NATIONAL JUNIOR COLLEGE
SENIOR HIGH 1 PROMOTIONAL EXAMINATIONS
HIGHER 2

NAME

SUBJECT CLASS: 1ma2

REGISTRATION NUMBER

MATHEMATICS 9758

01 October 2020

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

Writing Paper

READ THESE INSTRUCTIONS FIRST

This paper constitutes 70% of your overall score for SH1 H2 Mathematics.

Write your name, class and registration number on the work you hand in.
Please write clearly and use capital letters.

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	4	
2	6	
3	5	
4	4	
5	7	
6	9	
7	7	
8	7	
9	11	
10	11	
11	9	
12	8	
13	12	
Presentation Deduction		–1 / –2
TOTAL	100	

This document consists of **28** printed pages and **0** blank page.

[Turn over

1 A curve has equation $y = ax^3 + bx^2 + cx$ where a, b and c are constants. The curve passes through the point with coordinates $(1, 12)$ and the gradient of the curve is -112 at the point where $x = 3$. When the curve is reflected in the y-axis, the resulting curve passes through the point $(-2, 3)$. Find the values of a, b and c. [4]

2 Using partial fractions, find $\int \frac{2x^2 - x + 7}{(1-x)(3x^2+5)}\,dx$. [6]

3 Point P is a variable point on the circumference of a circle with fixed radius r cm. Two fixed points A and B lie on the circumference of the circle such that the line segment AB forms the diameter of the circle. Length BP is given by y cm and length AP is given by x cm. Given that the rate of increase of x is $0.3r$ cm per second, find, in terms of r, the rate of increase of the area of triangle APB when x is r cm. [5]

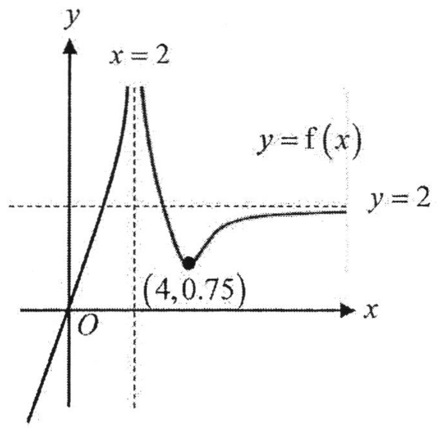
4 A curve has equation $y + \ln y = e^x$.

(i) Show that $\frac{d^2y}{dx^2}(y^2 + y) = y^2 e^x + \left(\frac{dy}{dx}\right)^2$. [2]

(ii) Explain why the curve is concave upwards for all real values of x. [2]

5 **(a)** Describe a sequence of transformations which transform the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{6-3x}{3x-4}$. [4]

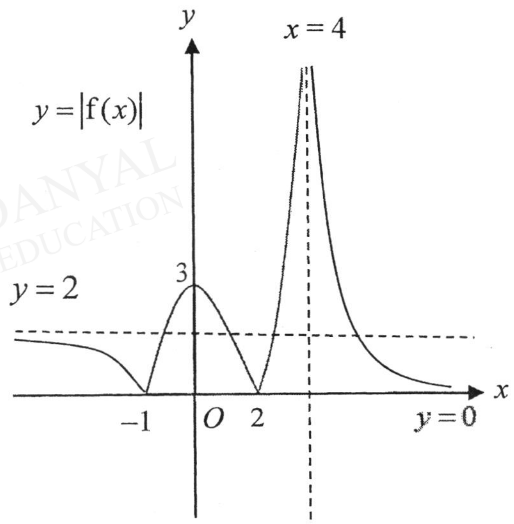
(b) The diagram shows the graph of $y = f(x)$ with asymptotes with equations $x = 2$ and $y = 2$. The curve passes through the origin and has a turning point at $(4, 0.75)$.

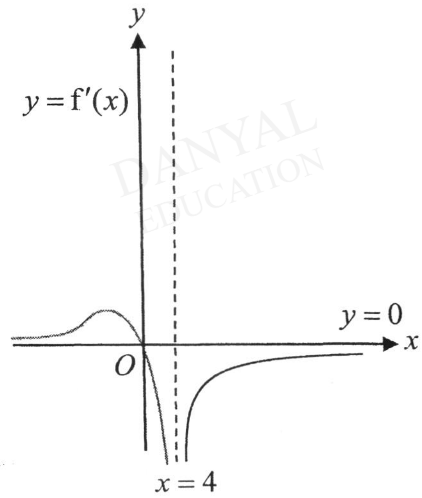


Sketch the graph of $y = \frac{1}{f(x)}$, clearly labelling the equations of the asymptotes and the coordinates of any turning points. [3]

6 **(a)** Without using a calculator, solve the inequality $\dfrac{4x}{x^2-3x-4} \geq -x$. Hence solve the inequality $\dfrac{4|x|}{x^2-3|x|-4} < -|x|$. [6]

(b) The diagrams below show the graphs of $y=|f(x)|$ and $y=f'(x)$.





Sketch the graph of $y=f(x)$, labelling the equations of any asymptotes and the coordinates of the turning point and the points where the curve crosses the axes. [3]

7 Referred to the origin, points A and B have non-zero position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively.

(i) Show that $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b}) = -2(\mathbf{a}\times\mathbf{b})$. [2]

It is given that $|\mathbf{b}| = 2|\mathbf{a}|$.

(ii) Find the maximum value of $|(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})|$ in terms of $|\mathbf{a}|$. [3]

It is further given that A is a fixed point on the x-axis and B is a variable point on the xz-plane.

(iii) Given that the value of $|(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})|$ is maximum, write down all possible expressions for $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})$ in terms of $|\mathbf{a}|$. [2]

8 Points A, B and C are non-collinear and origin O is not on the plane ABC. Point P has position vector given by $\overrightarrow{OP} = \lambda\overrightarrow{OA} + \mu\overrightarrow{OB} + (1-\lambda-\mu)\overrightarrow{OC}$, where $\lambda, \mu \in \mathbb{R}$.

(i) Prove that P lies in the plane ABC. [2]

It is now given that $\overrightarrow{OA} = \mathbf{i}+\mathbf{j}+\mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i}+2\mathbf{j}+4\mathbf{k}$ and $\overrightarrow{OC} = -2\mathbf{i}+\mathbf{k}$. Furthermore, P is the point of reflection of B in the line passing through A and C.

(ii) Find the value of λ and of μ. [5]

9 **(a)** The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{1}{1+x} - 2, \qquad \text{for } x \in \mathbb{R},\ x \geq 0,$$

$$\mathrm{g}: x \mapsto \left(x+\frac{4}{3}\right)^2 - \frac{1}{5}, \qquad \text{for } x \in \mathbb{R}.$$

(i) Determine whether the composite function fg exists. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ and find the range of f. [2]

(iii) Find the exact range of gf. [2]

(b) The function h is defined by

$$\mathrm{h}: x \mapsto (x+1)^2(x-2),\ x \in \mathbb{R}.$$

(i) Explain why the function h^{-1} does not exist. [1]

For the rest of the question, the domain of h is restricted to $x \in [-1,1]$.

(ii) State the domain of h^{-1}. [1]

(iii) Sketch on the same diagram the graphs of $y = \mathrm{h}(x)$, $y = \mathrm{h}^{-1}(x)$ and $y = \mathrm{h}^{-1}\mathrm{h}(x)$. [3]

10 A curve C has parametric equations

$$x = t^3, \quad y = 1+\sqrt{1-t^2} \quad \text{where } -1 \leq t \leq 1.$$

(i) Sketch C, labelling clearly the axial intercept. [2]

(ii) Use the substitution $t = \cos\theta$ to find the exact value of $\int_0^{\frac{\sqrt{3}}{2}} t^2\sqrt{1-t^2}\ \mathrm{d}t$. [5]

(iii) Hence, find the exact area bounded by C, the x-axis and the lines $x = 0$ and $x = \dfrac{\sqrt{27}}{8}$.

[4]

11 **(i)** Find $\int \frac{x}{\sqrt{1-k^2x^2}}\,\mathrm{d}x$, where k is a positive constant. [2]

(ii) Hence, find $\int \left(\sin^{-1} kx\right)\frac{x}{\sqrt{1-k^2x^2}}\,\mathrm{d}x$. [3]

(iii) Evaluate $\int_0^{\frac{1}{\sqrt{2}}} \left(\sin^{-1} x\right)\frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x$, giving your answer in the form $\frac{1}{\sqrt{a}}\left(1-\frac{\pi}{b}\right)$, where a and b are integers to be determined. [2]

(iv) Deduce the exact value of

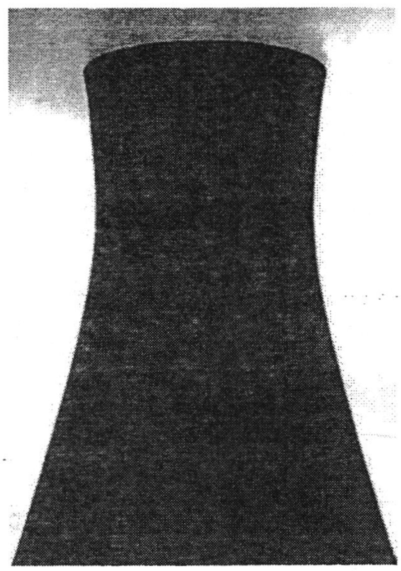
$$\int_m^{\frac{1}{\sqrt{2}}+m} \left[\sin^{-1}(x-m)\right]\frac{x-m}{\sqrt{1-(x-m)^2}}\,\mathrm{d}x$$

for any constant m. Explain your answer. [2]

12 A curve D has equation $3ax^2 - ay^2 = 1$, where $a > 0$ and $-60 \le y \le 30$.

(i) Sketch D, labelling clearly the axial intercepts and the coordinates of the end-points. [3]

Curve D traces the curved outline of the side view of a cooling tower as shown in the figure below. All units are in metres. The horizontal cross-sections of the cooling tower are circular planes whose centres lie on the y-axis. This hyperbolic form of the cooling tower allows it to withstand extreme winds while requiring less material than any other forms of their size and strength.



(Source: https://www.pleacher.com/mp/mlessons/calculus/mobaphyp.html)

An engineer is asked to design cooling towers for two different sites.

(ii) For the cooling tower at the first site, $a = 0.01$ is chosen. Find the exact volume contained by the tower. [You do not need to consider the thickness of the tower's walls.] [3]

(iii) For the cooling tower at the second site, another value of a is chosen such that the ratio of its smallest circumference to the circumference of its base is $1:2$. Determine the value of a to achieve this design. [2]

13 A curve C has equation $y = x^2 + 1$. A point Q on C has coordinates $\left(\frac{1}{\sqrt{2}}, \frac{3}{2}\right)$.

(i) Find the exact equation of the normal to C at Q. [3]

The **curvature** of a point P on a curve can be understood as a measure of the deviation of the curve from its tangent at P. The greater the curvature at P, the more the curve deviates from its tangent at P.

The curvature function of C is given by

$$\mathrm{k}(x) = \frac{2}{\left(1+4x^2\right)^{\frac{3}{2}}},$$

where $\mathrm{k}(a) = \dfrac{2}{\left(1+4a^2\right)^{\frac{3}{2}}}$ gives the curvature of C at the point $\left(a, a^2+1\right)$.

(ii) Find the exact curvature at Q. [1]

To visually represent the curvature at Q, mathematicians draw **calculation circles**. The calculation circles at Q are circles that pass through Q and have the same tangent as C at Q. The radii of the circles are equal to the reciprocal of the curvature at Q.

(iii) State the value of the radii of the calculation circles at Q. By considering this value and part **(i)**, find the exact equation of the calculation circle at Q whose centre has a negative x-coordinate. [4]

Mathematicians would like to investigate the calculation circles.

(iv) Sketch the graph of $y = \dfrac{2}{\left(1+4x^2\right)^{\frac{3}{2}}}$, labelling the coordinates of the turning point and the equation of the asymptote. [2]

(v) Hence, explain why there are four calculation circles on C with radius r for $r > \frac{1}{2}$. [2]

2020 SH1 H2 Math Promotional Examinations Markers' Comments

Qn.	Working	Comments
1	$\mathrm{f}(x)=ax^3+bx^2+cx$ $\mathrm{f}(1)=12$ $a+b+c=12----(1)$ $\mathrm{f}'(x)=3ax^2+2bx+c$ $\mathrm{f}'(3)=112$ $3a(3)^2+2b(3)+c=-112$ $\Rightarrow 27a+6b+c=-112----(2)$ $\mathrm{f}(-x)==a(-x)^3+b(-x)^2+c(-x)$ $=-ax^3+bx^2-cx$ At $x=-2, y=3$ $-a(-2)^3+b(-2)^2-c(-2)=3$ $\Rightarrow 8a+4b+2c=3----(3)$ System of equations: $a+b+c=12----(1)$ $27a+6b+c=-112----(2)$ $8a+4b+2c=3----(3)$ By GC, $a=-6.5, b=9, c=9.5$	Overall well done, a large majority of students scored full credit. Mistakes were generally on transformation, most common being wrongly replacing 'y' with '$-y$' to obtain $y=-ax^3-bx^2-cx$.

Qn.	Solution	Comments
2 (a)	$\dfrac{2x^2-x+7}{(1-x)(3x^2+5)}=\dfrac{A}{1-x}+\dfrac{Bx+C}{3x^2+5}$ $2x^2-x+7=A(3x^2+5)+(Bx+C)(1-x)$ When $x=1$: $8A=2-1+7$ $A=1$ Comparing coefficients:	Advice: It is imperative that you work on techniques central to A Level Mathematics such as the handling of partial fractions.

	x^2: $3-B=2$ $B=1$ $x^0\left(\text{constant}\right)$: $5+C=7$ $C=2$ Hence we get $\dfrac{2x^2-x+7}{(1-x)\left(3x^2+5\right)}=\dfrac{1}{1-x}+\dfrac{x+2}{3x^2+5}$ Therefore, $\displaystyle\int\frac{2x^2-x+7}{(1-x)\left(3x^2+5\right)}\,\mathrm{d}x$ $\displaystyle=\int\frac{1}{1-x}+\frac{x+2}{3x^2+5}\,\mathrm{d}x$ $\displaystyle=\int\frac{1}{1-x}\,\mathrm{d}x+\int\frac{x+2}{3x^2+5}\,\mathrm{d}x$ $\displaystyle=\int\frac{1}{1-x}\,\mathrm{d}x+\int\frac{x}{3x^2+5}\,\mathrm{d}x+\int\frac{2}{3x^2+5}\,\mathrm{d}x$ $\displaystyle=-\int\frac{-1}{1-x}\,\mathrm{d}x+\frac{1}{6}\int\frac{6x}{3x^2+5}\,\mathrm{d}x+\frac{2}{\sqrt{3}}\int\frac{\sqrt{3}}{\left(\sqrt{3}x\right)^2+\left(\sqrt{5}\right)^2}\,\mathrm{d}x$ $\displaystyle=-\ln\lvert1-x\rvert+\frac{1}{6}\ln\lvert3x^2+5\rvert+\frac{2}{\sqrt{3}}\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{5}}\right)+c$ $\displaystyle=-\ln\lvert1-x\rvert+\frac{1}{6}\ln\left(3x^2+5\right)+\frac{2}{\sqrt{15}}\tan^{-1}\left(\frac{\sqrt{15}x}{5}\right)+c$	Advice: It is absolutely crucial to recognise the need to split the integral in order to utilise existing integration techniques. Namely, you ought to start getting comfortable with recognising integrands of the form $\dfrac{f'(x)}{f(x)}$. In addition, the correct method for integration involving inverse tangent ought to be used! Method 1: Use $\displaystyle\int\frac{f'(x)}{a^2+\left[f(x)\right]^2}\,\mathrm{d}x$ $\displaystyle=\frac{1}{a}\tan^{-1}\left[\frac{f(x)}{a}\right]+C$

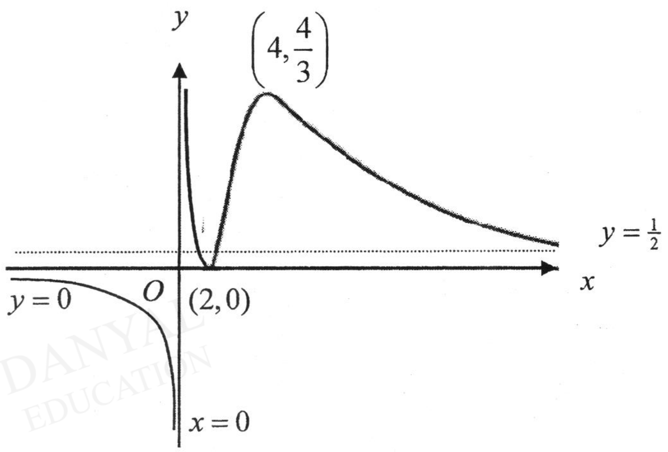
		OR Method 2: Use $\int \frac{1}{a^2+[x]^2}\,\mathrm{d}x$ $=\frac{1}{a}\tan^{-1}\left[\frac{x}{a}\right]+C$ Therefore, we have $\int \frac{2}{5+3x^2}\,\mathrm{d}x$ $=\frac{2}{3}\int \frac{1}{\frac{5}{3}+x^2}\,\mathrm{d}x$ $=\frac{2}{3}\int \frac{1}{\left(\sqrt{\frac{5}{3}}\right)^2+x^2}\,\mathrm{d}x$ $=\frac{2}{3}\left[\frac{1}{\sqrt{\frac{5}{3}}}\tan^{-1}\left(\frac{x}{\sqrt{\frac{5}{3}}}\right)\right]$ $=\frac{2\sqrt{3}}{3\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{5}}\right)$ $=\frac{2\sqrt{15}}{15}\tan^{-1}\left(\frac{\sqrt{15}x}{5}\right)+C$

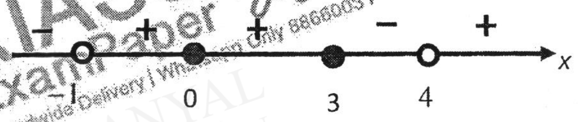
Qn.	Working	Comments
3	Since APB is a right-angle triangle, $x^2+y^2=(2r)^2$ $y=\sqrt{4r^2-x^2}$ (since $y>0$) $A=\frac{1}{2}xy=\frac{1}{2}x\sqrt{4r^2-x^2}$ $\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{2r^2-x^2}{\sqrt{4r^2-x^2}}$ When $x=r$, $\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{2r^2-r^2}{\sqrt{4r^2-r^2}}=\frac{r^2}{\sqrt{3r^2}}=\frac{r}{\sqrt{3}}$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$ $=\frac{r}{\sqrt{3}}\frac{3}{10}r$ $=r^2\frac{\sqrt{3}}{10}$ The rate of change of area is $\frac{\sqrt{3}}{10}r^2$ cm^2 / s	Many students are able to see that that they can write y in terms of x and r. From those who did, many realized that they need to differentiate the expression w.r.t x, however, some did not apply BOTH the product and Chain Rule. A more serious mistake when student substitute $x=r$ into $y=\sqrt{4r^2-x^2}$ and differentiate w.r.t r, this makes no sense as r is a constant. Most students are able to come up with the correct chain rule to compute connected rate of change. Some students didn't simplify their final answer.

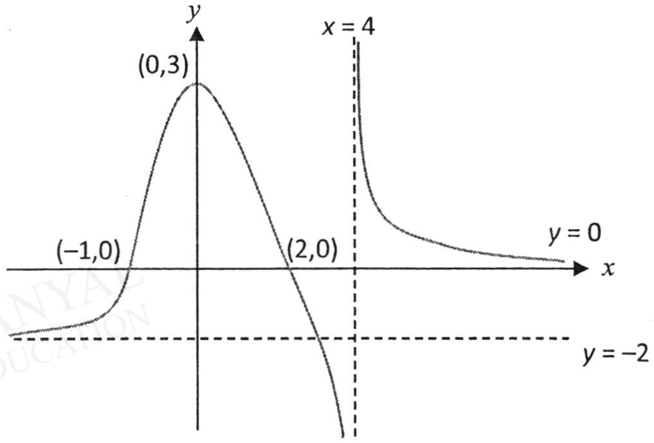
Qn.	Solution	Comments
4 (i)	$y+\ln y=\mathrm{e}^x$ $\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{\frac{\mathrm{d}y}{\mathrm{d}x}}{y}=\mathrm{e}^x$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}y-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{y^2}=\mathrm{e}^x$ $y^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}y-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=y^2\mathrm{e}^x$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(y^2+y\right)=y^2\mathrm{e}^x+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	Most students are able to differentiate $\ln y$ with respect to x to get $\frac{\frac{\mathrm{d}y}{\mathrm{d}x}}{y}$ or $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$. Many made $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject and differentiated implicitly to prove the second order differential equation. Some were not successful. One common mistake is that students did not apply chain rule when differentiating $\frac{1}{y}$ with respect to x. A few students wrote $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(y^2+y\right)=\frac{\mathrm{d}}{\mathrm{d}x}(2y+1)$ which is incorrect.

(ii)	$\frac{d^2y}{dx^2}(y^2+y)=y^2e^x+\left(\frac{dy}{dx}\right)^2$ Since $\ln y$ is defined, $y>0$, and $e^x>0$ for all $x\in\mathbb{R}$, $\frac{d^2y}{dx^2}=\frac{y^2e^x+\left(\frac{dy}{dx}\right)^2}{y^2+y}>0$ for $x\in\mathbb{R}$. The curve is concave upwards for $x\in\mathbb{R}$.	This part is not well done. Most students attempted to explain why $\frac{d^2y}{dx^2}>0$ but Very few students can write down $y>0$ since $\ln y$ **is defined.**

Qn.	Solution	Comments
5(a)	$y=\frac{-3x+6}{3x-4}=-1+\frac{2}{3x-4}$ $y=\frac{1}{x}\to y=\frac{1}{x-4}$ $\to y=\frac{1}{3x-4}\to y=\frac{2}{3x-4}$ $\to y=-1+\frac{2}{3x-4}$ 1: Translation by 4 units in the positive x direction, 2: Scaling parallel to the x-axis by factor of $\frac{1}{3}$, 3: Scaling parallel to the y-axis by a factor of 2, 4: Translation by 1unit in the negative y direction	It is important to express y in the form $A+\frac{B}{3x-4}$ first. Some students describe the transformations in the wrong order /wrong unit/factor for the x or y directions. Common mistake in the description of transformation : 1) use transformation, shift, instead of translation 2) to describe scaling parallel to x/y axis by a factor of k, common mistakes are factor is missing or students wrote k unit or did not write parallel to x or y axis.

	 	Many students did not draw the graph approaching the horizontal asymptote $y = 0$ when x tends to negative infinity. Some students drew the graph from $x = 0$ to 2 below the x axis and increasing to (2,0) which is incorrect. Use $(x, y) \to \left(x, \frac{1}{y}\right)$ to find the asymptotes, the corresponding turning points and axial intercepts.

Qn.	Solution	Comments
6(a)	$\frac{4x}{x^2 - 3x - 4} \geq -x$ $\Rightarrow \frac{4x}{x^2 - 3x - 4} + x \geq 0$ $\Rightarrow \frac{4x + x(x^2 - 3x - 4)}{x^2 - 3x - 4} \geq 0$ $\Rightarrow \frac{x^3 - 3x^2}{(x-4)(x+1)} \geq 0$ $\Rightarrow \frac{x^2(x-3)}{(x-4)(x+1)} \geq 0$  $\Rightarrow -1 < x \leq 3$ or $x > 4$ For $\frac{4\lvert x\rvert}{x^2 - 3\lvert x\rvert - 4} < -\lvert x\rvert$, Replace x by $\lvert x\rvert$, and from the previous part, $\lvert x\rvert \leq -1$ or $3 < \lvert x\rvert \leq 4$ (no solution) Since $x \neq -1$ and $x \neq 4$, we have $3 < \lvert x\rvert < 4$. Therefore, $-4 < x < -3$ or $3 < x < 4$	Most students are able to answer the first part of the question by achieving the fully factorised form and finding the critical values. Some students seem to still have an incorrect idea of what fully factorised mean, as they try to read critical values from completed square form of the quadratics. The latter part is generally not well done. While most students realized the replacement of x with $\lvert x\rvert$, they didn't take account of the fact that the inequality sign has changed compared to the first inequality.

(b)		Most students can get at least the part for $x>4$ and the asymptote $x=4$ correct. Many are also able to have the correct shape for $x<4$ but missing labels for intercepts and asymptotes are common for this part of the question. In particular, note the equation of the asymptote $y=0$.

Qn.	**Working**	**Comments**
7 (i)	$(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})$ $=\mathbf{a}\times\mathbf{a}+\mathbf{a}\times(-\mathbf{b})+\mathbf{b}\times\mathbf{a}+\mathbf{b}\times(-\mathbf{b})$ $=\mathbf{a}\times\mathbf{a}-\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{a}-\mathbf{b}\times\mathbf{b}$ $=\mathbf{0}-\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{a}-\mathbf{0}$ $=-\mathbf{a}\times\mathbf{b}-\mathbf{a}\times\mathbf{b}$ $=-2(\mathbf{a}\times\mathbf{b})$	The vector algebraic manipulations were generally well handled but many students wrote $\underset{\sim}{a}\times\underset{\sim}{a}=0$ instead of the zero vector $\underset{\sim}{0}$.
(ii)	$\lvert(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})\rvert$ $=\lvert-2(\mathbf{a}\times\mathbf{b})\rvert$ $=2\lvert(\mathbf{a}\times\mathbf{b})\rvert$ Maximum value of $\lvert(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})\rvert$ occurs when the value of $2\lvert(\mathbf{a}\times\mathbf{b})\rvert$ is maximum. $2\lvert(\mathbf{a}\times\mathbf{b})\rvert$ $=2\lvert(\mathbf{a}\times\mathbf{b})\rvert$ $=2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\lvert\sin\theta\rvert$ Maximum of $\lvert\sin\theta\rvert=1$ Maximum of $2\lvert(\mathbf{a}\times\mathbf{b})\rvert$	Some students wrote $\lvert\mathbf{a}\times\mathbf{b}\rvert=\lvert\mathbf{a}\rvert\times\lvert\mathbf{b}\rvert$ without clearly explaining how the RHS expression gave a maximum value. Very often, the wrong definition "$\mathbf{a}\times\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$" was written, failing to see that $\mathbf{a}\times\mathbf{b}$ is a vector rather than a scalar.

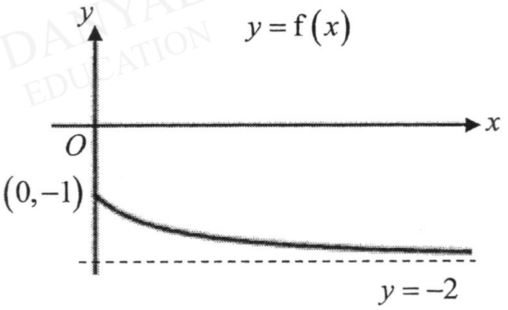
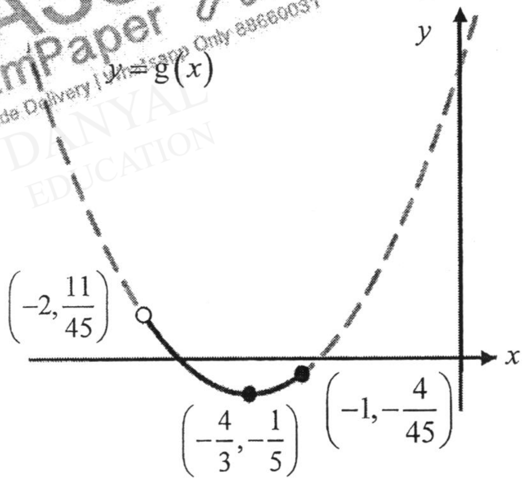
	$=2\|\mathbf{a}\|\|\mathbf{b}\|\|1\|$ $=2\|\mathbf{a}\|\|\mathbf{b}\|$ $=2\|\mathbf{a}\|(2\|\mathbf{a}\|)$ $=4\|\mathbf{a}\|^2$ Maximum value is $4\|\mathbf{a}\|^2$.	
(iii)	Maximum length of $\|(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})\|$ $=4\|\mathbf{a}\|^2$ By right-hand rule, $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})$ can be parallel to $-\mathbf{j}$ or $\mathbf{j}$. Possible expressions $=4\|\mathbf{a}\|^2\begin{pmatrix}0\\1\\0\end{pmatrix}$ or $4\|\mathbf{a}\|^2\begin{pmatrix}0\\-1\\0\end{pmatrix}$	Poorly done. Majority of the students were not able to observe that $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})$ is parallel to $-\mathbf{j}$ or $\mathbf{j}$. Once again, students who wrote $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})=4\|\mathbf{a}\|^2$ failed to see that the answer should be a vector, and not a scalar.

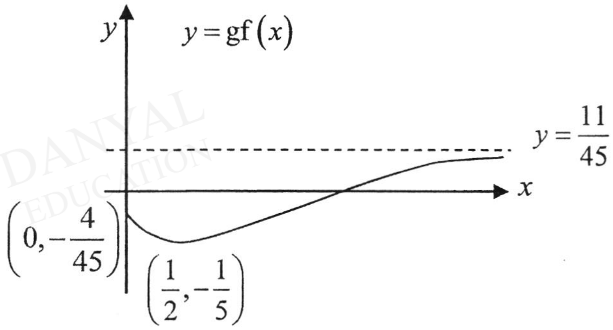
Qn.	Working	Comments
8 (i)	$\overrightarrow{OP}=\lambda\overrightarrow{OA}+\mu\overrightarrow{OB}+(1-\lambda-\mu)\overrightarrow{OC}$ $=\lambda\overrightarrow{OA}+\mu\overrightarrow{OB}+\overrightarrow{OC}-\lambda\overrightarrow{OC}-\mu\overrightarrow{OC}$ $=\overrightarrow{OC}+\lambda\left(\overrightarrow{OA}-\overrightarrow{OC}\right)+\mu\left(\overrightarrow{OB}-\overrightarrow{OC}\right)$ $=\overrightarrow{OC}+\lambda\overrightarrow{CA}+\mu\overrightarrow{CB}, \lambda,\mu\in\mathbb{R}$ Since $\overrightarrow{OP}$ is equal to the sum of $\overrightarrow{OC}$ and scalar multiples of $\overrightarrow{CA}$ and $\overrightarrow{CB}$, point P is a point on plane ABC.	Many students actually thought that $\overrightarrow{OP}$ lies on the plane although it is clear from the question that the origin is not on the plane. Similarly, there were students who wrote that $\overrightarrow{OA},\overrightarrow{OB},\overrightarrow{OC}$ lie on the plane. For students who tried to find the equation of the plane to show that it had the same expression as that for $\overrightarrow{OP}$, the parameters should be defined differently from λ and μ as they represent some specific real values for the point P whereas in the plane equation, the parameters are any real values.

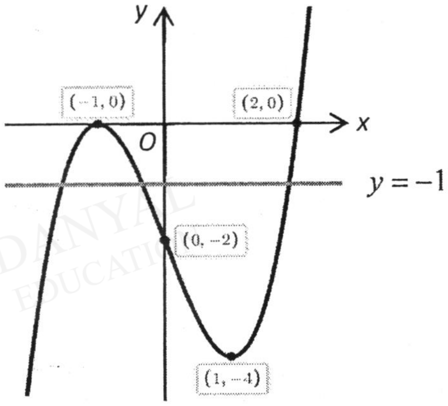
(ii)	**Method 1** $\overrightarrow{OB} = 2\mathbf{i}+2\mathbf{j}+4\mathbf{k}$ $l_{AC} : \mathbf{r} = \overrightarrow{OA} + \gamma\overrightarrow{AC}, \gamma \in \mathbb{R}$ $l_{AC} : \mathbf{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \gamma\begin{pmatrix}-3\\-1\\0\end{pmatrix}, \gamma \in \mathbb{R}$ Let N be the foot of perpendicular of B to l_{AC}. $\overrightarrow{ON} = \begin{pmatrix}1-3\gamma\\1-\gamma\\1\end{pmatrix}$, for some γ. $\overrightarrow{BN} = \begin{pmatrix}-1-3\gamma\\-1-\gamma\\-3\end{pmatrix}$ for some γ $\overrightarrow{BN} \bullet \overrightarrow{AC} = 0$ $\begin{pmatrix}-1-3\gamma\\-1-\gamma\\-3\end{pmatrix} \bullet \begin{pmatrix}-3\\-1\\0\end{pmatrix} = 0$ $3+9\gamma+1+\gamma=0$ $10\gamma+4=0$ $\gamma=-0.4$ $\overrightarrow{ON} = \begin{pmatrix}1-3(-0.4)\\1-(-0.4)\\1\end{pmatrix} = \begin{pmatrix}2.2\\1.4\\1\end{pmatrix}$ **Method 2: Projection vector method** Let N be the foot of perpendicular of B to l_{AC}. Let D be a point on l_{AC} with position vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$. $\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD} = \begin{pmatrix}2\\2\\4\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\3\end{pmatrix}$	A common error observed in writing the line equation was "$l_{AC} = \overrightarrow{OA} + \gamma\overrightarrow{AC}, \gamma \in \mathbb{R}$". Most students were able to interpret the question correctly and adopt suitable approaches to solve it, with Method 1 as the more common strategy. Some students wrongly assumed that B, C, P are collinear and applied ratio theorem to find $\overrightarrow{OP}$. For students who used the projection vector method, errors included • using vector product, instead of scalar product, to find the correct projection • adding modulus to the scalar product to find the projection vector • using vector in the wrong direction to find the projection e.g. projecting $\overrightarrow{BA}$ onto $\overrightarrow{AC}$ to find $\overrightarrow{AN}$

	$\overrightarrow{DN}=\left(\begin{pmatrix}1\\1\\3\end{pmatrix}\bullet\frac{\begin{pmatrix}-3\\-1\\0\end{pmatrix}}{\left\|\begin{pmatrix}-3\\-1\\0\end{pmatrix}\right\|}\right)\frac{\begin{pmatrix}-3\\-1\\0\end{pmatrix}}{\left\|\begin{pmatrix}-3\\-1\\0\end{pmatrix}\right\|}=\frac{1}{10}(-3-1)\begin{pmatrix}-3\\-1\\0\end{pmatrix}=-\frac{2}{5}\begin{pmatrix}-3\\-1\\0\end{pmatrix}$ $\overrightarrow{ON}=\overrightarrow{OD}+\overrightarrow{DN}=\begin{pmatrix}1\\1\\1\end{pmatrix}-\frac{2}{5}\begin{pmatrix}-3\\-1\\0\end{pmatrix}=\begin{pmatrix}2.2\\1.4\\1\end{pmatrix}$ **Cont'd** $\overrightarrow{ON}=\frac{\overrightarrow{OB}+\overrightarrow{OP}}{2}$ $\begin{pmatrix}2.2\\1.4\\1\end{pmatrix}=\frac{\begin{pmatrix}2\\2\\4\end{pmatrix}+\overrightarrow{OP}}{2}$ $\Rightarrow\overrightarrow{OP}=\begin{pmatrix}2.4\\0.8\\-2\end{pmatrix}$ $\begin{pmatrix}2.4\\0.8\\-2\end{pmatrix}=\lambda\begin{pmatrix}1\\1\\1\end{pmatrix}+\mu\begin{pmatrix}2\\2\\4\end{pmatrix}+(1-\lambda-\mu)\begin{pmatrix}-2\\0\\1\end{pmatrix}$ $\lambda+2\mu=0.8$ $\lambda+4\mu+1-\lambda-\mu=-2$ $\Rightarrow\mu=-1$ $\Rightarrow\lambda=2.8$	

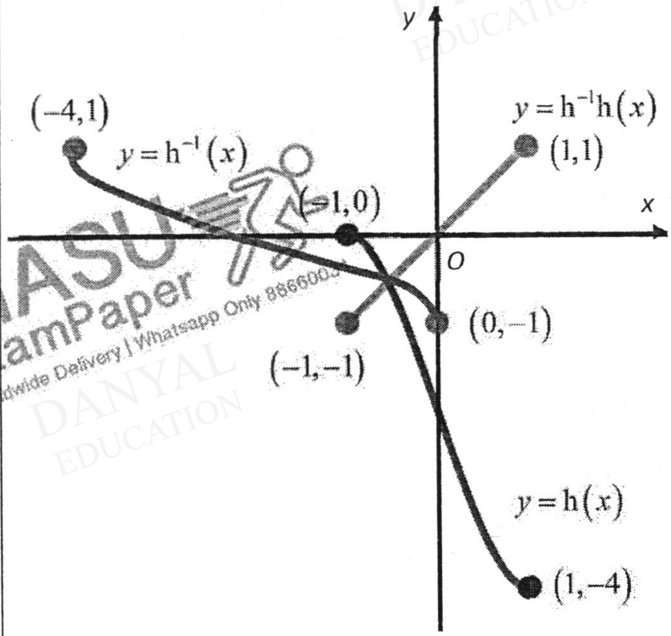
	Method 3 $\overrightarrow{OB}=2\mathbf{i}+2\mathbf{j}+4\mathbf{k}$ $l_{AC}:\mathbf{r}=\overrightarrow{OA}+\gamma\overrightarrow{AC},\gamma\in\mathbb{R}$ $l_{AC}:\mathbf{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\gamma\begin{pmatrix}-3\\-1\\0\end{pmatrix},\gamma\in\mathbb{R}$ Let N be the foot of perpendicular of B to l_{AC}. $\overrightarrow{ON}=\begin{pmatrix}1-3\gamma\\1-\gamma\\1\end{pmatrix}$, for some γ. $\overrightarrow{BN}=\begin{pmatrix}-1-3\gamma\\-1-\gamma\\-3\end{pmatrix}$ for some γ $\overrightarrow{BN}\bullet\overrightarrow{AC}=0$ $\begin{pmatrix}-1-3\gamma\\-1-\gamma\\-3\end{pmatrix}\bullet\begin{pmatrix}-3\\-1\\0\end{pmatrix}=0$ $3+9\gamma+1+\gamma=0$ $10\gamma+4=0$ $\gamma=-\frac{2}{5}$ $\overrightarrow{ON}=\frac{\overrightarrow{OB}+\overrightarrow{OP}}{2}$ $\overrightarrow{OP}=2\overrightarrow{ON}-\overrightarrow{OB}$ $=2\left(\overrightarrow{OA}+\overrightarrow{AN}\right)-\overrightarrow{OB}$ $=2\left(\overrightarrow{OA}-\frac{2}{5}\overrightarrow{AC}\right)-\overrightarrow{OB}$ $=2\left(\overrightarrow{OA}-\frac{2}{5}\left(\overrightarrow{OC}-\overrightarrow{OA}\right)\right)-\overrightarrow{OB}$ $=\frac{14}{5}\overrightarrow{OA}-\frac{4}{5}\overrightarrow{OC}-\overrightarrow{OB}$ $\Rightarrow \mu=-1$ $\Rightarrow \lambda=2.8$	

Qn.	Solution	Comments
9 **(a)(i)**	The function g is a quadratic function and has a minimum point at $\left(-\frac{4}{3},-\frac{1}{5}\right)$. $R_g=\left[-\frac{1}{5},\infty\right)$ $D_f=[0,\infty)$ Since $R_g \not\subseteq D_f$, fg does not exist.	Mostly well done. Some students wrongly used the x-coordinate of the minimum point instead and answered
(a)(ii)	 $x\to\infty, \frac{1}{1+x}-2\to-2$ At $x=0, y=-1$ $R_f=(-2,-1]$	It is a good observation that most students included the asymptote and axial intercept in their sketch. The common mistake is neglecting the domain $(x\geq 0)$ of the function f and instead sketched the full graph. This often led to the wrong range $(-\infty,-2)\cup(-2,\infty)$. Some students wrongly expressed the set as $[-1,-2)$.
(a)(iii)	<u>Method 1: Map R_f using graph of $y=g(x)$</u> From the graph of f: $[0,\infty)\xrightarrow{f}(-2,-1]$ From the graph of g: $(-2,-1]\xrightarrow{g}\left[-\frac{1}{5},\frac{11}{45}\right)$ 	Not well done. Many students did not show evidence of considering graph (whether it is $y=g(x)$ for Method 1 or $y=gf(x)$ for Method 2), and only substituted endpoints which led to the common wrong answer $\left[-\frac{4}{45},\frac{11}{45}\right)$.

	Therefore, $(0,\infty)\xrightarrow{f}(-2,-1]\xrightarrow{g}\left[-\frac{1}{5},\frac{11}{45}\right)$. i.e. $R_{gf}=\left[-\frac{1}{5},\frac{11}{45}\right)$ <u>Method 2: Map D_{gf} using graph of $y=gf(x)$</u>  $gf(x)=\left(\frac{1}{1+x}-\frac{2}{3}\right)^2-\frac{1}{5},\ x\in\mathbb{R},\ x\geq 0$ When $x\to\infty$, $\frac{1}{1+x}\to 0$. Therefore, $y=\left(\frac{1}{1+x}-\frac{2}{3}\right)^2-\frac{1}{5}\to\left(0-\frac{2}{3}\right)^2-\frac{1}{5}=\frac{11}{45}$ Using GC, the minimum point is $\left(\frac{1}{2},-\frac{1}{5}\right)$. From graph, $R_{gf}=\left[-\frac{1}{5},\frac{11}{45}\right)$	

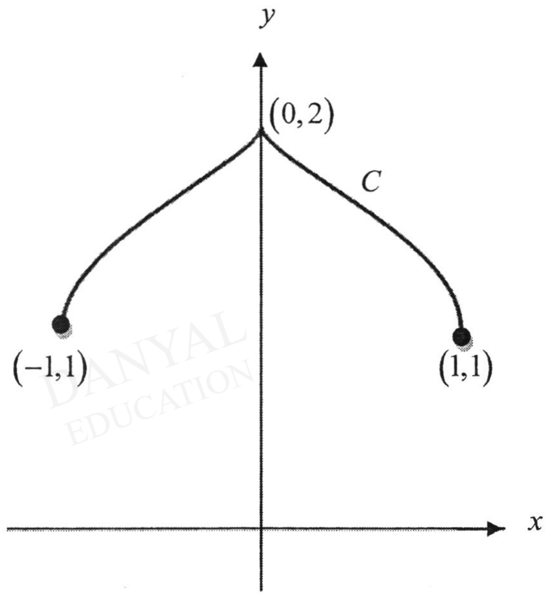
<table>
<tr><th>Qn.</th><th>Solution</th><th>Comments</th></tr>
<tr><td>9(b)
(i)</td><td><u>Method 1</u>
Since $\mathrm{h}(-1)=\mathrm{h}(2)=0$, hence h is not 1-1.
Therefore, h^{-1} does not exist.
<u>Method 2</u>


The horizontal line $y=-1$ cuts the graph of $y=\mathrm{h}(x)$ at more than one point. Hence, h is not 1-1.
Therefore, h^{-1} does not exist.</td><td>Good attempts but incomplete answers.

Most students attempted using Method 2. The main concern was that the sketch was not included to illustrate the intersections between the identified horizontal line and the graph of $y=\mathrm{h}(x)$.</td></tr>
<tr><td>(b)(ii)</td><td>$D_{\mathrm{h}^{-1}}=R_{\mathrm{h}}=[-4,0]$</td><td>Mostly well done. Some students wrongly stated $[0,-4]$.</td></tr>
<tr><td>(b)(iii)</td><td>
</td><td>$y=\mathrm{h}(x)$ and $y=\mathrm{h}^{-1}(x)$:
Shape and symmetry.
Correct endpoints $(-1,0)$ and $(1,-4)$, $(0,-1)$ and $(-4,1)$.

For the graph of $y=\mathrm{h}^{-1}\mathrm{h}(x)$:
Must label endpoints $(-1,-1)$ and $(1,1)$, the origin O and the line should pass through the point of intersection between $y=\mathrm{h}(x)$ and $y=\mathrm{h}^{-1}(x)$.

Position of points labelled should be accurate. For example, $(-1,-1)$ and $(0,-1)$ have the same y-coordinates.</td></tr>
</table>

<table>
<tr><th>Qn.</th><th>Solution</th><th>Comments</th></tr>
<tr><td>10
(i)</td><td>
</td><td>Most graphs were not well drawn. The sharp point was hard to see on the GC, so it is understandable that students did not sketch it properly. What was more alarming was that some graphs did not have the end points and some graphs only sketched the right hand side of the graph, which meant that some students sketched the graph without adjusting the range of values of t.</td></tr>
<tr><td>(ii)</td><td>$\frac{dt}{d\theta} = -\sin\theta$

$t=0, \cos\theta = 0, \theta = \frac{\pi}{2}; t = \frac{\sqrt{3}}{2}, \cos\theta = \frac{\sqrt{3}}{2}, \theta = \frac{\pi}{6}$

$\int_0^{\frac{\sqrt{3}}{2}} t^2\sqrt{1-t^2}\ dt$

$= \int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \cos^2\theta\sqrt{1-\cos^2\theta}\left(-\sin\theta\right)\ d\theta$

$= -\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \cos^2\theta\sqrt{1-\cos^2\theta}\sin\theta\ d\theta$

$= -\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \cos^2\theta\sqrt{\sin^2\theta}\sin\theta\ d\theta$

$= -\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \cos^2\theta\lvert\sin\theta\rvert\sin\theta\ d\theta$

$= -\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \cos^2\theta\sin\theta\sin\theta\ d\theta$ (since θ is acute)

$= -\frac{1}{4}\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} 4\cos^2\theta\sin^2\theta\ d\theta$

$= -\frac{1}{4}\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \left(2\cos\theta\sin\theta\right)^2\ d\theta$</td><td>These were the common mistakes made in the substitution process:
1. Did not do a full substitution, omitting to substitute the limits or "dt".
2. Placing the larger value of θ as the top limit of the definite integral.
3. Use degrees instead of radians for θ.
4. Confusion between cosine and inverse cosine. Common to see "$\cos\theta = 0 \Rightarrow \theta = 1$" in the working.

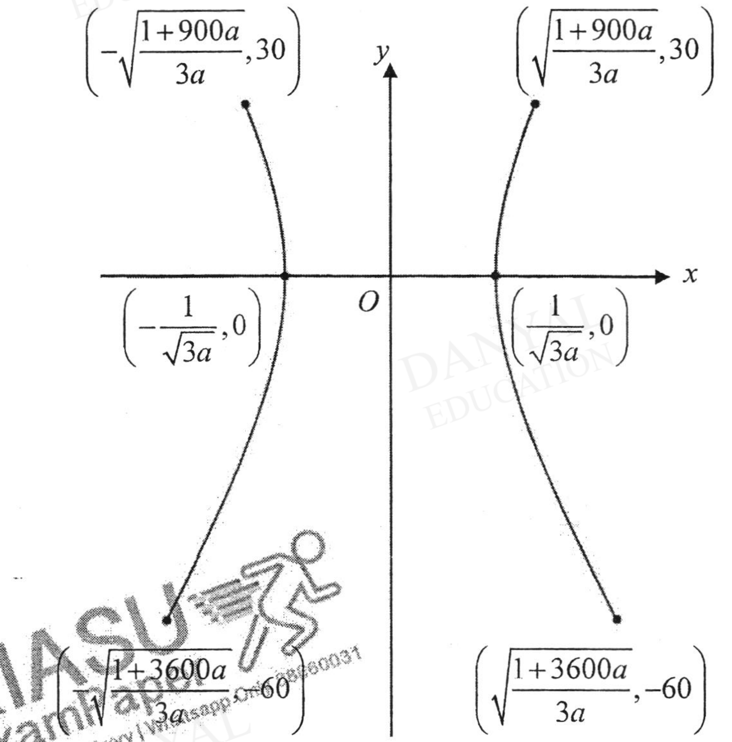
Students who were able to do the substitution correctly and get $\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \cos^2\theta\sin^2\theta\ d\theta$ often did not use the efficient method to evaluate this definite integral. A lot of students converted $\cos^2\theta$ and $\sin^2\theta$ individually to $\cos 2\theta$ and ended in a long expression which they had to do a lot of simplification before they reached $1-4\cos 4\theta$. Thus some made careless mistakes and often</td></tr>
</table>

	$= -\frac{1}{4}\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} (\sin 2\theta)^2 \, \mathrm{d}\theta$ $= -\frac{1}{4}\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \left(\frac{1-\cos 4\theta}{2}\right) \mathrm{d}\theta$ $= -\frac{1}{8}\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} 1-\cos 4\theta \, \mathrm{d}\theta$ $= -\frac{1}{8}\left[\theta - \frac{\sin 4\theta}{4}\right]_{\frac{\pi}{2}}^{\frac{\pi}{6}}$ $= -\frac{1}{8}\left(\frac{\pi}{6} - \frac{\sin 4\left(\frac{\pi}{6}\right)}{4} - \frac{\pi}{2}\right)$ $= \frac{\sqrt{3}}{64} + \frac{\pi}{24}$	could not get the correct answer. Some of these students, in my opinion, were exhausted by all the manipulations they had to do and simply integrated $\sin^2 2\theta$ as $\frac{\sin^3 2\theta}{3}$ or $\frac{\cos^3 2\theta}{3}$.
(iii)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2$ $x = 0, t = 0$ $x = \frac{\sqrt{27}}{8}$, $t^3 = \frac{\sqrt{27}}{8} \Rightarrow t = \frac{\sqrt{3}}{2}$ $\int_0^{\frac{3\sqrt{3}}{8}} y \, \mathrm{d}x$ $= \int_0^{\frac{\sqrt{3}}{2}} \left(1+\sqrt{1-t^2}\right)\left(3t^2\right) \mathrm{d}t$ $= \int_0^{\frac{\sqrt{3}}{2}} 3t^2 + 3t^2\sqrt{1-t^2} \, \mathrm{d}t$ $= 3\int_0^{\frac{\sqrt{3}}{2}} t^2 \, \mathrm{d}t + 3\int_0^{\frac{\sqrt{3}}{2}} t^2\sqrt{1-t^2} \, \mathrm{d}t$ $= \left[t^3\right]_0^{\frac{\sqrt{3}}{2}} + 3\left(\frac{\sqrt{3}}{64} + \frac{\pi}{24}\right)$ $= \frac{3\sqrt{3}}{8} + \left(\frac{3\sqrt{3}}{64} + \frac{\pi}{8}\right)$ $= \frac{27\sqrt{3}}{64} + \frac{\pi}{8}$	Students who were able to form the correct definite integral were able to use their answer in (ii) correctly. .

Qn.	Solution	Comments
11 (i)	$\int \frac{x}{\sqrt{1-k^2x^2}}\ \mathrm{d}x$ $=\int x\left(1-k^2x^2\right)^{-\frac{1}{2}}\ \mathrm{d}x$ $=\frac{-1}{2k^2}\int -2k^2x\left(1-k^2x^2\right)^{-\frac{1}{2}}\ \mathrm{d}x$ $=\frac{-1}{2k^2}\frac{\left(1-k^2x^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$ $=\frac{-1}{k^2}\left(1-k^2x^2\right)^{\frac{1}{2}}+C$	Advice: Imperative for you to recognise integrand of the form $\mathrm{f}'(x)\left[\mathrm{f}(x)\right]^n$.
(ii)	$\int\left(\sin^{-1}kx\right)\frac{x}{\sqrt{1-k^2x^2}}\ \mathrm{d}x$ $u=\left(\sin^{-1}kx\right),\quad \frac{\mathrm{d}v}{\mathrm{d}x}=\frac{x}{\sqrt{1-k^2x^2}}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{k}{\sqrt{1-k^2x^2}},\quad v=\frac{-1}{k^2}\left(1-k^2x^2\right)^{\frac{1}{2}}$ $\int\left(\sin^{-1}kx\right)\frac{x}{\sqrt{1-k^2x^2}}\ \mathrm{d}x$ $=\left(\sin^{-1}kx\right)\frac{-1}{k^2}\left(1-k^2x^2\right)^{\frac{1}{2}}-\int\frac{-1}{k^2}\left(1-k^2x^2\right)^{\frac{1}{2}}\frac{k}{\sqrt{1-k^2x^2}}\ \mathrm{d}x$ $=\frac{-\left(\sin^{-1}kx\right)\left(1-k^2x^2\right)^{\frac{1}{2}}}{k^2}+\int\frac{1}{k}\ \mathrm{d}x$ $=\frac{-\left(\sin^{-1}kx\right)\left(1-k^2x^2\right)^{\frac{1}{2}}}{k^2}+\frac{x}{k}+D$	Advice: Understanding of integration by parts displayed but you ought to take care of getting the correct $\frac{\mathrm{d}u}{\mathrm{d}x}$. Many candidates omitted the k at the numerator. Advice: $\int\frac{1}{k}\ \mathrm{d}x\neq\ln\lvert k\rvert$ k is a CONSTANT!

<table>
<tr>
<td>(iii)</td>
<td>When $k=1$, $\displaystyle\int \left(\sin^{-1} kx\right)\frac{x}{\sqrt{1-k^2x^2}}\,\mathrm{d}x$ becomes
$\displaystyle\int \left(\sin^{-1} x\right)\frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x$ so
$\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \left(\sin^{-1} x\right)\frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x$
$\displaystyle=\left[-\left(\sin^{-1} x\right)\left(1-x^2\right)^{\frac{1}{2}}+x\right]_0^{\frac{1}{\sqrt{2}}}$
$\displaystyle=-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\left(1-\frac{1}{2}\right)^{\frac{1}{2}}+\frac{1}{\sqrt{2}}$
$\displaystyle=-\frac{\pi}{4}\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}$
$\displaystyle=\frac{1}{\sqrt{2}}\left(1-\frac{\pi}{4}\right)$</td>
<td>Advice: Try to familiarise with the result of $\sin^{-1} x$ for the following:
$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$, $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)=-\frac{\pi}{4}$
$\sin^{-1}(1)=\frac{\pi}{2}$, $\sin^{-1}(-1)=-\frac{\pi}{2}$
$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$, $\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$
$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$, $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}$
Alternatively, observe these results from the graph of $\sin^{-1} x$.</td>
</tr>
<tr>
<td>(iv)</td>
<td>Method 1
$\displaystyle\int_m^{\frac{1}{\sqrt{2}}+m} \left[\sin^{-1}(x-m)\right]\frac{x-m}{\sqrt{1-(x-m)^2}}\,\mathrm{d}x$
$\displaystyle=\frac{1}{\sqrt{2}}\left(1-\frac{\pi}{4}\right)$
Both integrand and limits of integration underwent a translation of m units in the positive or negative x-direction so the area under the curve is preserved.
Or
Method 2
Let $u=x-m$
$\displaystyle\frac{\mathrm{d}u}{\mathrm{d}x}=1$
$x=m$, $u=m-m=0$
$\displaystyle x=\frac{1}{\sqrt{2}}+m$, $\displaystyle u=\frac{1}{\sqrt{2}}+m-m=\frac{1}{\sqrt{2}}$</td>
<td>Advice:
Note that a replacement of x with $x-m$ has taken place. It is insufficient to mention that the graph has been translated without making reference to the translation of the limit or the fact that the area under the curve remains the same.
For method 2, you must show the change of variable and substitution clearly.</td>
</tr>
</table>

	$\int_{m}^{\frac{1}{\sqrt{2}}+m}\left[\sin^{-1}(x-m)\right]\frac{x-m}{\sqrt{1-(x-m)^2}}\,\mathrm{d}x$ $=\int_{0}^{\frac{1}{\sqrt{2}}}\left(\sin^{-1}u\right)\frac{u}{\sqrt{1-u^2}}\,\mathrm{d}u$ $=\frac{1}{\sqrt{2}}\left(1-\frac{\pi}{4}\right)$	

Qn.	Solution	Comments
12(i)	 When $y=-60$, $3ax^2-a(-60)^2=1$ $\Rightarrow x=\sqrt{\frac{1+3600a}{3a}}$ or $-\sqrt{\frac{1+3600a}{3a}}$ When $y=30$, $3ax^2-a(30)^2=1$ $\Rightarrow x=\sqrt{\frac{1+900a}{3a}}$ or $-\sqrt{\frac{1+900a}{3a}}$	Students managed to recognise the equation that gives the shape of a hyperbola. **Skills to improve**: 1. Finding centre of hyperbola. 2. Finding axial intercepts of a hyperbola. 3. Finding the end points in questions with domain given. 4. Drawing to scale for conics questions. 5. Symmetry in a hyperbola.

(ii)	Given $a = 0.01$. $3(0.01)x^2 - (0.01)y^2 = 1,$ Volume of the tower $= \int_{-60}^{30} \pi x^2 \mathrm{d}y$ $= \pi \int_{-60}^{30} \frac{1}{3(0.01)} + \frac{y^2}{3} \mathrm{d}y$ $= \frac{\pi}{3}\left[100y + \frac{y^3}{3}\right]_{-60}^{30}$ $= 30000\pi \text{ m}^3$.	Students can remember the formula to find volume but are not proficient in its application. **Skills to improve**: 1. Identifying the limits in volume. 2. Rotation about the y -axis requires students to integrate with respect to y. 3. **Exact** solution is required. Hence the G.C. is not allowed in this question.
(iii)	Radius of base $= \sqrt{\frac{1+a(60)^2}{3a}}$ $\frac{\text{Smallest circumference}}{\text{Circumference of base}} = \frac{1}{2} = \frac{2\pi(\frac{1}{\sqrt{3a}})}{2\pi\left(\sqrt{\frac{1+a(60)^2}{3a}}\right)}$ $\frac{1}{4} = \frac{\frac{1}{3a}}{\frac{1+3600a}{3a}}$ $\frac{1}{4} = \frac{1}{1+3600a}$ $a = \frac{1}{1200}$	Most students recognised and used the formula for the circumference of a circle. However, some students did not realised that the smallest circumference is found at the x-axis instead of the end points.

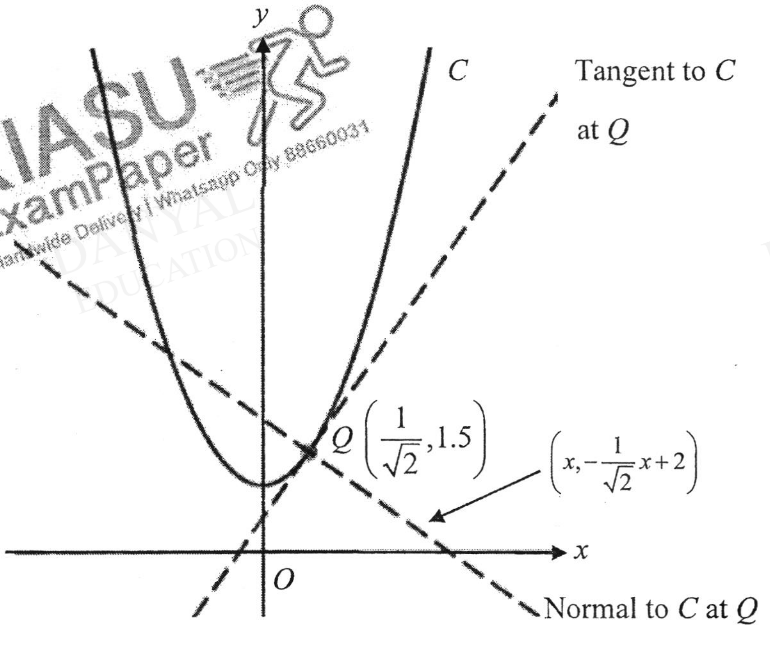
Qn.	Working	Comments
13 (i)	Gradient of C at Q $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{1}{\sqrt{2}}} = 2x\|_{x=\frac{1}{\sqrt{2}}}$ $= \sqrt{2}$ Gradient of normal at Q to C $= \frac{-1}{\sqrt{2}}$	Many students who did this part wrongly did not find the numerical value of the gradient of the normal and yet proceeded to do the substitution

<table>
<tr>
<td></td>
<td>Equation of normal: $\dfrac{y-\frac{3}{2}}{x-\frac{1}{\sqrt{2}}}=\dfrac{-1}{\sqrt{2}}$

$\Rightarrow y=-\dfrac{1}{\sqrt{2}}x+2$</td>
<td>$m=-\dfrac{1}{2x}$ into the line equation $\dfrac{y-\frac{3}{2}}{x-\frac{1}{\sqrt{2}}}=m$. The resulting equation is not that of a line.</td>
</tr>
<tr>
<td>(ii)</td>
<td>Curvature of C at $x=\dfrac{1}{\sqrt{2}}$
$=\dfrac{2}{\left(1+4\left(\frac{1}{\sqrt{2}}\right)^2\right)^{\frac{3}{2}}}=\dfrac{2}{3^{\frac{3}{2}}}=\dfrac{2}{\sqrt{27}}$ or $\dfrac{2}{3\sqrt{3}}$</td>
<td>This part is well-done.</td>
</tr>
<tr>
<td>(iii)</td>
<td>Radius of calculation circles at $Q=\dfrac{\sqrt{27}}{2}$.
Radius from the centre of the calculation circles to Q is perpendicular to the tangent to C at Q.

Method 1

It follows that the centre of the calculation circles lie on the normal to C at Q. Hence, the centre of the calculation circles must take the form $\left(x,-\dfrac{1}{\sqrt{2}}x+2\right)$.


</td>
<td>This part is poorly done. Students should draw out the diagram as shown and fill in the necessary details to find the centres of the calculation circles.</td>
</tr>
</table>

Since the distance from the centre of the calculation $\left(x, -\frac{1}{\sqrt{2}}x+2\right)$ to $Q\left(\frac{1}{\sqrt{2}}, 1.5\right)$ must be $\frac{\sqrt{27}}{2}$, we have the following equation:

$$\left(x-\frac{1}{\sqrt{2}}\right)^2+\left(-\frac{1}{\sqrt{2}}x+2-\frac{3}{2}\right)^2=\left(\frac{\sqrt{27}}{2}\right)^2$$

Method 2

The centres of the calculation circles must satisfy both

$$y=-\frac{1}{\sqrt{2}}x+2 \text{ and } \left(x-\frac{1}{\sqrt{2}}\right)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{\sqrt{27}}{2}\right)^2.$$

$$\Rightarrow \left(x-\frac{1}{\sqrt{2}}\right)^2+\left(-\frac{1}{\sqrt{2}}x+2-\frac{3}{2}\right)^2=\left(\frac{\sqrt{27}}{2}\right)^2$$

Cont'd

$$\left(x-\frac{1}{\sqrt{2}}\right)^2+\frac{1}{2}\left(x-\frac{1}{\sqrt{2}}\right)^2=\frac{27}{4}$$

$$\left(x-\frac{1}{\sqrt{2}}\right)^2\left(1+\frac{1}{2}\right)=\frac{27}{4}$$

$x=2\sqrt{2}$ or $x=-\sqrt{2}$

(rejected since $x<0$)

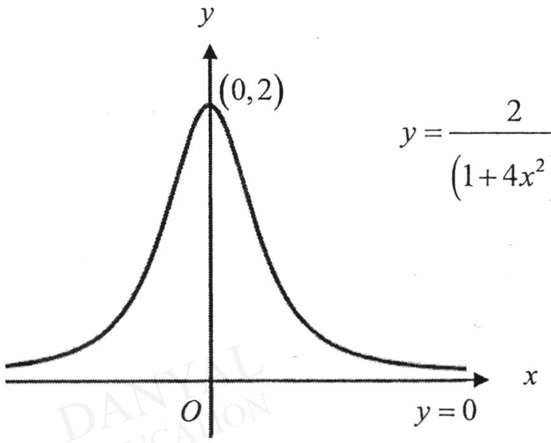
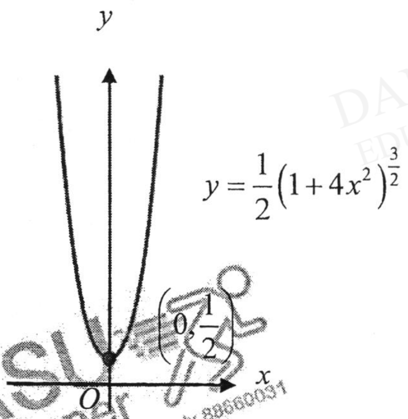
To find the *y*-coordinate of the centre of the calculating circle:

$$\Rightarrow y=-\frac{1}{\sqrt{2}}\left(-\sqrt{2}\right)+2=3$$

Equation of calculating circle:

$$\left(x+\sqrt{2}\right)^2+\left(y-3\right)^2=\frac{27}{4}$$

The solving of the equation is poorly done for many students who reached this part. The key learning point here is that students should not expand the equation when there is a common factor so that algebraic manipulation can be efficient.

(iv)		
(v)	Note that $r=\frac{1}{\mathrm{k}(x)}$ so a good start will be to sketch the graph of $y=\frac{1}{\mathrm{k}(x)}$ or $y=\frac{\left(1+4x^2\right)^{\frac{3}{2}}}{2}$, which is the reciprocal of the graph in (iv).  There are two roots to the equation $r=\frac{\left(1+4x^2\right)^{\frac{3}{2}}}{2}$ for $r>\frac{1}{2}$. Since each root is associated with two calculation circles, there will be four calculation circles with radius r for $r>\frac{1}{2}$.	Many students skipped this part. It is important to learn how to start on a non-routine question. Since this question asks about r and the question starts with a 'hence', we can think about how we can link r to the graph in part (iv).