12$. [4]

(ii) To make the sculpture more stable, plans are made to construct a vertical support from O to a point F on π, such that OF is perpendicular to π. Find the position vector of F. [3]

A pigeon is at a point $P\ (2,11,1)$. It flies in a straight line and rests on the sculpture along the edge DE.

(iii) Given that the pigeon rests at the point nearest to P, find the coordinates of this point. Hence find the distance the pigeon flew. [4]

Another pigeon has flight path with equation $\dfrac{x-1}{3}=\dfrac{2-y}{4}=z+3$.

(iv) Without using a calculator, determine if the flight paths of the two pigeons cross. [3]

NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

NAME

SUBJECT CLASS: 2ma2

REGISTRATION NUMBER

MATHEMATICS **9758/02**

Paper 2

21 September 2020

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)
Writing Paper

READ THESE INSTRUCTIONS FIRST

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Question Number	Marks Possible	Marks Obtained
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2	7	
3	8	
4	8	
5	11	
6	7	
7	8	
8	9	
9	10	
10	11	
11	2	
12	13	
Presentation Deduction		– 1 / – 2
TOTAL	100	

Section A: Pure Mathematics [40 marks]

1 **(i)** Given that $y=\sin^{-1}(2x)+1$, by using standard series from the List of Formulae (MF26), show that $\frac{\mathrm{d}y}{\mathrm{d}x} \approx 2+4x^2+12x^4+40x^6$, and state the range of values of x for which the expression is valid. [4]

(ii) Hence, find an approximate expression for y, up to and including the term in x^7. [2]

2 Referred to the origin O, points A, B and C have position vectors **a**, **b** and **c** respectively. B and C are points on a circle with diameter OA and circumference π units.

(i) Give a geometrical interpretation of $|\mathbf{a}\cdot\mathbf{b}|$ and show that $\mathbf{a}\cdot\mathbf{b}=|\mathbf{b}|^2$. [3]

The point D is on OA such that $OD:DA=4:5$. It is given that BD is perpendicular to OA.

(ii) Find the cosine of the acute angle BOD. [2]

(iii) Given that $|\mathbf{a}\times\mathbf{c}|=0.4$, find the area of triangle OCD. [2]

3 In the controlled environment of Laboratory (Lab) A where cells are cultivated, the number of cells increases by 250% from the start of the week to the end of the week. At the end of each week, 1.2 million cells are transferred to another laboratory for experiments. The number of cells (in millions) in Lab A at the start of the nth week is given by a_n.

(i) Show that $a_n=3.5^{n-1}a_1-0.48\left(3.5^{n-1}-1\right)$. [3]

It is given that Lab A begins operating at the start of the first week with 0.5 million cells.

(ii) Find the number of cells in Lab A at the end of the third week, before the cells are transferred to another laboratory. [2]

A research requires at least 60 million cells before it can commence. To facilitate this, Lab A and Lab B are to cultivate and combine all the cells in both the labs by the start of a certain week. Lab B begins operating one week after Lab A. Lab B begins operating at the start of that week with 0.5 million cells and cultivates 5 million cells by the end of that week. At the end of each subsequent week, Lab B cultivates 5 million cells more than in the previous week.

(iii) In which week can the research first commence? [3]

4 The curve C_1 has parametric equations

$$x = 4 + 4\sin\theta, \quad y = 2\cos\theta,$$

where $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$.

Sketch C_1, labelling any points where C_1 crosses the axes. [2]

The curve C_2 has equation $y = x^2 - ax$, where a is a positive constant.

(a) In the case where $a = \dfrac{7}{2}$, it is given that $(4, 2)$ is a point on both C_1 and C_2.

Sketch C_2 on the same diagram as C_1, labelling the points of intersection of C_1 and C_2. Hence, find the exact area of the region bounded by C_1 and C_2. [5]

(b) State the range of values of a such that C_1 and C_2 meet at two distinct points. [1]

5 The complex number z is given by $z = r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 1$ and $-\pi < \theta < -\frac{1}{2}\pi$.

The complex number w is such that $|w^* z| = r^2$ and $\arg\left(-\dfrac{w}{z^3}\right) = \pi - 4\theta$.

(i) Find $|w|$ in terms of r and show that $\arg w = -\theta$. [4]

(ii) State $|z + w|$ and indicate, on an Argand diagram, the points A, B, C and D that represent the complex numbers z, w, $z + w$ and zw respectively. [5]

(iii) Hence show that the area of the quadrilateral $ACBD$ is given by $r^2 \sin 2\theta - r^3 \sin\theta$. [2]

Section B: Probability and Statistics [60 marks]

6 A machine has previously been considered capable of producing spherical pellets with mean diameters 12.00 mm. After some modification to the machine, a random sample of 105 spherical pellets was taken and the diameter, x mm, of each pellet is measured. The results obtained are summarised as follows.

$$\sum x = 1261.05 \qquad \sum x^2 = 15145.47$$

(i) Calculate unbiased estimates of the population mean and variance of the diameter of spherical pellets after modification. [2]

(ii) Test, at the 3% level of significance, whether the mean diameter of the spherical pellets has increased after modification. [3]

(iii) State the meaning of the p-value of the test in part **(ii)** in context. [1]

(iv) Explain why there is no need for the production manager to know anything about the population distribution of the diameters of the spherical pellets. [1]

7 Independent events A and B are such that $\mathrm{P}(A) = 0.45$ and $\mathrm{P}(B) = b$.

(i) Prove that A' and B are independent events. [2]

For a third event C, it is given that A and C are mutually exclusive, $\mathrm{P}(B \cap C) = 0.2$ and $\mathrm{P}(A' \cap B' \cap C') = 0.1$.

(ii) Write down expressions for $\mathrm{P}(A \cap B')$ and $\mathrm{P}(A' \cap B \cap C')$ in terms of b. [2]

(iii) Find exactly the maximum and minimum possible values of $\mathrm{P}(A \cap B)$. [4]

8 Allysa has 20 blocks, consisting of 4 sets of 5 blocks. Each set consists of 1 triangle, 1 square, 1 parallelogram and 2 identical pentagons of the same colour. The colours of the sets are red, yellow, blue and green. So, for example, the blue set consists of 1 blue triangle, 1 blue square, 1 blue parallelogram and 2 blue pentagons.

(a) Suppose Allysa picks 4 blocks and arranges them in a row. In how many different ways can this be done so that the 4 blocks include all colours and all shapes? [2]

(b) Suppose instead Allysa chooses 5 blocks. Find the number of ways in which the 5 blocks can be chosen if they include exactly 2 pentagons, at least 1 triangle, and at least 1 square. [3]

(c) All the blocks are now arranged in a circle. Find the number of different arrangements that can be made with all the pentagons together and no two triangles adjacent. [4]

9 In a game, a dealer tosses three fair six-sided dice, each numbered from 1 to 6 and a player pays a stake of $\$k$ to guess the numbers that appear on the top sides of the dice. The number of correct guesses the player makes in one game is denoted by X.

(i) Show that $\mathrm{P}(X=2)=\dfrac{5}{72}$ and tabulate the probability distribution of X. [3]

(ii) A sample of 30 observations of X is taken. Find the probability that the mean of this sample exceeds 1. [4]

For each game, the dealer pays back the stake if the player makes one correct guess, twice the stake if the player makes two correct guesses and ten times the stake if the player makes three correct guesses. The player loses the stake if there are no correct guesses.

(iii) If 3 games are played, find the probability that the player makes a net loss. [3]

10 **(a)** In a barber shop, the number of male customers who dye their hair in a particular day is denoted by X. It is given that X has the distribution $\mathrm{B}(6, p)$.

(i) Given that $\mathrm{Var}(X)=0.8466$ and $p<0.5$, find the value of p. [1]

(ii) Given instead that the probability that not all the male customers dye their hair is $\dfrac{117648}{117649}$, find the exact value of p. [2]

(b) Analysis of the customer demographics of a hair salon over a long period shows that 65% of the customers are female. In a day, the first n customers that the salon receives are observed and the number of those customers who are female is denoted by Y.

(i) State, in context, two assumptions needed for Y to be well modelled by a binomial distribution. [2]

Assume now that Y has a binomial distribution.

(ii) Find the probability that, on a randomly chosen day, out of the first 15 customers, the thirteenth customer is the eighth and last female customer. [3]

(iii) Find the probability that, in seven randomly chosen days, there are exactly three days with at least 8 female customers among the first 15 customers. [3]

11 Research is being carried out into how the concentration of a drug in the bloodstream varies with time, measured from when the drug is given. Observations at successive times give the data shown in the following table.

Time (t minutes)	15	30	60	90	120	150	180	240	300
Concentration (x micrograms per litre)	82	65	43	37	22	19	12	6	2

Calculate the equation of the regression line of x on t. [2]

12 (a)

Weight Indicator / Age (years)	**Underweight**	**Acceptable Weight**	**Overweight**
	$<5^{th}$ percentile	**5^{th} - $<90^{th}$ percentiles**	**$\geq 90^{th}$ percentile**
13	≤ 15.2	$15.2 - 25.0$	≥ 25.0
14	≤ 15.5	$15.5 - 25.5$	≥ 25.5
15	≤ 15.9	$15.9 - 26.1$	≥ 26.1
16	≤ 16.2	$16.2 - 26.5$	≥ 26.5
17	≤ 16.4	$16.4 - 27.0$	≥ 27.0
18	≤ 16.7	$16.7 - 27.4$	≥ 27.4

The table contains information extracted from the health booklets for all new born babies in Singapore. It shows the BMI-for-age for boys aged 13-18 years in Singapore.

(i) Assuming that the BMI for 18-year-old boys in Singapore is normally distributed with mean μ and variance σ^2, use suitable information from the above chart to find the value of μ and show that $\sigma = 3.656$ correct to 3 decimal places. [4]

(ii) State, with a reason, whether the BMI of the combined group of boys aged 13 and 18 years can be assumed to be normally distributed. [1]

(b) During the circuit breaker period, a restaurant in a neighbourhood offers free delivery service to residents staying at Redwood Heights. On a randomly chosen day, the amount of time required for food preparation upon the placement of an order is normally distributed with mean 20 minutes and standard deviation 4 minutes. When the food is ready, a deliveryman collects the food immediately and delivers it on bicycle to Redwood Heights. The delivery time on bicycle from the restaurant to Redwood Heights is normally distributed with mean 18.1 minutes and standard deviation 2 minutes.

(i) Find the latest time a food order has to be placed so that there is a 95% chance that the food is delivered to Redwood Heights by 7.00 pm. [4]

The restaurant owner also engages deliverymen who deliver on motorbikes. The delivery time on motorbike from the restaurant to Redwood Heights is normally distributed with mean 15 minutes and standard deviation 4 minutes. Both types of deliverymen are paid only for their delivery trips. Deliverymen by bicycles and by motorbikes are paid at a rate of \$1.00/min and \$1.20/min respectively.

(ii) Find the probability that a deliveryman making 10 deliveries by motorbike gets paid more than a deliveryman making 10 deliveries by bicycle.
State, with a reason, which type of deliverymen would the cost-conscious restaurant owner employ. [4]

Q1	Suggested Solutions	Marker's Comments
	The first 3 terms of the GP are $a+5d, a+3d, a+2d$. common ratio, $r=\frac{a+3d}{a+5d}=\frac{a+2d}{a+3d}$ $(a+3d)^2=(a+2d)(a+5d)$ $a^2+6ad+9d^2=a^2+7ad+10d^2$ $d^2+ad=0$ $d=-a$ (Since $d\neq 0$) $r=\frac{a+3(-a)}{a+5(-a)}$ $=\frac{1}{2}$	Surprising poorly done question. Many students did not even know that they should start by forming an equation using common ratio obtained from 2 pairs of consecutive terms of the GP. Common mistakes: • Wrongly assuming that a is the 1st term of GP when question defined it to be the 1st term of AP instead. • Wrong calculation of common ratio, r, i.e. taking $r=2$ instead of $r=\frac{1}{2}$. Students should be mindful that the question stated that the series is "a **convergent** geometric series" and did not specify a last term, thus it is an infinite series with $\|r\|$ less than 1.
	Required sum $=T_{15}+T_{16}+...$ $=(a+5d)r^{14}+(a+5d)r^{15}+...$ $=\frac{(-4a)\left(\frac{1}{2}\right)^{14}}{1-\left(\frac{1}{2}\right)}$ $=-\frac{a}{2048}$	Many students were unable to interpret the phrasing of this modified A level question accurately. Refer to [2016/P1/4(ii)]. • Most students are <u>not proficient</u> at expressing series using the sigma notation. Please DO NOT ATTEMPT to do it unless it is required in the question. Please note that • since the question stated "Find, in terms of a,…", the final answer must and should only have a as an unknown constant. • The definition of commonly used letters such as a, d, r, n varies according to questions. Do not blindly apply the formula. The following answers are not accepted as they are not evaluated • $-\frac{a}{2^{11}}$ • $-8a\left(\frac{1}{2}\right)^{14}$ or $-\frac{8a}{2^{14}}$

Q1	Suggested Solutions	Marker's Comments
	Alternative Method: Required sum $= S_\infty - \left(T_1 + T_2 + ... + T_{14}\right)$ $= \dfrac{-4a}{1-\left(\frac{1}{2}\right)} - \dfrac{(-4a)\left(1-\left(\frac{1}{2}\right)^{14}\right)}{1-\left(\frac{1}{2}\right)}$ $= -8a + 8a\left(1-\left(\frac{1}{2}\right)^{14}\right)$ $= 8a\left(1-\left(\frac{1}{2}\right)^{14} - 1\right)$ $= -\dfrac{a}{2048}$	

Q2	Suggested Solutions	Marker's Comments
	$\displaystyle\sum_{r=1}^{n}\frac{1}{\sqrt{r}+\sqrt{r-1}}$ $\displaystyle=\sum_{r=1}^{n}\frac{\sqrt{r}-\sqrt{r-1}}{\left(\sqrt{r}+\sqrt{r-1}\right)\left(\sqrt{r}-\sqrt{r-1}\right)}$ $\displaystyle=\sum_{r=1}^{n}\frac{\sqrt{r}-\sqrt{r-1}}{r-(r-1)}$ $\displaystyle=\sum_{r=1}^{n}\left(\sqrt{r}-\sqrt{r-1}\right)$ $=\left[\begin{array}{l}\left(\sqrt{1}-\sqrt{0}\right)\\+\left(\sqrt{2}-\sqrt{1}\right)\\+\left(\sqrt{3}-\sqrt{2}\right)\\\\+\left(\sqrt{n-1}-\sqrt{n-2}\right)\\+\left(\sqrt{n}-\sqrt{n-1}\right)\end{array}\right]$ $=\sqrt{n}$	Well done. Most students were able to perform the rationalisation process as well as the method of differences correctly. Some of the mistakes observed, which could have easily been avoided, include: • omitting the brackets in $\sum_{r=1}^{n}\left(\sqrt{r}-\sqrt{r-1}\right)$, • not writing the addition signs in the method of differences, • writing "blank" summations: $\sum_{r=17}^{100} = \sum_{r=1}^{100} - \sum_{r=1}^{16}$ (for the second part) • reading "seventeenth" as "seventh"
	$u_{17} + \quad + u_{100} = \sqrt{100} - \sqrt{16} = 10 - 4 = 6$	

Q3	Suggested Solutions	Marker's Comments
	Volume of the hemisphere of radius r cm $=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)=\frac{2}{3}\pi r^3$ Volume of hemisphere of radius $r/2$ cm $=\frac{1}{2}\left(\frac{4}{3}\pi\left(\frac{r}{2}\right)^3\right)=\frac{1}{12}\pi r^3$ Volume of the rectangular cuboid $=2h^3$ Total volume of the model $=\frac{2}{3}\pi r^3+\frac{1}{12}\pi r^3+2h^3$ $=\frac{3}{4}\pi r^3+2h^3$ Given the volume of the model is a fixed value 500 m^3, $\frac{3}{4}\pi r^3+2h^3=500$ $\Rightarrow 2h^3=500-\frac{3}{4}\pi r^3$ $\Rightarrow h=\left(250-\frac{3}{8}\pi r^3\right)^{1/3}$ Surface area of hemisphere of radius r m $=\frac{1}{2}\left(4\pi r^2\right)=2\pi r^2$ Surface area of hemisphere of radius $r/2$ m $=\frac{1}{2}\left(4\pi\left(\frac{r}{2}\right)^2\right)=\frac{1}{2}\pi r^2$ Surface area of the walls and the roof $=2\left(2h^2\right)+2h^2+2h^2-\pi r^2-\frac{1}{4}\pi r^2$ $=8h^2-\frac{5}{4}\pi r^2$ Surface area of the floor $=2h^2$	Majority of students are able to find h in terms of r and hence express S in terms of r. However, either the carelessness in differentiation of S with respect to r or solving the $\frac{dS}{dr}=0$ led to the wrong value of r obtained.

Q3 — Suggested Solutions

Total surface area of the model, $S=8h^2-\frac{5}{4}\pi r^2+2h^2+2\pi r^2+\frac{1}{2}\pi r^2$

$$=10h^2+\frac{5}{4}\pi r^2$$

Substitute $h=\left(250-\frac{3}{8}\pi r^3\right)^{1/3}$ in S:

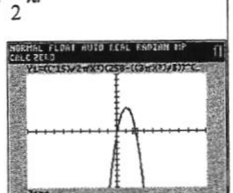
$$S=10\left(250-\frac{3}{8}\pi r^3\right)^{2/3}+\frac{5}{4}\pi r^2$$

Differentiating S with respect to r:

$$\frac{dS}{dr}=10\left(\frac{2}{3}\right)\left(-\frac{9}{8}\pi r^2\right)\left(250-\frac{3}{8}\pi r^3\right)^{-1/3}+\frac{5}{2}\pi r$$

$$=\left(-\frac{15}{2}\pi r^2\right)\left(250-\frac{3}{8}\pi r^3\right)^{-1/3}+\frac{5}{2}\pi r$$

Equating $\frac{dS}{dr}$ to 0 and using GC to solve,



$r=\sqrt[3]{\frac{2000}{3\pi+216}}=2.070186387=2.07$ (3 s.f.)

By 1st derivative test,

r	2	$\sqrt[3]{\frac{2000}{3\pi+216}}$	2.1
$\frac{dS}{dr}$	0.5541885	0	−0.248207
Slope	/	–	\

$\therefore r=2.07$ gives maximum S.

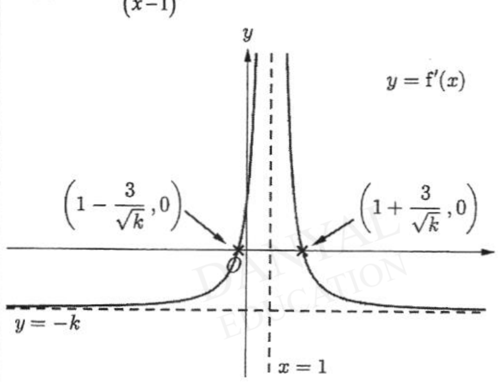
Marker's Comments

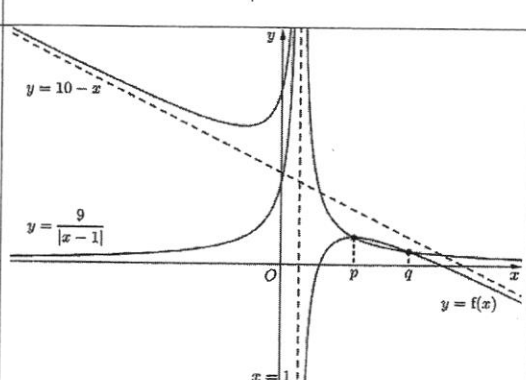
Many students did not make use of GC to solve $\frac{dS}{dr}=0$ or find the value of $\left.\frac{d^2S}{dr^2}\right\|_{x=2.0702}$.

Some students are not aware that they must put in specific values of $\frac{dS}{dr}$ when testing the sign of $\frac{dS}{dr}$ for different values of r. The following table is not accepted.

r	2	$\sqrt[3]{\frac{2000}{3\pi+216}}$	2.1
$\frac{dS}{dr}$	+	0	-
Slope	/	–	\

Q3	Suggested Solutions	Marker's Comments
	Alternatively, by 2^{nd} derivative test, By GC, $\left.\frac{\mathrm{d}^2S}{\mathrm{d}r^2}\right\|_{x=2.0702}=-8.1966725<0$ $\therefore$ Since $\frac{\mathrm{d}^2S}{\mathrm{d}r^2}<0$, $r=2.07$ gives maximum S.	

<table>
<tr><th>Q4</th><th>Suggested Solutions</th><th>Marker's Comments</th></tr>
<tr><td>(a)</td><td>$f'(x)=-k+\frac{9}{(x-1)^2}$
</td><td>The objective of this question is to check whether students know how to sketch $f'(x)$ graph, which happens to be a standard graph.
It should be a clear cut 3 marks to be performed given the level of competency of this cohort.
Many did not simplified $\sqrt{\frac{9}{k}}$ which may pose a potential issue that candidates are unable to simplify further.
For those who scored 2 marks, the common mistake is to mark the point $\left(1-\frac{3}{\sqrt{k}},0\right)$ on the positive x-axis.

Those who score 0 or 1 mark should review their learning on sketching $f'(x)$ graph and sketching of standard curves. One possible tip is to substitute k with a numerical value.</td></tr>
<tr><td>(b)</td><td></td><td>Learning objective:
Know how to use the GC to
(1) sketch the complete graph of a rational function.
(2) Solve inequality.

The specific GC skill involves the use of the zoom function which surprisingly was not well performed by many and resulted in the following mistakes :
(A) Stationary points incorrectly located (minimum point located where its x-coordinate is non-positive, or maximum located where the y-coordinate is non-positive)
(B) Unable to decipher the solution to the inequality, especially for $x<1$.

The use of "and" and comma in the final solution continues to stir confusion to those without strong concept and there is quite a number of such candidates.</td></tr>
</table>

Q4	Suggested Solutions	Marker's Comments
	The graphs of $y=10-x-\frac{9}{x-1}$ and $y=\frac{9}{\lvert x-1\rvert}$ intersects at $x=p$ and $x=q$. Using G.C. to solve, we have $p=4$ and $q=7$. Hence for $f(x)\geq\frac{9}{\lvert x-1\rvert}$, $x<1$ or $4\leq x\leq 7$.	In addition, there are many candidates who attempted to solve the inequality manually even though the question did not specifically state so, which also could have created unnecessary mistakes in their final answer. This could have also potentially resulted in time loss for other questions.

Q5	Suggested Solutions	Marker's Comments
(i)	Translate graph by b units in the positive x-direction Scale graph parallel to the x-axis by a factor of $\frac{1}{c}$. Scale graph parallel to the y-axis by a factor of a. Or Scale graph parallel to the x-axis by a factor of $\frac{1}{c}$. Translate graph by $\frac{b}{c}$ units in the positive x-direction. Scale graph parallel to the y-axis by a factor of a.	Majority of the students are proficient in the variable replacement to help them sequence the transformations correctly. Despite knowing the correct transformations, many are penalised for inaccurate phrasing of the descriptions. Here are some common ones: 1) Translate graph by b (missing "**units**") in positive x-**axis** (should be **direction**) 2) Scale graph by $\frac{1}{c}$ **units** (should be **factor of** $\frac{1}{c}$) **in** the x-**direction** (should be **parallel to the** x-**axis**) A few students did not read the question carefully and gave the transformations leading from C to $y=x^2$ instead.
(ii)	$147=a(-c-b)^2=ac^2+2abc+ab^2$ ---- (1) $3=a(2c-b)^2=4ac^2-4abc+ab^2$ ---- (2) $27=a(4c-b)^2=16ac^2-8abc+ab^2$ ---- (3) Using GC to solve the above equations, we get $ab^2=75,\quad abc=30,\quad ac^2=12$. Therefore $a=\frac{75}{b^2}$, $c=\frac{30}{ab}=\frac{30}{\left(\frac{75}{b^2}\right)b}=\frac{2}{5}b$	More than 95% of the students solve the system of equations manually (hence wasted precious time during the exam) since they did not realise that they could consider ac^2, abc, ab^2 as variables. Of these students, the following issues were observed: 1) Students did not check that the solutions obtained from solving 2 equations simultaneously must also satisfy the 3rd equation; 2) Students did not consider $\pm$ when taking square roots; 3) Students made a lot of careless algebraic mistakes when solving manually.
(iii)	$(-5,25)\to(-5+b,25)\to\left(\frac{-5+b}{c},25\right)\to\left(\frac{-5+b}{c},25a\right)$ Since $\frac{-5+b}{c}=0$, we have $b=5$. Hence $a=3,\ c=2$	Majority of students left this part blank. Of those who attempted, only a handful are successful. The others failed to use part (i) answer to see how the point $(-5, 25)$ is transformed to get the value of b and use part (ii) to get values of a and c. Students are not cognisant of the information given in the question. Many gave values of a, b and c that are not integers and/or negative.

Q6	Suggested Solutions	Marker's Comments
(i)	For $x\in[1,\infty)$, $\frac{\pi}{4}\le\tan^{-1}x<\frac{\pi}{2}$. Since $\tan^{-1}x\ge\frac{\pi}{4}>0$ and $1+x^2>0$, $\frac{\tan^{-1}x}{1+x^2}>0$. Thus, $\frac{\tan^{-1}x}{1+x^2}$ is positive. $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\tan^{-1}x}{1+x^2}\right)=\frac{(1+x^2)\left(\frac{1}{1+x^2}\right)-2x\tan^{-1}x}{(1+x^2)^2}$ $=\frac{1-2x\tan^{-1}x}{(1+x^2)^2}$ Since $2x\ge2$ and $\frac{\pi}{4}\le\tan^{-1}x<\frac{\pi}{2}$, we have $2x\tan^{-1}x\ge\frac{\pi}{2}\approx1.57$. Thus, $1-2x\tan^{-1}x\le1-\frac{\pi}{2}\approx-0.57<0$. Also, $(1+x^2)^2>0$ for all real values of x. This implies that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\tan^{-1}x}{1+x^2}\right)=\frac{1-2x\tan^{-1}x}{(1+x^2)^2}<0$. Hence, $\frac{\tan^{-1}x}{1+x^2}$ is decreasing.	Moderately well done. Candidates are required to show that $\tan^{-1}x>0$ for $x\ge1$, but not all did so convincingly. One common observed incorrect method was as follows: As $x\to\infty$, $\tan^{-1}x\to\frac{\pi}{2}$ and $1+x^2\to0$. Therefore, $\frac{\tan^{-1}x}{1+x^2}>0$. This is insufficient as the candidate has not shown that $\frac{\tan^{-1}x}{1+x^2}>0$ for ALL values of $x\ge1$. Similarly, this is also insufficient to show that $\frac{\tan^{-1}x}{1+x^2}$ is decreasing for all for ALL values of $x\ge1$ as this argument does not detect the presence of any stationary points or any parts of the function that may possibly be increasing. Most candidates were able to obtain $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\tan^{-1}x}{1+x^2}\right)=\frac{1-2x\tan^{-1}x}{(1+x^2)^2}$ correctly. However, a majority did not provide sufficient evidence as to why $1-2x\tan^{-1}x$ is negative. For this question, any methods of proof involving the sketch of the graph of $y=\frac{\tan^{-1}x}{1+x^2}$ are not accepted as this does not rule out the possibility of there being stationary points or parts of the graph that are increasing outside the sketch.

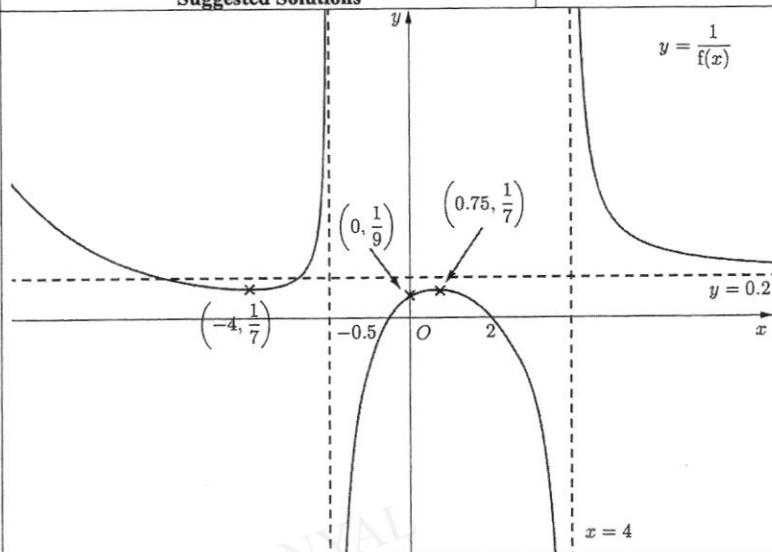
Q6	Suggested Solutions	Marker's Comments
(b)	$\int_1^N\frac{\tan^{-1}x}{1+x^2}\,\mathrm{d}x=\int_1^N\frac{1}{1+x^2}(\tan^{-1}x)\,\mathrm{d}x$ $=\left[\frac{(\tan^{-1}x)^2}{2}\right]_1^N$ $=\frac{(\tan^{-1}N)^2}{2}-\frac{(\tan^{-1}1)^2}{2}$ $=\frac{(\tan^{-1}N)^2}{2}-\frac{1}{2}\left(\frac{\pi}{4}\right)^2$ $=\frac{(\tan^{-1}N)^2}{2}-\frac{\pi^2}{32}$ $\int_1^\infty\frac{\tan^{-1}x}{1+x^2}\,\mathrm{d}x=\lim_{N\to\infty}\int_1^N\frac{\tan^{-1}x}{1+x^2}\,\mathrm{d}x$ $=\lim_{N\to\infty}\left(\frac{(\tan^{-1}N)^2}{2}-\frac{\pi^2}{32}\right)$ $=\frac{1}{2}\left(\frac{\pi}{2}\right)^2-\frac{\pi^2}{32}$ $=\frac{3\pi^2}{32}$, which is finite. Also, from **(i)**, $\frac{\tan^{-1}x}{1+x^2}$ is non-negative and non-increasing. Hence, by the Maclaurin-Cauchy test, $\sum_{r=1}^{\infty}\frac{\tan^{-1}r}{1+r^2}$ converges.	A decent proportion of candidates are able to obtain $\frac{(\tan^{-1}x)^2}{2}$ from standard integration techniques. However, the handling of the infinity sign ∞ was very poor. For example, instead of writing $\left[\frac{(\tan^{-1}x)^2}{2}\right]_1^\infty$, candidates should use the limit notation to write it as $\lim_{k\to\infty}\left[\frac{(\tan^{-1}x)^2}{2}\right]_1^k$. Similarly, the incorrect notation of $\tan^{-1}(\infty)$ should be written in a different manner, for example, $\lim_{x\to\infty}(\tan^{-1}x)=\frac{\pi}{2}$, or "as $x\to\infty$, $\tan^{-1}x\to\frac{\pi}{2}$". Please also note that in calculus, you should use **RADIANS** for all angles involved. The final mark for concluding that $\sum_{r=1}^{\infty}\frac{\tan^{-1}r}{1+r^2}$ converges is not given for most incorrectly calculated values of $\int_1^\infty\frac{\tan^{-1}x}{1+x^2}\,\mathrm{d}x$. Some who concluded that $\sum_{r=1}^{\infty}\frac{\tan^{-1}r}{1+r^2}$ converges made an additional error of concluding that "$\sum_{r=1}^{\infty}\frac{\tan^{-1}r}{1+r^2}$ converges to $\frac{3\pi^2}{32}$". The test does not tell us the value of the $\sum_{r=1}^{\infty}\frac{\tan^{-1}r}{1+r^2}$. (Additional optional note: the test does not allow us to conclude that $\sum_{r=1}^{\infty}\mathrm{f}(r)$ diverges if $\int_1^\infty\mathrm{f}(x)\,\mathrm{d}x$ is not finite.)

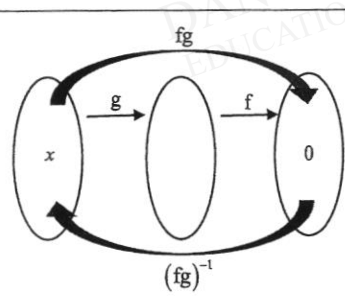
Q7	Suggested Solutions	Marker's Comments
(a)	$\int_0^{\pi} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right)+\pi \, dx$ $=\int_0^{\pi} \frac{1}{2}\cos\left(\frac{5x}{6}\right)+\frac{1}{2}\cos\left(\frac{x}{6}\right)+\pi \, dx$ $=\left[\frac{1}{2}\times\frac{6}{5}\sin\left(\frac{5x}{6}\right)+\frac{1}{2}\times 6\sin\left(\frac{x}{6}\right)+\pi x\right]_0^{\pi}$ $=\frac{3}{5}\times\frac{1}{2}+3\times\frac{1}{2}+\pi^2$ $=\frac{9}{5}+\pi^2$	Most well done. A few took some manipulation to get the sum correctly. A few used the wrong factor formula. A few did not put ½ or multiple the ½ to π. A few did the integration wrongly by multiplying 5/6 and 1/6 instead of dividing.
(b)(i)	Sum of area of $2n$ rectangles $=2$ (Sum of area of n rectangles from $0\le x\le \frac{\pi}{2}$) $=2\times\frac{\pi}{2n}\left\{f\left(\frac{\pi}{2n}\right)+f\left(\frac{2\pi}{2n}\right)+ \quad +f\left(\frac{n\pi}{2n}\right)\right\}$ $=\frac{\pi}{n}\left\{2\cos^2\left(\frac{\pi}{2n}\right)+1+2\cos^2\left(\frac{2\pi}{2n}\right)+1+ \quad +2\cos^2\left(\frac{(n-1)\pi}{2n}\right)+1+2\cos^2\left(\frac{n\pi}{2n}\right)+1\right\}$ $=\frac{\pi}{n}\left\{2\cos^2\left(\frac{\pi}{2n}\right)+2\cos^2\left(\frac{2\pi}{2n}\right)+ \quad +2\cos^2\left(\frac{(n-1)\pi}{2n}\right)+2\cos^2\left(\frac{n\pi}{2n}\right)+n\right\}$ $=\frac{2\pi}{n}\left\{\cos^2\left(\frac{\pi}{2n}\right)+\cos^2\left(\frac{2\pi}{2n}\right)+ \quad +\cos^2\left(\frac{(n-1)\pi}{2n}\right)+0\right\}+\frac{\pi}{n}\times n$ $=\frac{2\pi}{n}\left\{\cos^2\left(\frac{\pi}{2n}\right)+\cos^2\left(\frac{2\pi}{2n}\right)+ \quad +\cos^2\left(\frac{(n-1)\pi}{2n}\right)\right\}+\pi$	This is part was badly done. Many students failed to realise that $\cos\left(\frac{n\pi}{2n}\right)=\cos\left(\frac{\pi}{2}\right)=0$ so they only add $(n-1)$ rectangles to get the expression $\left\{2\cos^2\left(\frac{\pi}{2n}\right)+1+2\cos^2\left(\frac{2\pi}{2n}\right)+1+\cdots+2\cos^2\left(\frac{(n-1)\pi}{2n}\right)+1\right\}$. It would then be obvious that this approach is wrong because with only $n-1$ rectangles, there would only be $n-1$ number of 1s, which does not tally with the "show" expression. Instead of using symmetry, some students added the $2n$ rectangles $\frac{\pi}{n}\left\{2\cos^2\left(\frac{\pi}{2n}\right)+1+2\cos^2\left(\frac{2\pi}{2n}\right)+1+\cdots+2\cos^2\left(\frac{(2n-1)\pi}{2n}\right)+1\right\}$ which is not correct as they did not write $2\cos^2\left(\frac{n\pi}{2n}\right)+1+2\cos^2\left(\frac{n\pi}{2n}\right)+1$ in the middle. Many were able to sum the 1s to obtain $2n$. $2n-1$ is not accepted because question clearly states $2n$ rectangles. Students using this method must realise that they have to write 2 series and the sum of one series is equal to the other by symmetry.

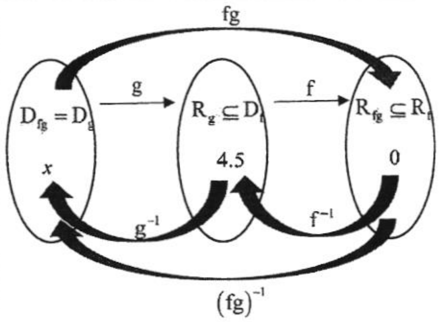
Q7	Suggested Solutions	Marker's Comments
		A few student obtained π indicating in their diagram by 1×π (area of the rectangles below the graph).
(b)(ii)	$S_\infty=\int_0^{\pi} 2\cos^2(x)+1 \, dx$ $=\int_0^{\pi}\cos 2x+2 \, dx$ $=\left[\frac{\sin 2x}{2}+2x\right]_0^{\pi}$ $=2\pi$	This is part was badly done. Many students write "As $n\to\infty$, $2\pi/n\to 0$, $\cos k(2\pi/n)\to 1$, etc. Hence $S_\infty=\pi$ or 2π or whatever". They failed to see S_∞ is $\int_0^{\pi} 2\cos^2(x)+1 \, dx$ Some student link S_∞ to $\int_0^{\infty} 2\cos^2(x)+1 \, dx$

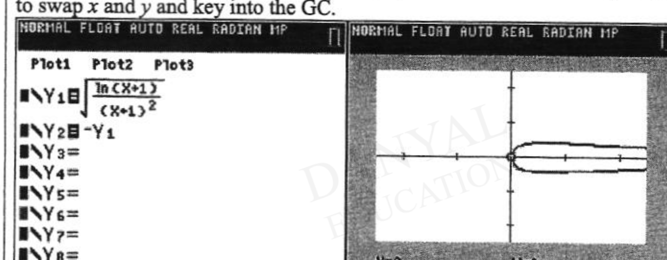
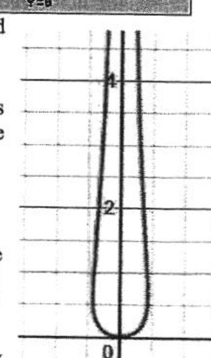
Q8	Suggested Solutions	Marker's Comments
(i)	$\dfrac{dx}{dt}=k(x-40)$ $\displaystyle\int \frac{1}{x-40}\,dx=\int k\,dt$ $\ln\lvert x-40\rvert=kt+C$ $x-40=e^{kt+C}\quad (\because x\ge 40)$ $x=40+Ae^{kt}$, where A is a constant When $t=0, x=60$, $A=60-40=20$ When $t=1800, x=50$, $50=40+20e^{1800k}\ \Rightarrow\ k=\dfrac{1}{1800}\ln\dfrac{1}{2}=-\dfrac{1}{1800}\ln 2$ Thus, we have $x=40+20e^{-\left(\frac{\ln 2}{1800}\right)t}$	This is a straightforward DE question. (Quantity) … at a rate proportional to (expression) is a standard phrasing that translate to $\dfrac{d(\text{quantity})}{dt}=k(\text{expression})$. There are students who still want to complicate matters by keeping k on LHS $\dfrac{1}{k}\displaystyle\int\frac{1}{x-40}\,dx=\int 1\,dt$ or by expanding $\displaystyle\int\frac{1}{kx-40k}\,dx=\int 1\,dt$. There are also stubborn students who did not remove modulus properly **BEFORE** substituting values. This should be done by either stating the condition $x\ge 40$ or through $\pm e^{C}=A$. For conditions, the most common mistake is not to take note of the units and proceed with $t=30$ instead of $t=1800$. For final answer, it is also common to have confusion over negative sign. When the k value is negative, and your form is $-k$, then your final answer should not have negative sign. For final answer, some convert it to $x=40+20\left(\frac{1}{2}\right)^{\frac{t}{1800}}$, which is unnecessary and definitely time-wasting if done wrongly. There are 2 mistakes that cannot be classified: (1) Creating a new condition to substitute - A significant number of students created a new condition of $t=1800$ and $\dfrac{dx}{dt}=\dfrac{50-60}{1800-0}$. This is a sign of doing too many practices and complicating the problem to fit what you know to do. (2) '40' is an outcast - Even though after integration, we have $x=40+Ae^{kt}$, students tend to leave out the 40 when substituting conditions.

Q8	Suggested Solutions	Marker's Comments
(ii)	$1000y=\displaystyle\int 40+20e^{\left(\frac{1}{1800}\ln\frac{1}{2}\right)t}\,dt$ $=40t+\dfrac{20}{\frac{1}{1800}\ln\frac{1}{2}}e^{\left(\frac{1}{1800}\ln\frac{1}{2}\right)t}+c$ $=40t+\dfrac{36000}{\ln 0.5}e^{\left(\frac{1}{1800}\ln\frac{1}{2}\right)t}+c$ Since $t=0, x=0$, we have $c+\dfrac{36000}{\ln 0.5}=0\ \Rightarrow\ c=-\dfrac{36000}{\ln 0.5}$ Converting t to hours, and equating y to 500, we have $40(3600t)+\dfrac{36000}{\ln 0.5}e^{\left(\frac{1}{1800}\ln\frac{1}{2}\right)(3600t)}-\dfrac{36000}{\ln 0.5}=500000$. Using GC, we have $t=3.1163446$. Thus, the user can stream for 3.1 hours continuously.	This is an interpretation-by-context DE question, which was not well done. Most students just equate x to 500 gigabytes to solve, which meant that they did not understand the relationship between x and y. Students who assume $1000y=xt$ most probably notice the clue from the units. Because x is megabytes per second and y is in gigabytes, which is easier to understand from the example of "speed in m/s versus distance in kilometres". However, based on Add Math Kinematics, we have learnt that we cannot find distance just by multiplying speed with time when speed is not constant. Thus, using this prior knowledge, we can now deduce that $1000y=\int x\,dt$. Some who managed to proceed have a misconception that c must be 0 (when $x=0$ and $y=0$) which is not the case in this question. Note that the step to convert t to hours in the suggested solution is not a necessary step to solve (can always do that after solving), but it does help to speed up the GC processing since the answer for t now will be small instead of a 5-digit answer. Also note that whether it is based on round-off to 1 d.p. or by context, the answer will be 3.1 hours and not 3.2 hours. An alternative solution to this question is to treat time as discrete and use GP sum to solve. A tricky thing to note is that if you treat t as discrete, then your final answer in seconds should be an integer, before you process your answer to hours. Otherwise, you are violating your own condition.

Q9	Suggested Solutions	Marker's Comments
(i)		
	Majority of the students got the correct asymptotes but missed out labelling the y-intercept. Many students are not alert to the fact that the minimum turning point and maximum turning point have the same y-coordinate. Some even sketched the maximum turning point (i.e $\left(0.75, \frac{1}{7}\right)$) above the horizontal asymptote $y = \frac{1}{5}$, without realising that $\frac{1}{7} < \frac{1}{5}$. A number of students got the graph wrong in $x < a$. They thought as $x \to -\infty, y \to \frac{1}{5}$.	

Q9	Suggested Solutions	Marker's Comments
(ii)	For the inverse of f to exist, smallest $k = 0.75$	This is well-done.
(iii)	$\mathrm{g} : x \mapsto \frac{2x+a}{x-b}$ for $x \in \mathbb{R}$, $x > b$, $a, b \in \mathbb{R}^+$. The graph of $y = \frac{2x+a}{x-b} = 2 + \frac{a+2b}{x-b}$ has vertical asymptote $x = b$ and horizontal asymptote $y = 2$. For the domain $x > b$, $\mathrm{R_g} = (2, \infty)$.	Many students applied long division to make the function into proper rational function to determine the correct equation of the horizontal asymptote, except that they did not note the given domain is $x > b$, thus stated the wrong range of $\mathrm{g}(x)$.
(iv)	 Let $(\mathrm{fg})^{-1}(0) = x$, from the mapping above, $\mathrm{fg}(x) = 0$, which implies $\mathrm{g}(x) = \mathrm{f}^{-1}(0)$.	This part is poorly attempted as many students left this blank. For those who attempted, many of them did not fully understand the concepts of the composite mapping. Common mistake made was that they took $(\mathrm{fg})^{-1}(0) = \mathrm{f}^{-1}\mathrm{g}^{-1}(0)$ and tried ways to find inverse of f. Another observation is that some students gave two answers without rejecting one of the answers. They should check that the value of x lies in the domain of g, i.e. $x > b$.

Q9	Suggested Solutions	Marker's Comments
	 From the graph of f in the domain $[0.75,\infty)$, $f^{-1}(0)=4$. Solve $g(x)=4$ $\dfrac{2x+a}{x-b}=4$ $\Rightarrow 2x+a=4x-4b$ $\Rightarrow 2x=a+4b$ $\Rightarrow x=0.5a+2b$	

Q10	Suggested Solutions	Marker's Comments
(i)	$x=0$ (or y-axis)	The most obvious method to notice the symmetry is through x^2. It means that any value x_1 and its negative version $-x_1$ will produce the same RHS value, which implies the same y-value. Thus, the curve is symmetrical in the y-axis.
(ii)(a)	Volume $=\pi\displaystyle\int_0^{e-1}\frac{\ln(y+1)}{(y+1)^2}\,dy$ $=\pi\displaystyle\int_0^{e-1}\frac{1}{(y+1)^2}\ln(y+1)\,dy$ $=\pi\left[\dfrac{-1}{(y+1)}\ln(y+1)\right]_0^{e-1}-\pi\displaystyle\int_0^{e-1}-\frac{1}{(y+1)}\frac{1}{(y+1)}\,dy$ $=\pi\left(-\dfrac{1}{e}\right)+\pi\displaystyle\int_0^{e-1}\frac{1}{(y+1)^2}\,dy$ $=\pi\left(-\dfrac{1}{e}\right)+\pi\left[\dfrac{-1}{(y+1)}\right]_0^{e-1}$ $=\pi\left(-\dfrac{1}{e}\right)+\pi\left(-\dfrac{1}{e}+1\right)$ $=\pi\left(1-\dfrac{2}{e}\right)$ *Those who have received P for this part need to review presentation for integration by parts.	We should try to get a sense of how the graph looks like and the way to do it is to swap x and y and key into the GC.  Thus, we know that the actual graph could be derived by reflecting in $y=x$ and look like this (see right).  A quick check and we would know the curve passes through the origin, which is the lowest point since $y<0$ causes $x^2=\dfrac{\ln(y+1)}{(y+1)^2}$ to be undefined. The rest is just pure integration techniques. One common error is splitting the integrand into $\dfrac{\ln(y+1)}{y+1}$ and $\dfrac{1}{y+1}$. Again, this is a sign of doing too many practices and complicating the problem to fit what you have done before.

Q10	Suggested Solutions	Marker's Comments
(ii)(b)	$x^2=\dfrac{\ln(y+1)}{(y+1)^2}$ $x^2(y+1)^2=\ln(y+1)$ $x^2\left[2(y+1)\dfrac{dy}{dx}\right]+2x(y+1)^2=\dfrac{1}{y+1}\dfrac{dy}{dx}$ When $y=e-1$, $x^2=\dfrac{\ln(e)}{(e)^2}=\dfrac{1}{e^2}\;\Rightarrow\; x=\pm\dfrac{1}{e}$ When $x=\dfrac{1}{e}$, $y=e-1$, we have $\dfrac{1}{e^2}[2e]\dfrac{dy}{dx}+\dfrac{2}{e}(e)^2=\dfrac{1}{e}\dfrac{dy}{dx}$ $\dfrac{1}{e}\dfrac{dy}{dx}=-2e$ $\dfrac{dy}{dx}=-2e^2$ Thus, the equation of normal is $y-(e-1)=\dfrac{1}{2e^2}\left(x-\dfrac{1}{e}\right)$. Since C is symmetrical about the y-axis, and P and Q share the same y-coordinate, their normals must meet at the y-axis. When $x=0$, $y-(e-1)=\dfrac{1}{2e^2}\left(-\dfrac{1}{e}\right)\;\Rightarrow\; y=e-\dfrac{1}{2e^3}-1$ $\therefore N\left(0,\, e-\dfrac{1}{2e^3}-1\right)$	Those who made mistakes in applying quotient rule and chain rule need to review their procedures. It is depressing to see the majority trying to make $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ the subject, which is an unnecessary step. The RECOMMENDED way is to sub x and y values once the implicit differentiation is completed. Because values are easier to manipulate than expressions. We emphasised this during Maclaurin Series. There are students who did not know that we are finding normal or they did not know that gradient of normal is $-1\div\dfrac{dy}{dx}$, which the $\dfrac{dy}{dx}$ should be a value (not an expression in x and y). It is heartening to see students making use of the clue that $x=0$ is the line of symmetry to solve for the final answer, rather than repeating the dreaded procedure of finding normal again. For those who went ahead to find the 2nd normal, the checking mechanism should be that the gradients of the 2 normals should be negative of each other due to the symmetry (not totally different values). It is also good to note that only a tiny minority did not give answer in coordinates. Most students are sensitive to the required form. Good!

Q11	Suggested Solutions	Marker's Comments
(a)	$w-6\alpha z=-5\alpha i+3$ --- (1) $\alpha z^2+iw=10\alpha+3i$ --- (2) Multiply (1) by i: $iw-6\alpha iz=5\alpha+3i$ --- (3) By elimination method, taking (2) − (3) : $\alpha z^2+6\alpha iz=5\alpha$ $\Rightarrow z^2+6iz-5=0$ $z=\dfrac{-6i\pm\sqrt{(6i)^2-4(1)(-5)}}{2(1)}$ $=\dfrac{-6i\pm\sqrt{-16}}{2}$ $=\dfrac{-6i\pm 4i}{2}$ $=\dfrac{-10i}{2}$ or $\dfrac{-2i}{2}$ $\therefore z=-5i$ or $-i$ (rejected since $\|z\|>1$) Substitute $z=-5i$ into (1): $w=6\alpha(-5i)-5\alpha i+3=3-35\alpha i$.	This was badly done. Students attempted the question by substitution method and then comparing the real parts and the imaginary parts. Careless mistakes are committed, hence many students lost the accuracy mark. Some students did not reject $z=-i$ as they did not realise that $\|z\|=\|-i\|=1$.

Q11	Suggested Solutions	Marker's Comments
(b)(i)	$\arg z_1 = -\tan^{-1}\frac{\sqrt{3}}{3} = -\frac{\pi}{6}$ and $\arg z_2 = \frac{\pi}{2}$ $\arg\left(\frac{z_1^{\ 3}}{z_2^{\ 2}}\right)^n = n\arg\left(\frac{z_1^{\ 3}}{z_2^{\ 2}}\right)$ $= n\left(\arg z_1^{\ 3} - \arg z_2^{\ 2}\right)$ $= n\left(3\arg z_1 - 2\arg z_2\right)$ $= n\left[3\left(-\frac{\pi}{6}\right) - 2\left(\frac{\pi}{2}\right)\right]$ $= -\frac{3\pi}{2}n$ $\arg\left(\frac{z_1^{\ 3}}{z_2^{\ 2}}\right)^n = \frac{\pi}{2}n$ (in principal range) For $\frac{z_1^{\ 3n}}{z_2^{\ 2n}} = \left(\frac{z_1^{\ 3}}{z_2^{\ 2}}\right)^n$ to be purely imaginary with a positive imaginary part, $\arg\left(\frac{z_1^{\ 3}}{z_2^{\ 2}}\right)^n = \ ,-\frac{3\pi}{2},\frac{\pi}{2},\frac{5\pi}{2},\frac{9\pi}{2},\ .=(4k+1)\frac{\pi}{2}, k\in\mathbb{Z}$ $\Rightarrow \frac{\pi}{2}n = (4k+1)\frac{\pi}{2}$ $\Rightarrow n = 4k+1$ For integer $n = \ldots,-3,1,5,9,13,\ldots = 4k+1, k\in\mathbb{Z}$	This part of the question is averagely done. Many students knew how to apply correct properties to find $\arg\left(\frac{z_1^{\ 3}}{z_2^{\ 2}}\right)^n$. But they did not express the $\arg\left(\frac{z_1^{\ 3}}{z_2^{\ 2}}\right)^n$ in principal range, hence did not realise that their final answer do not result in integer values of n. Some wrote $k\in\mathbb{R}$, which was wrong. Many did not observe that for argument of complex numbers which are purely imaginary with a positive imaginary part, the arguments are $\ldots,-\frac{3\pi}{2},\frac{\pi}{2},\frac{5\pi}{2},\frac{9\pi}{2},\ldots=(4k+1)\frac{\pi}{2}, k\in\mathbb{Z}$. They did not realise that the argument $(2k+1)\frac{\pi}{2}$, $k\in\mathbb{Z}$, include complex numbers which lie on the negative imaginary axis.

Q11	Suggested Solutions	Marker's Comments
(b)(ii)	$z_1^{\ 2} = 9-3-\mathrm{i}\left(6\sqrt{3}\right) = 6-\mathrm{i}\left(6\sqrt{3}\right)$ $z_1^{\ 3} = 27-\mathrm{i}\left(27\sqrt{3}\right)-27+\left(3\sqrt{3}\right)\mathrm{i} = -\mathrm{i}\left(24\sqrt{3}\right)$ Since $z_1 = 3-\mathrm{i}\sqrt{3}$ is a root of $px^3-9x^2+6x+36=0$, $p\left[-\left(24\sqrt{3}\right)\mathrm{i}\right]-9\left[6-\left(6\sqrt{3}\right)\mathrm{i}\right]+6\left(3-\mathrm{i}\sqrt{3}\right)+36=0$ $p\left[-\left(24\sqrt{3}\right)\mathrm{i}\right]=\left(48\sqrt{3}\right)\mathrm{i}$ $p=2$	Many students knew that they are to replace x with $3-\mathrm{i}\sqrt{3}$ to solve for p. The whole of Q11 stated that the use of calculator is not allowed in answering this question and yet a handful of students are not mindful and use GC in the calculation.
	Since the coefficients of the polynomial equation are real, by conjugate theorem, $z_1 = 3+\left(\sqrt{3}\right)\mathrm{i}$ is one of the roots. Thus, the remaining root is a real root. Let the remaining factor be $2x+a$. We have $2x^3-9x^2+6x+36=(2x+a)\left(x-3-\left(\sqrt{3}\right)\mathrm{i}\right)\left(x-3+\left(\sqrt{3}\right)\mathrm{i}\right)$ $=(2x+a)\left(x^2-6x+12\right)$ By observation, $a=3$. $2x^3-9x^2+6x+36=(2x+3)\left(x-3-\left(\sqrt{3}\right)\mathrm{i}\right)\left(x-3+\left(\sqrt{3}\right)\mathrm{i}\right)$ Note: $2x^3-9x^2+6x+36\neq\left(x+\frac{3}{2}\right)\left(x-3-\left(\sqrt{3}\right)\mathrm{i}\right)\left(x-3+\left(\sqrt{3}\right)\mathrm{i}\right)$ as $\left(x+\frac{3}{2}\right)$ is not accepted.	Common mistake made is they left the factor as $\left(x+\frac{3}{2}\right)$. Some students expressed their answer as $(2x+3)\left(3-\left(\sqrt{3}\right)\mathrm{i}\right)\left(3+\left(\sqrt{3}\right)\mathrm{i}\right)$ which they failed to understand what is meant by factor.

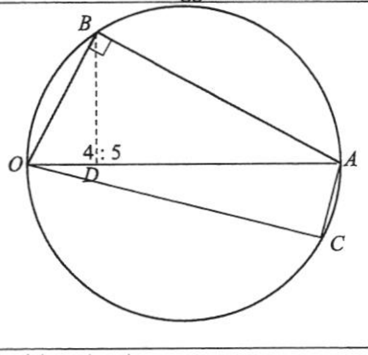
Q11	Suggested Solutions	Marker's Comments
	Alternatively, Since the coefficients of the polynomial equation are real, by conjugate theorem, $z_1 = 3+\left(\sqrt{3}\right)\text{i}$ is one of the roots. Thus, the remaining root is a real root. $px^3-9x^2+6x+36=(Ax+B)\left(x-3-\left(\sqrt{3}\right)\text{i}\right)\left(x-3+\left(\sqrt{3}\right)\text{i}\right)$ $\Rightarrow px^3-9x^2+6x+36=(Ax+B)\left(x^2-6x+12\right)$ By comparing coefficient of $x^3:\ p=A$ constant term : $36=12B \Rightarrow B=3$ $x^2: -9=-6A+B \Rightarrow A=2$ $\therefore p=2$ $\therefore 2x^3-9x^2+6x+36=(2x+3)\left(x-3-\left(\sqrt{3}\right)\text{i}\right)\left(x-3+\left(\sqrt{3}\right)\text{i}\right)$	

Q12	Suggested Solutions	Marker's Comments
(i)	$\overrightarrow{OA}=\begin{pmatrix}0\\4\\-8\end{pmatrix}$ $\quad\overrightarrow{OB}=\begin{pmatrix}8\\0\\-4\end{pmatrix}$ $\quad\overrightarrow{OC}=\begin{pmatrix}4\\8\\0\end{pmatrix}$ $\overrightarrow{AB}=\begin{pmatrix}8\\-4\\4\end{pmatrix}=4\begin{pmatrix}2\\-1\\1\end{pmatrix}$ $\quad\overrightarrow{AC}=\begin{pmatrix}4\\4\\8\end{pmatrix}=4\begin{pmatrix}1\\1\\2\end{pmatrix}$ Normal of plane $\pi=\begin{pmatrix}2\\-1\\1\end{pmatrix}\times\begin{pmatrix}1\\1\\2\end{pmatrix}=\begin{pmatrix}-3\\-3\\3\end{pmatrix}=-3\begin{pmatrix}1\\1\\-1\end{pmatrix}$ Equation of π: $\mathbf{r}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}=\begin{pmatrix}0\\4\\-8\end{pmatrix}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}=12$ $x+y-z=12$	The question was well done on the whole. Students know what to do but some were unable to find the position vectors of A, B and C. Nevertheless marks are awarded for the method of finding the equation of the plane.
(ii)	Equation of line consisting of O and F $\mathbf{r}=\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$ For coordinates of F, $\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}.\begin{pmatrix}1\\1\\-1\end{pmatrix}=12$ $\lambda=4$	This part is also well done even for students who were unable to find the correct position vectors earlier on. It is not recommended for students to memorize and use the formula for projection vector for a simple question of finding the foot of perpendicular. Some students remembered the wrong formula for finding the projection vector and misuse it in this case.

Q12	Suggested Solutions	Marker's Comments
	$\overline{OF}=4\begin{pmatrix}1\\1\\-1\end{pmatrix}$ Coord of F is $(4,4,-4)$.	
(iii)	Equation of line DE: $\mathbf{r}=\begin{pmatrix}8\\0\\4\end{pmatrix}+t\begin{pmatrix}1\\0\\0\end{pmatrix}$ Let the point on DE nearest to P be Q whose position vector is $\begin{pmatrix}8+\lambda\\0\\4\end{pmatrix}$. $\left(\begin{pmatrix}8+\lambda\\0\\4\end{pmatrix}-\begin{pmatrix}2\\11\\1\end{pmatrix}\right)\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=0$ $6+\lambda=0$ $\lambda=-6$ Hence, $OQ=\begin{pmatrix}2\\0\\4\end{pmatrix}$. Shortest distance from P to line $ED=PQ$ $=\left\|\begin{pmatrix}2\\0\\4\end{pmatrix}-\begin{pmatrix}2\\11\\1\end{pmatrix}\right\|=\left\|\begin{pmatrix}0\\-11\\3\end{pmatrix}\right\|$ $=\sqrt{130}$ m The shortest distance the pigeon has to fly is $\sqrt{130}$ m.	This part is also well done. However, many students did not read the question carefully and did not use " hence" to find the shortest distance. No marks are awarded for such students even when the answer is correct.

Q12	Suggested Solutions	Marker's Comments
(iv)	$\frac{x-1}{3}=\frac{2-y}{4}=\frac{z+3}{1}\Rightarrow\mathbf{r}=\begin{pmatrix}1\\2\\-3\end{pmatrix}+\mu\begin{pmatrix}3\\-4\\1\end{pmatrix}$ $\mathbf{r}=\begin{pmatrix}2\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}0\\-11\\3\end{pmatrix}$ Equating the two flight paths' equations, $\begin{pmatrix}1+3\mu\\2-4\mu\\-3+\mu\end{pmatrix}=\begin{pmatrix}2\\-11\lambda\\4+3\lambda\end{pmatrix}$ Solving the first 2 equations, we have $\mu=\frac{1}{3}$ and $\lambda=-\frac{2}{33}$. However, substituting in the third equation, we have $-3+\frac{1}{3}=-\frac{8}{3}\neq\frac{42}{11}=4-\frac{2}{11}$. Thus, the flight paths do not cross.	Some students are unable to write down the correct equation of the lines. Nevertheless, full method marks are awarded if they equate the 2 equations and show that there is no common point of intersection. No marks are awarded if the students use GC. Almost all the students know the method to do this question.

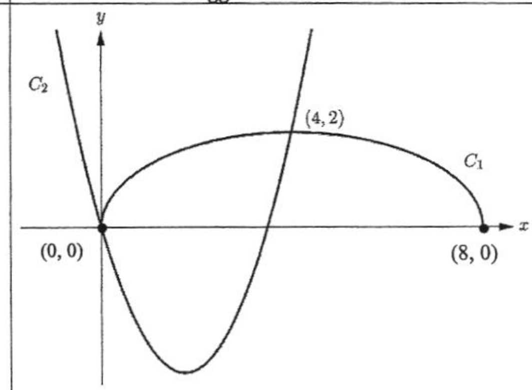
Q1	Suggested Solutions	Marker's Comments
(i)	$y=\sin^{-1}(2x)+1$ $\frac{dy}{dx}=\frac{2}{\sqrt{1-(2x)^2}}$ $=2(1-4x^2)^{-\frac{1}{2}}$ $=2\left[1+\left(-\frac{1}{2}\right)(-4x^2)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-4x^2)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(-4x^2)^3+...\right]$ $=2\left[1+2x^2+6x^4+20x^6+...\right]$ $\approx 2+4x^2+12x^4+40x^6$ The expansion of $(1-4x^2)^{-\frac{1}{2}}$ up to and including the term in x^6 is valid if $\lvert 4x^2\rvert<1$ $-\frac{1}{2}<x<\frac{1}{2}$	Critical error: $\sin^{-1}(2x)=\frac{1}{\sin 2x}$. Note that inverse functions for trigonometry are not reciprocal functions. There is a reason why we have the cosec function. The 2 clues to know how to approach are: - Question says standard series (not by differentiating), and sin inverse is not one of the standard series. - Questions asks to find series for $\frac{dy}{dx}$, not y. So we should at least find out what is $\frac{dy}{dx}$ first. Because this is a "show" question, it is important to preserve the intermediate working, especially between the coefficients and the variables. E.g. $1+\left(\frac{1}{2}\right)(4x^2)+\frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{2!}(4x^2)^2+\frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{3!}(4x^2)^3$ is not acceptable. There are a lot of errors and carelessness in solving the inequalities. Non-simplified final answers are not accepted, i.e. $-\sqrt{\frac{1}{4}}<x<\sqrt{\frac{1}{4}}$ and $\lvert x\rvert<\frac{1}{2}$ (question ask for x, not $\lvert x\rvert$).
(ii)	$y\approx\int 2+4x^2+12x^4+40x^6\,dx$ $=2x+\frac{4}{3}x^3+\frac{12}{5}x^5+\frac{40}{7}x^7+c$ When $x=0, \sin^{-1}(2x)+1=1 \Rightarrow c=1.$ $\therefore \sin^{-1}(2x)\approx 1+2x+\frac{4}{3}x^3+\frac{12}{5}x^5+\frac{40}{7}x^7$	Most students who use the integration approach left c as it is or totally left out c. Students who did repeated differentiation usually get the full 2 marks, but the effort/time spent to get this 2 marks is not worth it.

Q2	Suggested Solutions	Marker's Comments
		
	As $\|\mathbf{a}\|=1$, $\|\mathbf{a}\cdot\mathbf{b}\|$ represents either (1) the length of projection of **b** onto **a**, or (2) the length of OD. As AB and **b** are perpendicular (properties of semi-circle) $\mathbf{b}\cdot(\mathbf{b}-\mathbf{a})=0$ $\|\mathbf{b}\|^2-\mathbf{a}\cdot\mathbf{b}=0$ $\mathbf{a}\cdot\mathbf{b}=\|\mathbf{b}\|^2$	The common mistake is "the length of projection of **a** onto **b**", which shows that there was no identification of whether **a** or **b** is a unit vector. For these geometrical interpretation questions, identifying unit vector is an important step. Some did not state that $\overline{AB}$ and **b** are perpendicular before $\mathbf{b}\cdot(\mathbf{b}-\mathbf{a})=0$.
	$OD:DA=4:5$ and $\|\overline{OA}\|=\|\mathbf{a}\|=1$ Let angle $AOB=\theta$. $\cos\theta=\dfrac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}=\dfrac{\|\mathbf{b}\|^2}{\|\mathbf{a}\|\|\mathbf{b}\|}=\|\mathbf{b}\|$.	Most students are able to show $\cos\theta=\|\mathbf{b}\|$ or $\dfrac{4}{9\|\mathbf{b}\|}$ or something, after applying $\mathbf{a}\cdot\mathbf{b}=\|\mathbf{b}\|^2$

Q2	Suggested Solutions	Marker's Comments
	Considering triangle AOB, $\cos\theta=\dfrac{\|\mathbf{b}\|}{\|\mathbf{a}\|}$. $\dfrac{\|\mathbf{b}\|}{\|\mathbf{a}\|}=\dfrac{\frac{4}{9}\|\mathbf{a}\|}{\|\mathbf{b}\|}\Rightarrow\|\mathbf{b}\|^2=\dfrac{4}{9}\|\mathbf{a}\|^2=\dfrac{4}{9}$ as **a** is a unit vector Hence $\cos\theta=\dfrac{2}{3}$.	
	Area of triangle $OCD=\frac{1}{2}(OD)$ (height) $=\frac{1}{2}\left(\frac{4}{9}\right)(0.4)=\frac{8}{90}$ OR $=\dfrac{1}{2}\left\|\overline{OC}\times\overline{OD}\right\|=\dfrac{1}{2}\left\|\mathbf{c}\times\dfrac{4}{9}\mathbf{a}\right\|=\dfrac{1}{2}\left(\dfrac{4}{9}\right)(0.4)=\dfrac{8}{90}=\dfrac{4}{45}$	Mostly well done. The most common mistake is to take the ratio to be 4/5, rather than 4/9.

Q3	Suggested Solutions	Marker's Comments
(i)	$a_2 = 3.5a_1 - 1.2$ $a_3 = 3.5a_2 - 1.2$ $= 3.5(3.5a_1 - 1.2) - 1.2$ $= 3.5^2 a_1 - 1.2(3.5) - 1.2$ $= 3.5^2 a_1 - 1.2(1+3.5)$ $a_4 = 3.5\left[3.5^2 a_1 - 1.2(1+3.5)\right] - 1.2$ $= 3.5^3 a_1 - 1.2(1+3.5+3.5^2)$ $a_n = 3.5^{n-1} a_1 - 1.2(1+3.5+3.5^2+...+3.5^{n-2})$ $= 3.5^{n-1} a_1 - 1.2\left(\frac{(1)(3.5^{n-1}-1)}{3.5-1}\right)$ $= 3.5^{n-1} a_1 - 0.48(3.5^{n-1}-1)$ (Shown)	Learning points: • "the number of cells <u>increases by</u> 250% from the start of the week to the end of the week". So before the transfer is done, the (total) number of cells at the end of the week is 3.5 times the amount from the start of the week. • There is a need to show clear GP pattern before the application of formula. i.e. writing out of the following line is the preferred $a_n = 3.5^{n-1} a_1 - 1.2(1+3.5+3.5^2+...+3.5^{n-2})$ Common mistake: • The number of terms for GP $1+3.5+3.5^2+...+3.5^{n-1}$ is n, not $n-1$. Writing this wrong working in the previous line before application of formula will not give the correct expression for a_n.
(ii)	By the end of week 3, the number of cells have increased by 250% in from the beginning of week 3. No. of particles in <u>millions</u> before the cells are transferred to another laboratory $= a_4 + 1.2$ $= 3.5^{4-1}(0.5) - 0.48(3.5^{4-1}-1) + 1.2$ $= 2.5375$ <u>Alternatively</u> No. of particles in <u>millions</u> before the cells are transferred to another laboratory $= a_3 \times 3.5$ $= \left(3.5^{3-1}(0.5) - 0.48(3.5^{3-1}-1)\right) \times 3.5$ $= 2.5375$	This part is not well done. Candidates were not able to interpret the question correctly. Common mistakes: • a_3 is the number of cells (in millions) in Lab A at the <u>start of the 3rd</u> week. • a_4 is the number of cells (in millions) in Lab A at the <u>start of the 4th</u> week i.e. the number of cells (in millions) in Lab A at the <u>end of the 3rd</u> week <u>after</u> the cells are transferred.

Q3	Suggested Solutions	Marker's Comments
(iii)	Let the number of cells (in millions) in Lab B at the start of the nth week is given by d_n $d_1 = 0$ $d_2 = 0.5$ $d_3 = 0.5 + 5$ $d_4 = 0.5 + 5 + (5+5)$ $= 0.5 + 5 + 2(5)$ $d_n = 0.5 + 5 + 2(5) + 3(5) + ... + (n-2)(5)$ $= 0.5 + \frac{n-2}{2}(10 + (n-2-1)(5))$ $= 0.5 + \frac{n-2}{2}(10 + 5(n-3))$ $3.5^{n-1}(0.5) - 0.48(3.5^{n-1}-1) + 0.5 + \frac{n-2}{2}(10+5(n-3)) \geq 60$ By G.C., least $n = 6$ $\therefore$ The earliest week is week 6.	Very poorly done part. Common mistakes: • Failed to interpret question correctly and did not attempt to construct a table or do listing to recognise the pattern. • Did not recognise that for $d_n = 0.5 + 5 + 2(5) + 3(5) + ... + (n-2)(5)$, the formula for sum of the first n terms of an arithmetic progression is only applicable for $5 + 2(5) + 3(5) + ... + (n-2)(5)$, with first term = 5 (not 0.5), total number of terms = $n-2$. • Taking the sum of a_n when formulating the inequality when it is already a formula which gives the (total) number of cells (in millions) in Lab A at the start of the nth week.

<table>
<tr><th>Q4</th><th>Suggested Solutions</th><th>Marker's Comments</th></tr>
<tr><td></td><td>
</td><td>The most common mistakes made in this question are
<table>
<tr><th>Issue</th><th>Remarks</th></tr>
<tr><td>The relative position of (4,2) is not at the maximum point at C_1.</td><td>Candidates should pay attention to such basic details.</td></tr>
<tr><td>C_1 is still drawn as a full ellipse and not taking the range of the parameter into consideration.</td><td>This is a <u>serious</u> issue should candidates continue to commit the mistake because this issue is highlighted umpteen times throughout the past 1.5 years. Repeating the same mistake over and over again indicates a lack of awareness of the importance of sketching curves in parametric form.</td></tr>
</table></td></tr>
<tr><td>(a)</td><td>Area bounded by C_1 and C_2</td><td>The assessment objective of this question is to see if students are able to
- set up the correct definite integral that represents the area to be considered
- use the parametric equations provided to perform substitution to find part of the definite integrals.
- Use of appropriate trigonometric identity to evaluate $\int \cos^2\theta \, d\theta$.

Many candidates have difficulty setting up the correct definite integral. This is especially so when there are confusions where the region to consider "crosses over" the x-axis, which will not be a concern if students consider the difference of integral "upper curve minus lower curve". This concept has been emphasised in lessons during integration lessons.

As a result, many versions came in which answers may not make sense and they make a big mess in their working. It is suggested that students who score 3 marks or less in this part of the question attempt the same question again a few weeks later.</td></tr>
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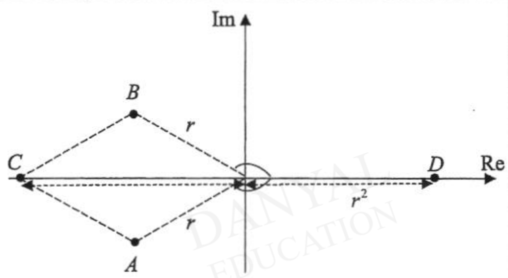
<table>
<tr><th>Q4</th><th>Suggested Solutions</th><th>Marker's Comments</th></tr>
<tr><td></td><td>$=\int_0^4 y_1 - y_2 \, dx$
$=\int_{-\frac{\pi}{2}}^{0} 2\cos\theta\left(4\cos\theta\right) d\theta - \int_0^4 x^2 - \frac{7}{2}x \, dx$
$=4\int_{-\frac{\pi}{2}}^{0} 2\cos^2\theta \, d\theta - \left[\frac{x^3}{3} - \frac{7x^2}{4}\right]_0^4$
$=4\int_{-\frac{\pi}{2}}^{0} \left(1+\cos 2\theta\right) d\theta - \left[\frac{x^3}{3} - \frac{7x^2}{4}\right]_0^4$
$=4\left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{0} - \left[\frac{64}{3} - \frac{7\times 16}{4}\right]$
$=\left(2\pi + \frac{20}{3}\right) \text{units}^2$</td><td>Another <u>serious</u> issue that is commonly seen is students are failure to use the substitution to change the upper and lower limits of the definite integral. This part is definitely emphasised during lessons and yet not correctly applied in this question.</td></tr>
<tr><td>(b)</td><td>Since C_2 is a quadratic curve with x-intercepts 0 and a, in order for C_1 and C_2 to meet at two distinct points, $0 < a \le 8$.</td><td>This part of the question is higher ordered, where some students relate to discriminant because they probably noted "two distinct points".

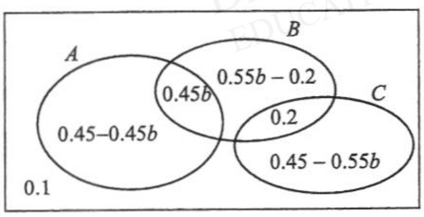
The easiest way actually is to look at the sketch the students themselves draw and 'vary' the value of a.</td></tr>
</table>

Q5	Suggested Solutions	Marker's Comments
(i)	$\lvert w^{*}z\rvert = r^2 \Rightarrow \lvert w^{*}\rvert\lvert z\rvert = r^2$ $\Rightarrow \lvert w\rvert\lvert z\rvert = r^2$ $\Rightarrow \lvert w\rvert = r$ $\arg\left(-\frac{w}{z^3}\right) = \pi - 4\theta \Rightarrow \arg(-w) - \arg(z^3) = \pi - 4\theta$ $\Rightarrow \arg(-1) + \arg(w) - 3\arg(z) = \pi - 4\theta$ $\Rightarrow \pi + \arg(w) - 3\theta = \pi - 4\theta$ $\Rightarrow \arg(w) = -\theta$	This is generally done well by students as majority showed their ability to apply relevant properties of modulus and argument. For $\arg\left(-\frac{w}{z^3}\right)$, a significant number of students have difficulty with managing the negative sign. Of these students, some pretended that it did not exist while the rest simply invented their own property: $\arg(-z) = -\arg(z)$. There is a small group of students who applied $\arg(-z^3) = 3\arg(-z)$ which is correct for the case of odd power but not true in general. Credit is given to these students but do note the conditions above.
(ii)	 Note that since z and w are conjugates of each other, $z+w = 2\,\mathrm{Re}(z) = 2r\cos\theta$ and $zw = \lvert z\rvert^2 = r^2$ must therefore lie on the real axis. $\lvert z+w\rvert = -2r\cos\theta$ or $2r\lvert\cos\theta\rvert$.	A significant majority of students did not get credit for $\lvert z+w\rvert$. Common mistakes observed: - $\lvert z+w\rvert = \lvert z\rvert + \lvert w\rvert = 2r$ - since $z+w = 2r\cos\theta$, then $\lvert z+w\rvert = 2r$ (how bewildering) - $\lvert z+w\rvert = 2r\cos\theta$ (not realising that $\cos\theta$ is negative in this question) As for the sketching of points on the Argand diagram, many students seemed to know that $zw = r^2$ and hence many placed point D on the positive real axis with some not getting credit for it because length of OD is shorter than that of OB or OA. Most showed that they know points A and B are reflection of each other in the real axis. Common mistakes observed are: - C on the positive real axis resulting in a concave quadrilateral $ACBD$ because they thought that $z+w = 2r\cos\theta$ is positive; - $OACB$ is not a rhombus; - Point A is not in 3rd quadrant.

Q5	Suggested Solutions	Marker's Comments
(iii)	Area of the quadrilateral $ACBD$ $= 2\times\frac{1}{2}(-r\sin\theta)(-2r\cos\theta + r^2)$ $= -r\sin\theta(-2r\cos\theta + r^2)$ $= 2r^2\sin\theta\cos\theta - r^3\sin\theta$ $= r^2\sin 2\theta - r^3\sin\theta$	This part surprisingly proves to be a "killer" as a significant majority of students left this part blank or could not score full marks mainly due to the following reasons: - Based on the points A, B, C and D, they had a wrong quadrilateral and hence a different area - They are confused with the signs of the trigonometric terms since θ being in the 3rd quadrant added complexity in the manipulation.

Q6	Suggested Solutions	Marker's Comments
(i)	$\bar{x}=\dfrac{1261.05}{105}=12.01$ $s^2=\dfrac{1}{n-1}\left[\sum x^2-\dfrac{\left(\sum x\right)^2}{n}\right]$ $=\dfrac{1}{104}\left[15145.47-\dfrac{(1261.05)^2}{105}\right]$ $=0.0024951923$ $=0.00250$ (3 s.f.)	Most students managed to get the correct numerical values of the unbiased estimates of the population mean and variance. Many of them, however, wrote that these values are for population mean and variance. This is quite a disappointment as they did not demonstrate understanding of sample versus population. A small number of students did not get the correct value for s^2 and this is disturbing to the marker as the formula to calculate this quantity is in MF26!
(ii)	$\mathrm{H_0}:\mu=12$ versus $\mathrm{H_1}:\mu>12$ Under $\mathrm{H_0}$, $\bar{X}$ $\mathrm{N}\left(12,\ \dfrac{s^2}{105}\right)$ approximately by Central Limit Theorem since $n=105$ is large. Upper-tailed test at 3% level of significance. Critical region: Reject $\mathrm{H_0}$ if p-value <0.03 p-value $=0.0201157096<0.03$ Reject $\mathrm{H_0}$ and conclude that there is sufficient evidence at 3% level of significance that the mean diameter, after the modification, of the spherical pellets has increased.	A significant majority of students are able to do this part well except for some who did not demonstrate any evidence of learning. Common mistakes include: - $X\sim\mathrm{N}(12,\ s^2)$, $\bar{X}\sim\mathrm{N}\left(12.01,\ \dfrac{s^2}{105}\right)$, $\bar{X}\sim\mathrm{N}(12,\ s^2)$ - Conclusion is incomplete, for example, many did not state "at 3% level of significance", or simply just state that there is sufficient evidence to reject H_0, etc A handful of students omitted "mean" and wrote "… to claim that the diameter of the spherical pellet has increased". Another handful wrote "… to claim that the mean diameter of the spherical pellet *has changed*".
(iii)	There is a probability of 0.0201 that the sample mean diameter is more than 12.01 mm, given that the population mean diameter is 12 mm.	Only a very small number of students gained credit for this part. Common mistakes include: - Relating this with level of significance;

Q6	Suggested Solutions	Marker's Comments
		- Using vague terms like "mean is more extreme than the one observed"; - Missing out "given that the population mean diameter is 12 mm"; - Inaccurate content like "probability that mean is more than 12mm"; - Confused with the meaning of level of significance; - Gibberish statements that marker could not comprehend.
(iv)	Since the sample size is 105 and it is considered as large, hence we can apply Central Limit Theorem to approximate the distribution of $\bar{X}$ to be normal.	Only a very small number of students gained credit for this part. Common mistakes include: - Using vague terms like "it is normal by Central Limit Theorem", "distribution can be normal by ….", "mean diameter can be normally distributed", etc; - Central Limit Theorem approximates the population distribution to be normal; - Population size is large; - Simply stated that Central Limit Theorem can be applied without saying the result of the application.

Q7	Suggested Solutions	Marker's Comments
(i)	$\mathrm{P}(A'\cap B)$ $=\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=b-0.45b$ $=0.55b$ $\mathrm{P}(A')\times\mathrm{P}(B)$ $=(1-0.45)\times b$ $=0.55b$ Or $\mathrm{P}(A'\cap B)$ $=\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=\mathrm{P}(B)-\mathrm{P}(A)\mathrm{P}(B)$ $=(1-\mathrm{P}(A))\mathrm{P}(B)$ $=\mathrm{P}(A')\mathrm{P}(B)$ As $\mathrm{P}(A'\cap B)=\mathrm{P}(A')\times\mathrm{P}(B)$, A' and B are independent events.	Most students are able to show. A few students started with $\mathrm{P}(A'\cap B)=\mathrm{P}(A')\mathrm{P}(B)$ or used words "since". Some used conditional probabilities with similar mistakes. Most common mistakes is $\mathrm{P}(A'\cap B)=0.55b$.
OR	$\mathrm{P}(A'\mid B)=1-\mathrm{P}(A\mid B)=1-\mathrm{P}(A)=\mathrm{P}(A')$ As $\mathrm{P}(A'\mid B)=\mathrm{P}(A')$, A' and B are independent events.	
(ii)	$\mathrm{P}(A\cap B')=0.45-0.45b$ $\mathrm{P}(A'\cap B\cap C')=0.55b-0.2$	Well done except for some calculation errors.
(iii)	$\mathrm{P}(B'\cap C)=(1-0.1)-(0.45-0.45b)-b$ $=0.45-0.55b$  $0.55b-0.2\geq 0$ and $0.45-0.45b\geq 0$ and $0.45-0.55b\geq 0$	Poorly done. Most students did not find $\mathrm{P}(B'\cap C)$ or setup inequalities before arriving at maximum or minimum values.

Q7	Suggested Solutions	Marker's Comments
	$\Rightarrow b\geq\frac{4}{11}$ and $b\leq 1$ and $b\leq\frac{9}{11}$ $\Rightarrow\frac{4}{11}\leq b\leq\frac{9}{11}$ $\Rightarrow\frac{9}{55}\leq 0.45b\leq\frac{81}{220}$ Maximum possible value of $\mathrm{P}(A\cap B)$ is $\frac{81}{220}$. Minimum possible value of $\mathrm{P}(A\cap B)$ is $\frac{9}{55}$.	

Q8	Suggested Solutions	Marker's Comments
(a)	$\underbrace{\binom{4}{1}}_{\substack{\text{select 1}\\\text{shape of}\\\text{colour 1}}} \times \underbrace{\binom{3}{1}}_{\substack{\text{select 1}\\\text{remaining}\\\text{shape of}\\\text{colour 2}}} \times \underbrace{\binom{2}{1}}_{\substack{\text{select 1}\\\text{remaining}\\\text{shape of}\\\text{colour 3}}} \times \underbrace{\binom{1}{1}}_{\substack{\text{select 1}\\\text{remaining}\\\text{shape of}\\\text{colour 4}}} = 24$ No. of arrangements of 4 chosen blocks $= 4!$ $\therefore$ No. of arrangements $= 24 \times 4! = 576$.	Poorly done, most candidates scored 1 out of 2 marks. Note that once a block a chosen, the subsequent block cannot be of the same colour or shape. Also, the two pentagons of the same colour are considered identical, while many candidates treated them as distinct in their calculations.
(b)	<u>Case 1: 2 pentagons of different colours or same colour, 1 triangle and 1 parallelogram, 1 square</u> $\underbrace{\left[\binom{4}{2}+\binom{4}{1}\right]}_{\substack{\text{Two cases: select 2}\\\text{pentagons of different}\\\text{colours and of the}\\\text{same colour}}} \times \underbrace{\binom{4}{1}}_{\substack{\text{select 1}\\\text{triangle}}} \times \underbrace{\binom{4}{1}}_{\substack{\text{select 1}\\\text{parallelogram}}} \times \underbrace{\binom{4}{1}}_{\substack{\text{select 1}\\\text{square}}} = 640$ <u>Case 2: 2 pentagons of different colours or same colour, 2 triangles and 1 parallelogram</u> $\underbrace{\left[\binom{4}{2}+\binom{4}{1}\right]}_{\substack{\text{Two cases: select 2}\\\text{pentagons of different}\\\text{colours and of the}\\\text{same colour}}} \times \underbrace{\binom{4}{2}}_{\substack{\text{select 2}\\\text{triangles}\\\text{of}\\\text{different}\\\text{colours}}} \times \underbrace{\binom{4}{1}}_{\substack{\text{select 1}\\\text{parallelogram}}} = 240$ <u>Case 3: 2 pentagons of different colours or same colour, 1 triangle and 2 parallelograms</u> $\underbrace{\left[\binom{4}{2}+\binom{4}{1}\right]}_{\substack{\text{Two cases: select 2}\\\text{pentagons of different}\\\text{colours and of the}\\\text{same colour}}} \times \underbrace{\binom{4}{1}}_{\substack{\text{select 1}\\\text{triangle}}} \times \underbrace{\binom{4}{2}}_{\substack{\text{select 2}\\\text{parallelograms}\\\text{of}\\\text{different}\\\text{colours}}} = 240$ Total number of ways $= 640 + 240 + 240$ $= 1120$	Poorly done. A majority of candidates treated the pentagons of the same colour as distinct objects, which is incorrect as the pentagons of the same colour are identical. Recall that $\binom{n}{r}$ is the number of ways to choose r objects from n **distinct** objects. Most candidates were able to consider the correct 3 cases for the 3 blocks apart from the two pentagons, with a minority missing out the case with 2 pentagons, 1 triangle, 1 square and 1 parallelogram. Some candidates attempted to do the complement method, which is more difficult. Most who used this method identified the complement cases wrongly, or missed out the highlighted case below: Required no. of ways = no. of ways to choose exactly 2 pentagons – no. of ways to choose exactly 2 pentagons and no triangles – no. of ways to choose exactly 2 pentagons and no squares + no. of ways to choose exactly 2 pentagons and no squares and no triangles. A handful of candidates mistook this question as an arrangement question and included a multiplication of 5! in their solutions.

Q8	Suggested Solutions	Marker's Comments
Last part	Step 1: Number of ways to arrange the pentagons (as one unit), parallelograms and squares in a circle $= \dfrac{9!}{9}$ Step 2: Number of ways to arrange all pentagons next to each other $= \dfrac{8!}{2!2!2!2!}$ Step 3: Number of ways to slot in the distinct triangles $= \binom{9}{4} \times 4!$ Total number of arrangements $= \dfrac{9!}{9} \times \dfrac{8!}{2!2!2!2!} \times \binom{9}{4} \times 4!$ $= 3.072577536 \times 10^{11}$ $= 3.07 \times 10^{11}$	This part was moderately well done. A decent proportion of candidates were able to score at least half the marks in this part. Some common mistakes observed were: • Not dividing 8! by 2!2!2!2! when arranging the pentagons (again, pentagons of the same colour are considered identical!). • Arranging the blocks in an **incorrect order**, which results in undercounting or overcounting. • Attempting the complement method and producing the incorrect complement cases. Due to the large final answer, candidates are allowed to present their answer as 3.07×10^{11}. (Refer to 2019 A-Level Paper 2 Q6(iii))

Q9	Suggested Solutions	Marker's Comments

(i)

$\mathrm{P}(X=2)=\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times 3=\frac{15}{216}=\frac{5}{72}$

$\mathrm{P}(X=0)=\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}=\frac{125}{216}$

$\mathrm{P}(X=1)=\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times 3=\frac{75}{216}=\frac{25}{72}$

$\mathrm{P}(X=3)=\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{216}$

X	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{125}{216}$	$\frac{25}{72}$	$\frac{5}{72}$	$\frac{1}{216}$

Marker's Comments: Well-performed by many.

(ii)

$\mathrm{E}(X)=0\left(\frac{125}{216}\right)+1\left(\frac{25}{72}\right)+2\left(\frac{5}{72}\right)+3\left(\frac{1}{216}\right)$

$=\frac{1}{2}$

$\mathrm{Var}(A)=\mathrm{E}(A^2)-\left(-\frac{101}{216}\right)^2$

$=0^2\left(\frac{125}{216}\right)+1^2\left(\frac{25}{72}\right)+2^2\left(\frac{5}{72}\right)+3^2\left(\frac{1}{216}\right)-\left(\frac{1}{2}\right)^2$

$=\frac{5}{12}$ or 0.417

Let $\bar{A}=\frac{A_1+\ldots+A_{30}}{30}$

Since the distribution of A is not a normal distribution and $n=30$ is large enough, by Central Limit Theorem,

Marker's Comments:

The part on finding the expectation of X and the method to find variance of X is well performed by many.

The justification part is not well performed. There are students who still use Central Limit Theorem to approximate X to a normal distribution, clearly demonstrating their weak concept the use of this theorem.

In addition, in a justification question, it is necessary for students to state clearly what "n" is and the significance of this "n". The following phrase is vague :

Statement/Phrase	Issue
Since n is large, …	What is n? Not properly defined.
Since $n=30$,	Here we can assume n is the sample size but, what is the significance of this size?

Q9	Suggested Solutions	Marker's Comments

$\bar{A}\sim \mathrm{N}\left(\frac{1}{2},\frac{5/12}{30}\right)$ approximately.

$\mathrm{P}(\bar{A}>1)=0.00001105257393$

$=0.0000111$ (3 s.f.)

(iii)

Let A be the net gains in dollars for each game

a	$-k$	0	k	$9k$
$\mathrm{P}(A=a)$	$\frac{125}{216}$	$\frac{25}{72}$	$\frac{5}{72}$	$\frac{1}{216}$

$\mathrm{P}(A_1+A_2+A_3<0)$

$=\mathrm{P}(\text{all "}-k\text{"})+\mathrm{P}(2\text{"}-k\text{","}0\text{"})+\mathrm{P}(2\text{"}-k\text{","}k\text{"})+\mathrm{P}(\text{"}-k\text{",}2\text{"}0\text{"})$

$=\left(\frac{125}{216}\right)^3+\frac{3!}{2!}\left(\frac{125}{216}\right)^2\left(\frac{25}{72}\right)+\frac{3!}{2!}\left(\frac{125}{216}\right)^2\left(\frac{5}{72}\right)+\frac{3!}{2!}\left(\frac{125}{216}\right)\left(\frac{25}{72}\right)^2$

$=0.821740406$

$=0.822$ (3 s.f.)

Marker's Comments:

Many students make the assumption that this question is about finding expectations and variances and went ahead to find, **without reading the question carefully**. As a result, unnecessary time is wasted and no marks are awarded.

Another common issue shown in students' work is failure to consider all 4 cases. Hence many score 1 to 2 marks.

Q10	Suggested Solutions	Marker's Comments
(a)(i)	$6p(1-p)=0.8466$ $p^2-p-0.1411=0$ $\therefore p=0.17$ (since $p<0.5$)	Almost all the candidates were able to score for this part.
(a)(ii)	$\mathrm{P}(X\neq 6)=\dfrac{117648}{117649}$ $1-\mathrm{P}(X=6)=\dfrac{117648}{117649}$ $\mathrm{P}(X=6)=\dfrac{1}{117649}$ $\binom{6}{6}p^6(1-p)^0=\dfrac{1}{117649}$ $p^6=\dfrac{1}{117649}$ $\therefore p=\dfrac{1}{7}$	Learning points: • Other than interpreting the phrase "exact value" literally, it simply means that all algebraic manipulations to get to the final answer must be shown clearly. Candidates can still use calculator to check that the final numerical values obtained are evaluated and simplified. In this question, candidates should have used the calculator to evaluate $\sqrt[6]{117649}$. Common mistakes • Wrongly interpreted the question as $\mathrm{P}(X\neq 0)=\dfrac{117648}{117649}$ • Used calculator and expressed the answer in terms of 3 s.f. instead of an exact value.
(b)(i)	The event that a customer is female is independent of the event that another customer is female. The probability that any one customer is female is the same throughout for the first n customers.	Candidates need to note that if any underlined keywords are missing, the answers are incomplete. Unacceptable answers (this list is not exhaustive): • The event that the salon receives a <u>female customer</u> is independent of the event that the salon receives another <u>female customer</u>. / The probability that the salon receives a <u>female customer</u> is the same throughout for the first n customers. • The event that the <u>customers are</u> female is independent of the event that other customers are female. / The probability that the <u>customers are</u> female is the same throughout for the first n customers.

Q10	Suggested Solutions	Marker's Comments
		• The probability that a customer is female is the equal for the first n customers. (this implies that the probability is $\dfrac{1}{n}$) • The probability that a customer is female is the same <u>for all female / woman / the female customers.</u>
(b)(ii)	Let Y be the number of female customers out of 12. $Y\sim \mathrm{B}(12,0.65)$ Required probability $=\mathrm{P}(Y=7)\times 0.65\times(1-0.65)^2$ $=0.2039195668\times 0.65\times 0.35^2$ $=0.0162370955$ $=0.0162$ (3 s.f.)	Candidates were able to comprehend the question with varying degree of success, with most noting that n should not be taken to be 15.
(b)(iii)	Let F be the number of female customers out of 15. $F\sim \mathrm{B}(15,0.65)$ $\mathrm{P}(F\geq 8)=1-\mathrm{P}(F\leq 7)$ $=1-0.1132311308$ $=0.8867688692$ Let W be the number of days out of 7 with at least 8 female customers who visits the salon within the 1st hour. $W\sim \mathrm{B}(7,\mathrm{P}(F\geq 8))$ $\mathrm{P}(W=3)=0.0040120168$ $=0.00401$ (3 s.f.)	Question is well done by most candidates.

Q12	Suggested Solutions	Marker's Comments
(a)	Let X be the random variable denoting the BMI for a 18 year old boy in Singapore. $X \sim \mathrm{N}\left(\mu,\sigma^2\right)$ $\mathrm{P}\left(X<16.7\right)=0.05$ $\mathrm{P}\left(Z<\dfrac{16.7-\mu}{\sigma}\right)=0.05$ $\dfrac{16.7-\mu}{\sigma}=-1.644853626$ $\mu-1.6449\sigma=16.7---(1)$ $\mathrm{P}\left(X>27.4\right)=0.1$ $\mathrm{P}\left(Z>\dfrac{27.4-\mu}{\sigma}\right)=0.1$ $\dfrac{27.4-\mu}{\sigma}=1.281551567$ $\mu+1.2816\sigma=27.4---(2)$ Solving (1) and (2) simultaneously, we have $\mu=22.71418212$ and $\sigma=3.656363113$ $=22.7$ (3 s.f.) $=3.656$ (3 d.p.)	Some students are unable to interpret the information from the set of data given and were unable to start off with the question. Quite a number of students assumed that the distribution is symmetrical. It is important for them to remember not to make assumptions based on what they have done on the past and to read every question carefully. Quite a number of students ignore the word "show" and concluded that σ=3.656 immediately from the GC without showing any working or more decimal places. 1 mark is deducted for such cases.
(a)(ii)	No. based on the data, the group will have a distribution that is bimodal and thus cannot be normal.	This part is badly done. Many students assumed that it is a test on CLT and wrongly concluded that CLT can be applied. Some students mention that the means are too far apart or means are different but did not mention that this will lead to a bimodal distribution. No marks are given. Students who do not know the term bimodal and wrote the distribution has 2 peaks are also given full credit.

Q12	Suggested Solutions	Marker's Comments
	Alternative : It cannot be assumed that the BMI for 13 year old boys in Singapore is normally distributed. Hence no conclusion can be made about the combined sample.	Though our testing objective was intended to be the knowledge of a bimodal distribution, it wasn't stated in the question that the BMI for a 13 year old is assumed to be normally distributed. As such the alternative answer is also given full credit.
(b)(i)	Let X be the time taken for food preparations. Let Y be the time taken for the delivery time by a delivery man on bicycle. Let t be the time taken for food prep such that there is a 95% chance that the food is delivered by 7pm. $X \sim N\left(20,4^2\right)$ $Y \sim N\left(18.1,2^2\right)$ $X+Y \quad N\left(38.1,2^2+4^2\right)$ $\mathrm{P}(X+Y<t)\geq 0.95$ $t \geq 45.5$ mins She should place her order latest by 6.14pm.	This part is well done. Some students do not understand what they are doing and found $\mathrm{P}\left(X+Y>t\right)=0.95$ instead. Most students use $\mathrm{P}\left(X+Y<t\right)=0.95$ and obtained $t=45.5$. As such they were confused if they should conclude 6.14pm or 6.15pm. Students who concluded correctly are given the full credit in this case. Almost the entire cohort is penalised 1 presentation mark for the inability to define the random variable precisely for this part. "Let Y be the time taken for the delivery time by **a delivery man** on bicycle." The point about **"a delivery man"** is required. Many defined it as "time taken for the delivery time by bicycle." In some cases even more vague "delivery time". Please note that "delivery time on bicycle for a randomly selected food order" is also not accepted.
(b)(ii)	Let W be the time taken for the delivery time by a deliveryman on motorbike.	Many students are unable to find the correct mean and variance for this part. Some students still do not know the concept $\mathrm{Var}\left(W-Y\right)=\mathrm{Var}\left(W\right)+\mathrm{Var}\left(Y\right)$. Some students use the wrong random variable $12Y-10W$. It is important to understand that $1.2(W_1+W_2+\ldots W_{10})$ does not equate to 12W.

Q12	Suggested Solutions	Marker's Comments
	$W \sim N\left(15,4^2\right)$ $Y \sim N\left(18.1,2^2\right)$ $D=1.2\left(W_1+\ldots W_{10}\right)-1\left(Y_1+\ldots Y_{10}\right) \sim \mathrm{N}\left(-1,270.4\right)$ $\mathrm{P}(D>0)=0.476$	Similar to the previous part, the random variable is badly defined. "Let *Y* be the time taken for the delivery time by **a delivery man** on motorcycle." The point about **"a delivery man"** is required. A total of 1 presentation mark is deducted for both parts.
	Since there is a higher probability that a delivery man on bicycle earns more than one on motorbike. The restaurant owner should employ those on motorbike to save more money.	Students have to make the judgement based on the probability they have found and not based on the expected salary of a delivery man. Marks are also awarded to students who mentioned that it is less likely that a delivery man on motorbike earns more than a delivery man on bicycle. No marks are awarded when the students mention average salary of a delivery man on motorbike is lesser than that of a delivery man on bicycle.