Name:________________________________ **Class:**____________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2020

MATHEMATICS **9758/01**

Higher 2 **18 September 2020**

Paper 1 **3 hours**

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question	1	2	3	4	5	6	7	8	9	10	11	12	0 / –1	/ 100
Marks														

This document consists of **23** printed pages and **1** blank page.

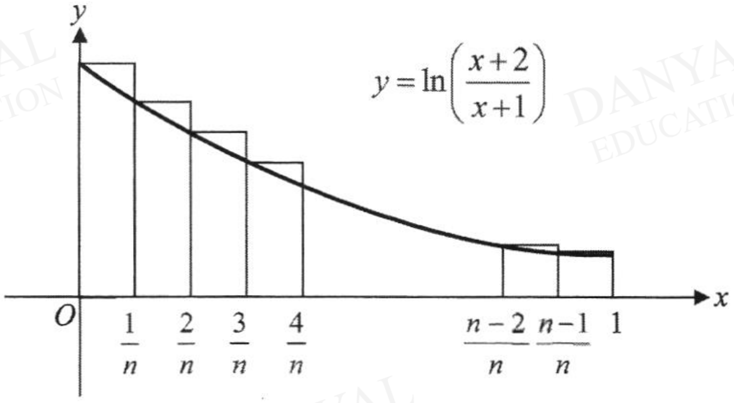
[Turn over

1 **(i)** Solve the inequality $\frac{9x+2}{x-2} \geq 2$, leaving your answer in exact form. [3]

(ii) Hence, solve the inequality $\frac{9\mathrm{e}^x+2}{2-\mathrm{e}^x} \leq -2$. [2]

2 Use method of differences to find $\sum_{n=1}^{N} \ln\left(\frac{n+1}{n+2}\right)$. Determine, stating your reason, if the series is convergent. [4]

3



The diagram shows part of the graph of $y = \ln\left(\frac{x+2}{x+1}\right)$, with rectangles, each of width $\frac{1}{n}$, approximating the area under the curve between $x = 0$ and $x = 1$.

(i) Show that the total area A, of these n rectangles, is given by

$$A = \frac{1}{n}\sum_{r=0}^{n-1} \ln\left(\frac{r+2n}{r+n}\right).$$ [2]

4 The usual selling price of a bottle of Vitamin C, Grape Seed Extract and Super Antioxidant is \$365 in total. During the year-end sale, two companies, Union and Vatson, offered the following discounts to their customers:

Company	**Discounts given for each item**			**Total price after the discount**
	Vitamin C	**Grape Seed Extract**	**Super Antioxidant**	
Union	10%	15%	15%	\$314.75
Vatson	8%	25%	15%	\$301.55

(i) Find the usual selling price of each bottle of Vitamin C, Grape Seed Extract and Super Antioxidant. [3]

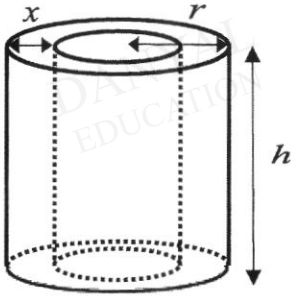
(ii) The employees from Union are offered a further 5% discount on the usual selling price of each bottle of Vitamin C and Grape Seed Extract. Determine if this additional discount will make it more attractive for the employees to purchase all the 3 items from their own company rather than from Vatson. [2]

5 The equation of a curve is $x^2 - kxy + y^2 = 3k + 2$, where k is a constant.

(i) Show that the x-coordinates of the points on the curve where the tangents to the curve are parallel to the y-axis satisfy the equation $\left(4-k^2\right)x^2 = 12k+8$. [5]

(ii) Determine, algebraically, the set of values of k for the two tangents in part **(i)** to exist. [4]

6 A fixed volume of k cm^3 material is used to make a hollow glass pipe.



The cost of polishing the glass pipe is \$3 per cm^2 for the inner and outer curved surfaces and \$2 per cm^2 for the top and the base. The outer radius of the pipe is r cm and the thickness of the pipe is x cm.

(i) Show that the cost of polishing the entire pipe, \$$C$, is $\dfrac{6k}{x} + 8\pi xr - 4\pi x^2$. [4]

(ii) Suppose that the volume of the material used is 40 cm^3 and $r = 8$ cm. The glass-maker wants to minimise C by varying x. Using differentiation, find the minimum value of C and prove that it is a minimum. [4]

7 **(i)** Given that $\mathrm{f}(x)=\sin\left(3x+\frac{\pi}{6}\right)$, find $\mathrm{f}(0)$, $\mathrm{f}'(0)$ and $\mathrm{f}''(0)$. Hence find the first three terms in the Maclaurin series of $\mathrm{f}(x)$. Give the coefficients in exact form. [5]

(ii) Given that the first two terms in the Maclaurin series of $\mathrm{f}(x)$ are equal to the first two non-zero terms in the series expansion of $\frac{1}{a+bx}$, where a and b are constants, find a and b, leaving your answers in exact form. [4]

(iii) Find the set of values of x for which the expansion in **(ii)** is valid. [1]

8 A curve C has parametric equations

$$x=t^3-4t,$$
$$y=t^2,$$

for $t \le 0$.

(i) Sketch C, stating clearly its axial intercepts. [2]

(ii) Show that the area, R, enclosed by the curve C and the y-axis is given by

$$k\int_{t_1}^{t_2}\left(t^4-4t^2\right)\,\mathrm{d}t,$$

where k, t_1 and t_2 are constants to be stated. Hence, find the exact area of R. [4]

(iii) By finding the Cartesian equation of C, find the volume generated when R is rotated completely about the y-axis. [4]

9 It is given that $\mathrm{f}(x)=x+\frac{m^2}{x-1}$, where $0.5<m<1$.

(i) Sketch the graph of $y = \mathrm{f}(x)$, showing clearly the coordinates of the turning points, axial intercept(s) and the equation(s) of any asymptote(s). [5]

(ii) By inserting a suitable graph to your sketch in **(i)**, find the set of values of k, in terms of m, for which the equation $x^2-(1+k)x+m^2+k=0$ has two distinct positive roots. [3]

(iii) The curve $y=\mathrm{f}(x)$ undergoes the transformations A, B and C in succession:

A: A translation of 1 unit in the negative x-direction,

B: A stretch with scale factor $\frac{1}{3}$ parallel to the x-axis and

C: A translation of 1 unit in the negative y-direction.

Given that the resulting curve is $y=3x+\frac{1}{9x}$, find the value of m. [3]

10 Functions g and h are defined by

$$\mathrm{g}: x \rightarrow \ln x, \qquad x \in \mathbb{R},\ x > 0,$$

$$\mathrm{h}: x \rightarrow |x|, \qquad x \in \mathbb{R},\ x \neq 0.$$

(i) Show that gh exists. Find gh in a similar form and state its range.

Determine whether the composite function hg exists. [4]

The function f is defined by

$$\mathrm{f}: x \rightarrow \mathrm{gh}(x), \qquad x \in \mathbb{R},\ x < 0.$$

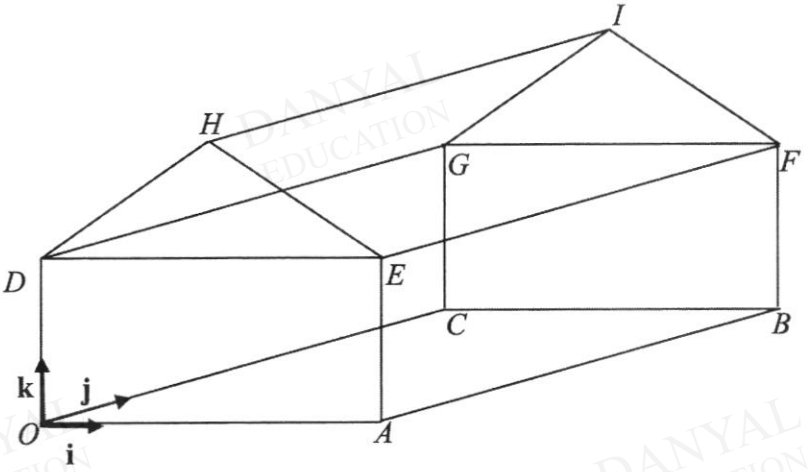
(ii) Find f^{-1} and state the domain of f^{-1}. [3]

The composite function fq is defined by

$$\mathrm{fq}: x \rightarrow \ln x^2, \qquad x \in \mathbb{R},\ x \neq 0.$$

(iii) Find q in a similar form and state the range of q. [3]

11



Farmer Joe built a barn with a horizontal rectangular base *OABC*, where $OA = 4$ m, $AB = 5$ m, and vertical walls. There are two rectangular walls *OCGD* and *ABFE* where $OD = AE = BF = CG = 3\text{ m}$. The point *O* is the origin and vectors **i**, **j**, **k** each of length 1 m, are taken along *OA*, *OC* and *OD* respectively. The roof consists of the planes *DGIH* and *EFIH* where *HI* is a horizontal beam. *DGIH* is part of the plane p_1 and *EFIH* is part of the plane p_2, where the planes p_1 and p_2 have equations

$$p_1\!: \ -x + 2z = 6,$$

$$p_2\!: \ x + 2z = 10.$$

(i) Find the acute angle between p_1 and p_2. [2]

(ii) Find the equation of the line passing through *H* and *I* and deduce the height of the horizontal beam *HI* from the ground. [3]

(iii) Farmer Joe installed a wire from D to I. Find the equation of the line passing through D and I. [2]

(iv) Farmer Joe wanted to place a light bulb at $Q(2, s, k)$, where $0 < k < 4$ and $0 < s < 5$. Show that Q is equidistant from $DGIH$ and $EFIH$. [5]

12 On 1st June 2015, Tom obtained a study loan of $50 000 from a bank for his university education starting in mid-August the same year. No interest on the loan was charged until after he graduated from the university. Tom graduated on 30th June 2018 and from 1st August 2018, the bank started charging him an interest of 0.4% per month on the amount owed to the bank on the first day of each month.

In the same month that he obtained the loan, Tom decided to set aside some money each month for the repayment of his loan after graduation. Hence, on 15th June 2015, he set aside $120 and on the 15th of each subsequent month, he set aside $10 more than in the previous month.

(i) How much money did Tom set aside by the day he graduated? [2]

Tom decided to repay the bank $10 000 upon graduation such that the outstanding amount owed was reduced to $40 000. He made a monthly repayment of $800 to the bank on the second day of each month from 2nd August 2018 onwards.

(ii) Show that the amount of money owed after the interest was charged at the beginning of the n^{th} month is given by

$$40\ 000(1.004)^n - 200\ 800(1.004^{n-1} - 1).$$ [4]

(iii) On which date would Tom be able to repay his loan completely? [3]

(iv) If Tom decides to completely repay his loan in exactly 4 years, how much more should he repay the bank each month? Leave your answer to the nearest whole number. [3]

Name:________________________________ **Class:**__________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2020

MATHEMATICS **9758/02**

Higher 2 **21 September 2020**

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Marks												

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Section A: Pure Mathematics [40 marks]

1 **(a)** It is given that $\mathrm{f}(x)=(x-1)^3$.

(i) By sketching the graphs of $y=\mathrm{f}(x)$ and $y=|\mathrm{f}(x)|$ on the same diagram, state the set of values of *x* for which $\mathrm{f}(x)=|\mathrm{f}(x)|$. [2]

(ii) Find $\int_0^n |\mathrm{f}(x)|\,\mathrm{d}x$ in terms of *n*, where $n>1$. [3]

(b) Use the substitution $x=2\sec\theta$ to find $\int \frac{\sqrt{x^2-4}}{x}\,\mathrm{d}x$. [5]

2 **(i)** Show, by means of the substitution $w=xy$, that the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}+y-xy=1$$

can be reduced to the form $\frac{\mathrm{d}w}{\mathrm{d}x}=1+w$. [2]

Hence find *y* in terms of *x*, given that $y=1$ when $x=0$. [4]

(ii) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=x\mathrm{e}^{-x},$$

giving your answer in the form $y=\mathrm{f}(x)$. [4]

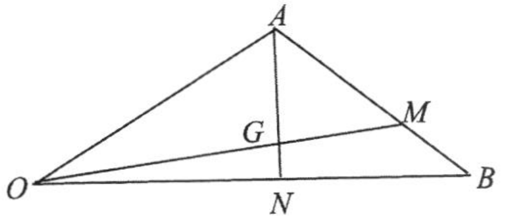
3 **(a)** Two of the roots of the equation $az^4-z^3+26z^2+bz-34=0$, where *a* and *b* are real, are $1+4i$ and -2. Find the values of *a* and *b* and the remaining roots of the equation. [4]

(b) It is given that $w=\left(\sqrt{3}+\mathrm{i}\right)^{10}$.

(i) Without using a graphing calculator, find the modulus and argument of *w*. [2]

(ii) Hence find the smallest positive integer *n* for which $\frac{w^n}{w^*}$ is real. [4]

4



With respect to the origin *O*, the position vectors of the points *A* and *B* are **a** and **b** respectively. The point *M* is on *AB* such that $AM : MB = k : 1-k$ and the point *N* is on *OB* such that $ON : NB = k : 1-k$, where *k* is a constant, $0 < k < 1$.

(i) Find in terms of **a** and **b** the position vectors of the points *M* and *N*. Hence, show that the position vector of point *G*, the point of intersection of *OM* and *AN*, is $\frac{1}{2-k}\left[(1-k)\mathbf{a}+k\mathbf{b}\right]$. [6]

(ii) Given that $k = \frac{1}{2}$, show that the area of triangle *OAB* is six times the area of triangle *AGM*. [4]

Section B: Probability and Statistics [60 marks]

5 For events *A*, *B* and *C* it is given that $P(A) = 0.5$, $P(B) = 0.45$, $P(C) = 0.3$ and $P(A \cap B \cap C) = 0.05$. It is also given that the events *A* and *B* are independent and that events *A* and *C* are independent.

(i) Given also that events *B* and *C* are independent, find $P(A' \cap B' \cap C')$. [3]

(ii) Given instead that the events *B* and *C* are not independent, find the greatest and least values of $P(A' \cap B' \cap C')$. [3]

6

7 The eleven letters in the word CORONAVIRUS are rearranged to form 'words' which may not make sense. Find the probability that

(i) the two 'R's are together; [3]

(ii) the vowels (O, A, I, U) are separated; [3]

(iii) either the first letter is a 'C' or the last letter is an 'S' or both. [3]

7 On average, 8% of surgical masks sold online are defective. The masks are sold in boxes of 20.

(i) Find the expected number of defective masks in a randomly chosen box. [2]

(ii) Find the probability that there are at least 2 and less than 10 defective masks in a randomly chosen box. [2]

(iii) 4 boxes are randomly picked for a customer. Find the probability that one box contains no more than 3 defective masks and the remaining boxes contain at least 2 defective masks each. [4]

8 Paul and Mary play a game. For each round of the game, they each throw an unbiased die. The one who obtains the higher score is given, by the other player, the difference in dollars between that score and the score of the other player. If the scores are equal, then neither player receives anything. The random variable X is Paul's winnings at each round.

(i) Show that $\mathrm{P}(X=-1)=\dfrac{5}{36}$ and find the probability distribution of X. [3]

(ii) Find the expectation and variance of X. [4]

(iii) Find the probability that Paul neither wins nor loses after two rounds of the game. [3]

9 Hey Day Farm claims that chicken eggs from their farm have a mean mass of at least 55 grams. A random sample of 30 eggs is selected. The masses, x, in grams, are summarised by

$$\sum x = 1615, \quad \sum x^2 = 87173.$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Determine if the claim is valid by carrying out a test at the 5% significance level. [4]

(iii) State the meaning of p-value in the context of this question. [1]

After the introduction of a new type of feed to their hens, Hey Day Farm takes a new random sample of 60 eggs. The masses of the sample have mean m grams and variance 9 grams^2. A test is carried out using a 5% significance level with the null hypothesis $\mu = 55$ and the alternative hypothesis $\mu \neq 55$, where μ grams is the population mean mass. The test indicates that there is sufficient evidence to deduce that the mean mass of the population differs from 55 grams.

(iv) Find the set of values within which the mean mass m of this sample must lie, giving your answer to 2 decimal places. [4]

10 [In this question, you should state clearly the values of the parameters of any distribution you use.]

Mr Lim is a tuition teacher who charges his students according to the duration of different topics taught during the lesson. The duration, in minutes, of Pure Math and Statistics sessions in a single lesson are modelled as having independent normal distributions with means and standard deviations as shown in the table below.

	Mean	Standard deviation
Pure Math	55.4	3.6
Statistics	36.2	2.3

(i) Find the probability that the total duration of two randomly chosen Pure Math sessions differs from three times the duration of a randomly chosen Statistics session by at least 3 minutes. [4]

(ii) The probability of the average duration of 5 Statistics sessions exceeding r minutes is at least 0.3. Find the range of values of r, giving your answer correct to 2 decimal places. [3]

Students pay \$2.50 and \$1.50 for each minute of Pure Math and Statistics sessions during a lesson respectively. In addition, students pay a fixed charge of \$24 per lesson. Assume that one lesson of Mr Lim consists of a Pure Math and a Statistics session.

(iii) **(a)** Calculate the mean and variance of the total cost paid in a randomly chosen lesson. [2]

(b) Find the probability that out of 15 randomly chosen lessons, a student pays less than \$220 in at most 5 lessons. [3]

Another tuition teacher, Miss Lee, charges her students according to the duration of the lesson. The duration, in minutes, in a single lesson is normally distributed with mean 88.5 and standard deviation 3.8.

(iv) 20 lessons from Mr Lim and 16 lessons from Miss Lee are randomly selected. Find the probability that the mean duration of these lessons is between 90 and 91 minutes. [3]

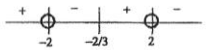
Qn	Solution
1(i)	$\frac{9x+2}{x-2}-2\geq 0$ $\frac{7x+6}{x-2}\geq 0$ $x\leq -\frac{6}{7}$ or $x>2$
(ii)	$\frac{9e^x+2}{2-e^x}\leq -2$ $\frac{9e^x+2}{e^x-2}\geq 2$ Replacing x with e^x in (i) $e^x\leq -\frac{6}{7}$ (reject) or $e^x>2$ $\Rightarrow x>\ln 2$

Qn	Solution
2	$\sum_{n=1}^{N}\ln\left(\frac{n+1}{n+2}\right)=\sum_{n=1}^{N}[\ln(n+1)-\ln(n+2)]$ $=\ \ln 2-\cancel{\ln 3}$ $+\cancel{\ln 3}-\cancel{\ln 4}$ $+\cancel{\ln 4}-\cancel{\ln 5}$ $+\ldots$ $+\cancel{\ln N}-\cancel{\ln(N+1)}$ $+\cancel{\ln(N+1)}-\ln(N+2)$ $=\ln 2-\ln(N+2)$
	When $N\to\infty,\ \ln(N+2)\to\infty$. Hence, $\sum_{n=1}^{N}\ln\left(\frac{n+1}{n+2}\right)\to -\infty$. The series is not convergent.

Qn	Solution
3(i)	$A=\frac{1}{n}\left[\ln\left(\frac{2}{1}\right)+\ln\left(\frac{\frac{1}{n}+2}{\frac{1}{n}+1}\right)+\ln\left(\frac{\frac{2}{n}+2}{\frac{2}{n}+1}\right)+\ldots+\ln\left(\frac{\frac{n-1}{n}+2}{\frac{n-1}{n}+1}\right)\right]$
	$=\frac{1}{n}\left[\ln\left(\frac{0+2n}{0+n}\right)+\ln\left(\frac{1+2n}{1+n}\right)+\ln\left(\frac{2+2n}{2+n}\right)+\ldots+\ln\left(\frac{(n-1)+2n}{(n-1)+n}\right)\right]$ $=\frac{1}{n}\sum_{r=0}^{n-1}\ln\left(\frac{r+2n}{r+n}\right)$
(ii)	$\lim_{n\to\infty}A=\int_0^1\ln\left(\frac{x+2}{x+1}\right)\,dx=0.523$ (3s.f.)

Qn	Solution
4(i)	Let x represent selling price of a bottle of Vitamin C y represent selling price of a bottle of Grape Seed Extract z represent selling price of a bottle of Super Antioxidant $x+y+z=365$ $0.9x+0.85y+0.85z=314.75$ $0.92x+0.75y+0.85z=301.55$ Hence $x=\$90,\ y=\$150,\ z=\$125$
(ii)	$0.85x+0.8y+0.85z=\$302.75>\$301.55$ Not attractive.

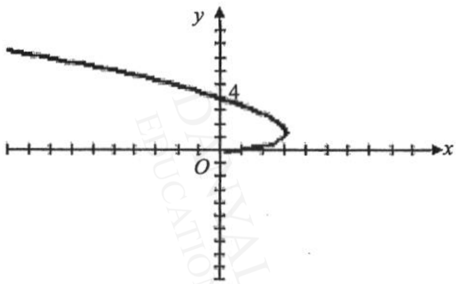
Qn	Solution
5(i)	$x^2 - kxy + y^2 = 3k + 2$ Differentiating with respect to x, $2x - kx\frac{\mathrm{d}y}{\mathrm{d}x} - ky + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $(2y - kx)\frac{\mathrm{d}y}{\mathrm{d}x} = ky - 2x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ky - 2x}{2y - kx}$ Tangent parallel to y-axis $\Rightarrow$ $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undefined $2y - kx = 0$ $y = \frac{kx}{2}$ Substitute $y = \frac{kx}{2}$ into the equation of curve: $x^2 - kx\left(\frac{kx}{2}\right) + \left(\frac{kx}{2}\right)^2 = 3k + 2$ $x^2 - \frac{1}{4}k^2x^2 = 3k + 2$ $\left(1 - \frac{1}{4}k^2\right)x^2 = 3k + 2$ $\left(4 - k^2\right)x^2 = 12k + 8$

Qn	Solution
5(ii)	**Method 1: Using definition of square root** $\left(4 - k^2\right)x^2 = 12k + 8$ $x^2 = \frac{12k + 8}{4 - k^2}$ $x = \pm\sqrt{\frac{12k + 8}{4 - k^2}}$ For $\sqrt{\mathrm{f}(x)}$ to exist and to be defined, , $\frac{12k + 8}{4 - k^2} > 0$ $\frac{4(3k + 2)}{(2 - k)(2 + k)} > 0$  $k < -2$ or $-\frac{2}{3} < k < 2$
	Alternative Method (Using Discriminant) For the tangents to exist, equation in (i) has real and distinct roots $\Rightarrow D > 0$. $\left(4 - k^2\right)x^2 = 12k + 8$ $0^2 - 4\left(4 - k^2\right)\left(-12k - 8\right) > 0$ $-4(2 - k)(2 + k)\left[-4(3k + 2)\right] > 0$ $16(2 - k)(2 + k)(3k + 2) > 0$ $(2 - k)(2 + k)(3k + 2) > 0$ $k < -2$ or $-\frac{2}{3} < k < 2$

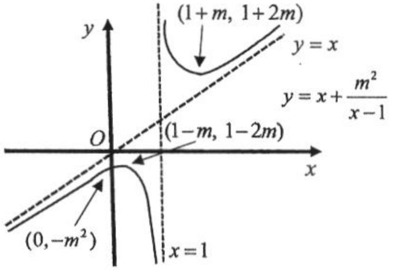
Qn	Solution
6(i)	For volume of material to make the hollow pipe: $k=\pi r^2h-\pi(r-x)^2h$ $=\pi h\left(r^2-r^2+2rx-x^2\right)$ $=\pi hx(2r-x)$ $\therefore\ h=\dfrac{k}{\pi x(2r-x)}$ Inner and outer curved surface areas: $2\pi rh+2\pi(r-x)h=2\pi h(2r-x)$ $=2\pi\left(\dfrac{k}{\pi x(2r-x)}\right)(2r-x)$ $=\dfrac{2k}{x}$ Top and base surface area: $2\left[\pi r^2-\pi(r-x)^2\right]=2\pi\left[r^2-r^2+2rx-x^2\right]=2\pi x(2r-x)$ $C=3\left[\dfrac{2k}{x}\right]+2\left[2\pi x(2r-x)\right]=\dfrac{6k}{x}+8\pi xr-4\pi x^2$

Qn	Solution
6(ii)	When $k=40$ and $r=8$, $C=\dfrac{240}{x}+64\pi x-4\pi x^2$ For minimum cost, $\dfrac{\mathrm{d}C}{\mathrm{d}x}=-\dfrac{240}{x^2}+64\pi-8\pi x=0$ $\Rightarrow \pi x^3-8\pi x^2+30=0$ Using GC, $x=-1.0284$ or $x=1.1836$ or $x=7.8448$ Since $x>0$ we reject $x=-1.0284$. $\dfrac{\mathrm{d}^2C}{\mathrm{d}x^2}=\dfrac{480}{x^3}-8\pi$ For $x=1.1836$, $\dfrac{\mathrm{d}^2C}{\mathrm{d}x^2}=\dfrac{480}{(1.1836)^3}-8\pi=264.35\quad(>0)$ For $x=7.8448$, $\dfrac{\mathrm{d}^2C}{\mathrm{d}x^2}=\dfrac{480}{(7.8448)^3}-8\pi=-24.14\quad(<0)$ Therefore C is minimum when $x=1.1836$ when $x=1.1836$, $C=\dfrac{240}{1.1836}+64\pi(1.1836)-4\pi(1.1836)^2=423.14$ Therefore minimum value of C is \$423.14

Qn	Solution
7 (i)	$\mathrm{f}(x)=\sin\left(3x+\frac{\pi}{6}\right)$ $\mathrm{f}'(x)=3\cos\left(3x+\frac{\pi}{6}\right)$
	$\mathrm{f}''(x)=-9\sin\left(3x+\frac{\pi}{6}\right)$
	Substitute $x=0$ into the above, $\mathrm{f}(0)=\frac{1}{2}$, $\mathrm{f}'(0)=\frac{3\sqrt{3}}{2}$, $\mathrm{f}''(0)=-\frac{9}{2}$
	$\mathrm{f}(x)=\frac{1}{2}+x\left(\frac{3\sqrt{3}}{2}\right)+\frac{x^2}{2}\left(-\frac{9}{2}\right)+\ldots$
	$=\frac{1}{2}+\frac{3\sqrt{3}}{2}x-\frac{9}{4}x^2+\ldots$
(ii)	$\frac{1}{a+bx}=(a+bx)^{-1}$ $=\frac{1}{a}\left(1+\frac{b}{a}x\right)^{-1}$ $=\frac{1}{a}\left(1-\frac{b}{a}x+\ldots\right)$
	$=\frac{1}{a}-\frac{b}{a^2}x+\ldots$
	$\frac{1}{a}=\frac{1}{2}\Rightarrow a=2$
	$-\frac{b}{a^2}=\frac{3\sqrt{3}}{2}\Rightarrow b=-6\sqrt{3}$
(iii)	$\left\|\frac{-6\sqrt{3}}{2}x\right\|<1\Rightarrow\|x\|<\frac{\sqrt{3}}{9}$ $\therefore -\frac{\sqrt{3}}{9}<x<\frac{\sqrt{3}}{9}$

Qn	Solution
8 (i)	
(ii)	Area, $R=\int_0^4 x\ \mathrm{d}y$ $=\int_0^{-2}\left(t^3-4t\right)(2t)\ \mathrm{d}t$
	$=-2\int_{-2}^{0}\left(t^4-4t^2\right)\ \mathrm{d}t$, where $k=-2, t_1=-2, t_2=0$
	$=-2\left[\frac{t^5}{5}-\frac{4t^3}{3}\right]_{-2}^{0}$
	$=-2\left[0-\left(\frac{-32}{5}+\frac{32}{3}\right)\right]$ $=\frac{128}{15}$
(iii)	$y=t^2\Rightarrow t=-\sqrt{y}$ $x=-y^{\frac{3}{2}}+4y^{\frac{1}{2}}$
	Volume required $=\pi\int_0^4\left(-y^{\frac{3}{2}}+4y^{\frac{1}{2}}\right)^2\ \mathrm{d}y$
	$=67.0$

	Alternative method $x=t^3-4t$ $=t\left(t^2-4\right)$ $=t(y-4)$ $\Rightarrow t=\dfrac{x}{y-4}$
	Subst. into $y=t^2$, $y=\left(\dfrac{x}{y-4}\right)^2$
	$\Rightarrow x^2=y\left(y-4\right)^2$ Volume required $=\pi\int_0^4 y\left(y-4\right)^2\mathrm{d}y$
	$=67.0$

Qn	Solution
9(i)	$\mathrm{f}\left(x\right)=x+\dfrac{m^2}{x-1}$ $\mathrm{f}'(x)=1-\dfrac{m^2}{\left(x-1\right)^2}=0$ $\left(x-1\right)^2-m^2=0$ $x=1\pm m$ When $x=1+m$, $\mathrm{f}\left(x\right)=1+m+\dfrac{m^2}{1+m-1}=1+2m$ When $x=1-m$, $\mathrm{f}\left(x\right)=1-m+\dfrac{m^2}{1-m-1}=1-2m$ The stationary points are $(1+m,1+2m)$ and $(1-m,1-2m)$ 
(ii)	$x^2-(1+k)x+m^2+k=0$ $x^2-x-kx+m^2+k=0$ $x(x-1)+m^2=k(x-1)$ $x+\dfrac{m^2}{x-1}=k$ Draw horizontal line $y=k$ on the graph of C, for two distinct positive roots, then $-m^2<k<1-2m$ or $k>1+2m$

(iii)	$y=3x+\frac{1}{9x}$ Before C: $y=3x+\frac{1}{9x}+1$ Before B: $y=3\left(\frac{1}{3}x\right)+\frac{1}{9\left(\frac{1}{3}x\right)}+1$ $=x+\frac{1}{3x}+1$ Before A: $y=(x-1)+\frac{1}{3(x-1)}+1$ $=x+\frac{1}{3(x-1)}$
	Let $x+\frac{m^2}{x-1}=x+\frac{1}{3(x-1)}$
	$m^2=\frac{1}{3}$ $m=\frac{1}{\sqrt{3}}$ since $0.5<m<1$
	Alternative method $y=\mathrm{f}(x)=x+\frac{m^2}{x-1}$ After A: $y=\mathrm{f}(x+1)=x+1+\frac{m^2}{x}$ After B: $y=\mathrm{f}(3x+1)=3x+1+\frac{m^2}{3x}$ After C: $y=\mathrm{f}(3x+1)-1=3x+\frac{m^2}{3x}$ Given that $3x+\frac{m^2}{3x}=3x+\frac{1}{9x}$ $\frac{m^2}{3}=\frac{1}{9}$ $m=\frac{1}{\sqrt{3}}$ since $0.5<m<1$

Qn	**Solution**
10(i)	For gh to exist, $R_h \subseteq D_g$ $R_h=\mathbb{R}^+$ $D_g=\mathbb{R}^+$ Hence $R_h=D_g \Rightarrow gh$ exists.
	$\mathrm{g}(\mathrm{h}(x))=\mathrm{g}(\|x\|)$ $=\ln\|x\|, x\neq 0$
	$R_{gh}=\mathbb{R}$
	For hg to exist, $R_g \subseteq D_h$ $R_g=\mathbb{R}$ $D_h=(-\infty,0)\cup(0,\infty)$ $R_g \not\subset D_h$ hg does not exist.
(ii)	$\mathrm{f}(x)=\mathrm{gh}(x),\quad x<0$ $\mathrm{f}(x)=\ln\|x\|,\quad x<0$ Let $y=\ln\|x\|$ $\|x\|=\mathrm{e}^y$ $x=\pm\mathrm{e}^y$
	Since $x<0$, $x=-\mathrm{e}^y$ $\mathrm{f}^{-1}(x)=-\mathrm{e}^x, x\in\mathbb{R}$
(iii)	$\mathrm{fq}(x)=\ln\left(x^2\right), x\neq 0$ $\mathrm{q}(x)=\mathrm{f}^{-1}\left(\ln\left(x^2\right)\right)$ $=-e^{\ln\left(x^2\right)}$ $=-x^2$ Since $D_{\mathrm{fq}}=D_{\mathrm{q}}$, $\therefore \mathrm{q}(x)=-x^2,\quad x\neq 0$
	$R_q=(-\infty,0)$

Qn	Solution
11(i)	$\cos\theta=\dfrac{\left\lvert\begin{pmatrix}1\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-1\\0\\2\end{pmatrix}\right\rvert}{\sqrt{1^2+2^2}\sqrt{(-1)^2+2^2}}$ $=\dfrac{3}{5}$ $\theta=\cos^{-1}\left(\dfrac{3}{5}\right)$ $=53.1^\circ$
(ii)	$-x+2z=6$ $x+2z=10$ Using GC, $x=2-0\lambda$ $y=\lambda$ $z=4+0\lambda$ $\mathbf{r}=\begin{pmatrix}2\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\0\end{pmatrix},\lambda\in\mathbb{R}$
	Since (2, 0, 4) is a point on the line *HI*, the *z*-coordinate is 4. Height of the horizontal beam from the ground is 4 metres.
(iii)	$OI=\begin{pmatrix}2\\5\\4\end{pmatrix},\ OD=\begin{pmatrix}0\\0\\3\end{pmatrix}$ $DI=\begin{pmatrix}2\\5\\4\end{pmatrix}-\begin{pmatrix}0\\0\\3\end{pmatrix}=\begin{pmatrix}2\\5\\1\end{pmatrix}$
	Equation of the line passing through *DI* is $\mathbf{r}=\begin{pmatrix}0\\0\\3\end{pmatrix}+\mu\begin{pmatrix}2\\5\\1\end{pmatrix},\mu\in\mathbb{R}$

(iv)	$\overline{DQ}=\begin{pmatrix}2\\s\\k\end{pmatrix}-\begin{pmatrix}0\\0\\3\end{pmatrix}=\begin{pmatrix}2\\s\\k-3\end{pmatrix}$ $\overline{EQ}=\begin{pmatrix}2\\s\\k\end{pmatrix}-\begin{pmatrix}4\\0\\3\end{pmatrix}=\begin{pmatrix}-2\\s\\k-3\end{pmatrix}$
	Perpendicular distance from *Q* to *DGIH* $=\dfrac{\lvert\overline{DQ}\bullet\mathbf{n}\rvert}{\lvert\mathbf{n}\rvert}$ $=\dfrac{\left\lvert\begin{pmatrix}2\\s\\k-3\end{pmatrix}\bullet\begin{pmatrix}-1\\0\\2\end{pmatrix}\right\rvert}{\sqrt{(-1)^2+2^2}}$ $=\dfrac{\lvert -2+2k-6\rvert}{\sqrt{5}}$ $=\dfrac{\lvert 2k-8\rvert}{\sqrt{5}}$ Perpendicular distance from *Q* to *EFIH* $=\dfrac{\lvert\overline{EQ}\bullet\mathbf{n}\rvert}{\lvert\mathbf{n}\rvert}$ $=\dfrac{\left\lvert\begin{pmatrix}-2\\s\\k-3\end{pmatrix}\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}\right\rvert}{\sqrt{1^2+0^2+2^2}}$ $=\dfrac{\lvert -2+2k-6\rvert}{\sqrt{5}}$ $=\dfrac{\lvert 2k-8\rvert}{\sqrt{5}}$ Since the two distances are equal, *Q* is equidistant from both *DGIH* and *EFIH*.

Qn	Solution
12(i)	Tom's savings follow an AP with first term 100 and common difference 10. Hence, by end of June 2018, he would have saved $\frac{37}{2}(2(120)+36(10))=11100$

(ii)

Month	Amt owed on 1st of the month	Amt owed on 2nd of the month
1 (Aug 2018)	$40000(1.004)$	$40000(1.004)-800$
2 (Sep 2018)	$(40000(1.004)-800)(1.004)$ $=40000(1.004)^2-800(1.004)$	$40000(1.004)^2-800(1.004)-800$
3 (Oct 2018)	$40000(1.004)^3-800(1.004)^2-800(1.004)$	
n	$40000(1.004)^n-800(1.004)^{n-1}-....-800(1.004)$	

Hence, amount owed on 1st of the n^{th} month

$$=40000(1.004)^n-800(1.004)^{n-1}-....-800(1.004)$$

$$=40000(1.004)^n-800[(1.004)+....+800(1.004)^{n-1}]$$

$$=40000(1.004)^n-800\left(\frac{1.004(1.004^{n-1}-1)}{1.004-1}\right)$$

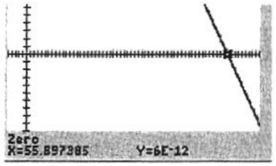
$$=40000(1.004)^n-800\left(\frac{1.004(1.004^{n-1}-1)}{1.004-1}\right)$$

$$=40000(1.004)^n-200800(1.004^{n-1}-1) \text{ (shown)}$$

(iii)

$$40000(1.004)^n-200800(1.004^{n-1}-1)\le 800$$

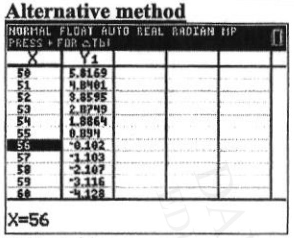
$$50(1.004)^n-251(1.004^{n-1}-1)-1\le 0$$



From the graph, $n\ge 55.897$

Hence, least $n=56$, i.e. the date that Tom would be able to repay his loan completely is 2 Mar 2023.

Alternative method



```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS ▸ FOR △Tbl
  X     Y1
  50    5.8169
  51    4.8401
  52    3.8595
  53    2.8749
  54    1.8864
  55    0.894
  56    ⁻0.102
  57    ⁻1.103
  58    ⁻2.107
  59    ⁻3.116
  60    ⁻4.128
X=56
```

Using table,

for $50(1.004)^n-251(1.004^{n-1}-1)-1\le 0$,

$n\ge 56$

Hence, least $n=56$, i.e. the date that Tom would be able to repay his loan completely is 2 Mar 2023.

(iv)

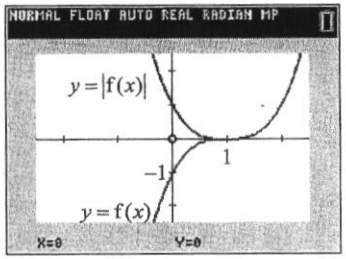
Let the new monthly repayment be x.

$$40000(1.004)^{48}-x\left(\frac{1.004(1.004^{47}-1)}{1.004-1}\right)\le x$$

$$x\left(1+\frac{1.004(1.004^{47}-1)}{1.004-1}\right)\ge 40000(1.004)^{48}$$

$$x\ge\frac{40000(1.004)^{48}}{1+251(1.004^{47}-1)}=917.5522$$

Hence, he needs to pay \$918 - \$800= \$118 more per month.

Qn	Solution
1(a)	$\mathrm{f}(x)=(x-1)^3$
(i)	 From graphs, $\mathrm{f}(x)=\lvert\mathrm{f}(x)\rvert$ when $x\geq 1$.
(ii)	$\int_0^n \lvert\mathrm{f}(x)\rvert\,\mathrm{d}x=\int_0^1 -(x-1)^3\,\mathrm{d}x+\int_1^n (x-1)^3\,\mathrm{d}x$
	$=-\frac{1}{4}\left[(x-1)^4\right]_0^1+\frac{1}{4}\left[(x-1)^4\right]_1^n$
	$=\frac{1}{4}+\frac{1}{4}(n-1)^4$
(b)	$\int \frac{\sqrt{x^2-4}}{x}\,\mathrm{d}x$ $=\int \frac{\sqrt{(2\sec\theta)^2-4}}{2\sec\theta}(2\sec\theta\tan\theta)\,\mathrm{d}\theta$ $\boxed{\begin{array}{c} x=2\sec\theta \\ \frac{\mathrm{d}x}{\mathrm{d}\theta}=2\sec\theta\tan\theta \end{array}}$
	$=2\int \tan^2\theta\,\mathrm{d}\theta$
	$=2\int \sec^2\theta-1\,\mathrm{d}\theta$
	$=2(\tan\theta-\theta)+c$
	$=2\left(\frac{\sqrt{x^2-4}}{2}-\cos^{-1}\frac{2}{x}\right)+c$ $=\sqrt{x^2-4}-2\cos^{-1}\frac{2}{x}+c$

2(i)	$w=xy$ ---(1) $\frac{\mathrm{d}w}{\mathrm{d}x}=x\frac{\mathrm{d}y}{\mathrm{d}x}+y$ --- (2)
	Subst. (1) and (2) into $x\frac{\mathrm{d}y}{\mathrm{d}x}+y-xy=1$ $\frac{\mathrm{d}w}{\mathrm{d}x}-w=1$ $\frac{\mathrm{d}w}{\mathrm{d}x}=1+w$ (shown)
	$\int \frac{1}{1+w}\,\mathrm{d}w=\int 1\,\mathrm{d}x$
	$\ln\lvert 1+w\rvert=x+c$
	$1+w=Ae^x$, where $A=\pm e^c$ $w=Ae^x-1$ $xy=Ae^x-1$
	Subst. $x=0$ and $y=1$, $A=1$ $xy=e^x-1$ $y=\frac{e^x-1}{x}$
(ii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=x\mathrm{e}^{-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\int x\mathrm{e}^{-x}\mathrm{d}x$ $=-x\mathrm{e}^{-x}+\int \mathrm{e}^{-x}\mathrm{d}x$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-x\mathrm{e}^{-x}-\mathrm{e}^{-x}+c$
	$y=\int -x\mathrm{e}^{-x}-\mathrm{e}^{-x}+c\,\mathrm{d}x$ $=-\int x\mathrm{e}^{-x}\mathrm{d}x-\int \mathrm{e}^{-x}\mathrm{d}x+\int c\,\mathrm{d}x$ $=-\left(-x\mathrm{e}^{-x}-\mathrm{e}^{-x}\right)+\mathrm{e}^{-x}+cx+d$
	$=x\mathrm{e}^{-x}+2\mathrm{e}^{-x}+cx+d$

2020 J2 H2 Mathematics Preliminary Examination P2 (Solutions)

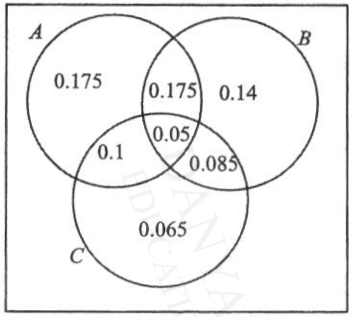
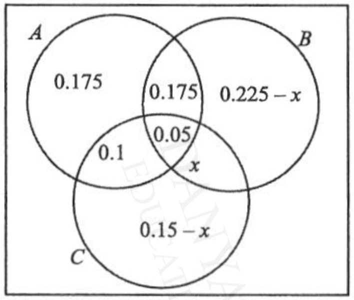
Qn	Solution
3(a)	Since $1+4i$ is a root of the equation $az^4-z^3+26z^2+bz-34=0$, $a(1+4i)^4-(1+4i)^3+26(1+4i)^2+b(1+4i)-34=0$ $a(161-240i)-(-47-52i)+26(-15+8i)+b(1+4i)-34=0$
	Comparing real and imaginary parts, $161a+b=377$ ... (1) $240a-4b=260$ ... (2)
	Solving (1) and (2) using GC, $a=2, b=55$
	$2z^4-z^3+26z^2+55z-34=0$ $z=1+4i,\ 1-4i,\ -2,\ \frac{1}{2}$
	Alternative method $(z-(1+4i))(z-(1-4i))(z+2)=(z^2-2z+17)(z+2)$ $=z^3+13z+34$
	$az^4-z^3+26z^2+bz-34=(z^3+13z+34)(az-1)$ Comparing coefficients, $26=13a \Rightarrow a=2$ $b=-13+34a=55$
	Hence, the remaining 2 roots are $1-4i$ and $\frac{1}{2}$.
(b) (i)	$\left\|\left(\sqrt{3}+\mathrm{i}\right)^{10}\right\|=\left\|\sqrt{3}+\mathrm{i}\right\|^{10}=\left(\sqrt{\sqrt{3}^2+1^2}\right)^{10}=2^{10}=1024$ $10\arg\left(\sqrt{3}+\mathrm{i}\right)=10\left(\frac{\pi}{6}\right)=\frac{5}{3}\pi$ $\therefore \arg\left(\sqrt{3}+\mathrm{i}\right)^{10}=-\frac{1}{3}\pi$

2020 J2 H2 Mathematics Preliminary Examination P2 (Solutions)

(ii)	$\arg\left(\frac{w^n}{w^*}\right)=\arg\left(w^n\right)-\arg\left(w^*\right)$ $=n\arg\left(w\right)+\arg\left(w\right)$ $=-\frac{\pi}{3}(n+1)$
	$\frac{w^n}{w^*}$ is real $\Rightarrow \arg\left(\frac{w^n}{w^*}\right)=k\pi$, where $k\in\mathbb{Z}$ $-\frac{\pi}{3}(n+1)=...,-2\pi,-\pi,0,\pi,2\pi,...$ $n=...,5,2,-1,-4,-7,...$ Smallest positive integer value for $n=2$

Qn	Solution
4(i)	$\overrightarrow{OM}=k\mathbf{b}+(1-k)\mathbf{a}$, by ratio theorem $\overrightarrow{ON}=k\mathbf{b}$.
	$\overrightarrow{AN}=\overrightarrow{ON}-\overrightarrow{OA}=k\mathbf{b}-\mathbf{a}$ line AN: $\mathbf{r}=\mathbf{a}+\lambda\left(k\mathbf{b}-\mathbf{a}\right), \lambda\in\mathbb{R}$----(1) line OM: $\mathbf{r}=\ \mu\left(k\mathbf{b}+(1-k)\mathbf{a}\right),\ \mu\in\mathbb{R}$---(2) To find position vector of point G, equate (1) and (2) $\mathbf{a}+\lambda\left(k\mathbf{b}-\mathbf{a}\right)=\ \mu\left(k\mathbf{b}+(1-k)\mathbf{a}\right)$ $(1-\lambda)\mathbf{a}+\lambda k\mathbf{b}=\mu k\mathbf{b}+\mu(1-k)\mathbf{a}$ $\therefore\ \lambda k=\mu k$---(3) and $(1-\lambda)=\mu(1-k)$-----(4) From (3) $\lambda=\mu$ Subt $\lambda=\mu$ into (4) $(1-\lambda)=\lambda(1-k)$ $2\lambda-\lambda k=1$ $\lambda=\frac{1}{2-k}=\mu$ $\therefore\ \overrightarrow{OG}=\frac{1}{2-k}\left(k\mathbf{b}+(1-k)\mathbf{a}\right)=\frac{1}{2-k}\left((1-k)\mathbf{a}+k\mathbf{b}\right)$ (Shown)

(ii)	$\overrightarrow{OG}=\frac{1}{2-\frac{1}{2}}\left((1-\frac{1}{2})\mathbf{a}+\frac{1}{2}\mathbf{b}+\right)+=\frac{1}{3}(\mathbf{a}+\mathbf{b})$ $\overrightarrow{AG}=\overrightarrow{OG}-\overrightarrow{OA}=\frac{1}{3}(\mathbf{a}+\mathbf{b})-\mathbf{a}=\frac{1}{3}(\mathbf{b}-2\mathbf{a})$ $\overrightarrow{AM}=\overrightarrow{OM}-\overrightarrow{OA}=\left(\frac{1}{2}\mathbf{b}+\frac{1}{2}\mathbf{a}\right)-\mathbf{a}=\frac{1}{2}(\mathbf{b}-\mathbf{a})$ Area of $AGM=$ $\frac{1}{2}\left\|\overrightarrow{AG}\times\overrightarrow{AM}\right\|=\frac{1}{2}\left\|\frac{1}{3}(\mathbf{b}-2\mathbf{a})\times\frac{1}{2}(\mathbf{b}-\mathbf{a})\right\|$ $=\frac{1}{12}\|(\mathbf{b}-2\mathbf{a})\times(\mathbf{b}-\mathbf{a})\|$ $=\frac{1}{12}\|\mathbf{b}\times\mathbf{b}-\mathbf{b}\times\mathbf{a}-2\mathbf{a}\times\mathbf{b}+2\mathbf{a}\times\mathbf{a}\|$ $=\frac{1}{12}\|-\mathbf{b}\times\mathbf{a}-2\mathbf{a}\times\mathbf{b}\|$, Since $\mathbf{b}\times\mathbf{b}=2\mathbf{a}\times\mathbf{a}=\mathbf{0}$ $=\frac{1}{12}\|-\mathbf{b}\times\mathbf{a}+2\mathbf{b}\times\mathbf{a}\|$ $=\frac{1}{12}\|\mathbf{b}\times\mathbf{a}\|$ $=\frac{1}{6}\left(\frac{1}{2}\|\mathbf{b}\times\mathbf{a}\|\right)$ $=\frac{1}{6}(\text{Area of triangle } OAB)$ Area of triangle $AGM=\frac{1}{6}(\text{Area of triangle } OAB)$ Area of triangle $OAB=6\times(\text{Area of triangle } AGM)$ (Shown)

Qn	Solution
5(i)	 $\text{P}(A)=0.5$, $\text{P}(B)=0.45$, $\text{P}(C)=0.3$, $\text{P}(A\cap B\cap C)=0.05$ $\text{P}(A\cap B)=\text{P}(A)\times\text{P}(B)=0.5\times0.45=0.225$ $\text{P}(A\cap C)=\text{P}(A)\times\text{P}(C)=0.5\times0.3=0.15$ $\text{P}(B\cap C)=\text{P}(B)\times\text{P}(C)=0.45\times0.3=0.135$ $\text{P}(A'\cap B'\cap C')=1-\text{P}(A\cup B\cup C)$ $=1-0.5-0.065-0.085-0.14$ $=0.21$
(ii)	 $x\ge0$ and $0.15-x\ge0\Rightarrow x\le0.15$ and $0.225-x\ge0\Rightarrow x\le0.225$ Hence $0\le x\le0.15$. $\text{P}(A'\cap B'\cap C')=1-\text{P}(A\cup B\cup C)$ $=1-(0.5+0.15-x+x+0.225-x)$ $=0.125+x$ When $x=0$, least $\text{P}(A'\cap B'\cap C')=0.125$ When $x=0.15$, greatest $\text{P}(A'\cap B'\cap C')=0.125+0.15=0.275$

Qn	Solution
6(i)	No. of ways where the two 'R's are together $= \dfrac{10!}{2!} = 1814400$
	Required probability $= \dfrac{1814400}{\dfrac{11!}{2!2!}} = \dfrac{1814400}{9979200} = \dfrac{2}{11}$ or 0.182
(ii)	No. of ways to arrange the consonants $= \dfrac{6!}{2!} = 360$
	No. of ways to slot in the vowels $= {}^7C_5 \times \dfrac{5!}{2!} = 1260$
	Required probability $= \dfrac{360 \times 1260}{\dfrac{11!}{2!2!}} = \dfrac{453600}{9979200} = \dfrac{1}{22}$ (or 0.0455)
(iii)	No of ways that the first letter is a C or last letter is an S $= \dfrac{10!}{2!2!} \times 2 = 1814400$
	No of ways that the first letter is a C and last letter is an S $= \dfrac{9!}{2!2!} = 90720$
	Required probability $= = \dfrac{1814400 - 90720}{\dfrac{11!}{2!2!}} = \dfrac{1723680}{9979200} = \dfrac{19}{110}$ (or 0.173)

Qn	Solution
7(i)	Let X = No of defective masks per box of 20 $X \sim \mathrm{B}(20, 0.08)$ Expected number of defective masks=20(0.08)=1.6
(ii)	$\mathrm{P}(2 \le X < 10) = \mathrm{P}(X \le 9) - \mathrm{P}(X \le 1) = 0.48314$ ≈ 0.483
	Alternatively, $\mathrm{P}(2 \le X < 10) = \mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \quad + \mathrm{P}(X = 8) + \mathrm{P}(X = 9)$ $= 0.48314 \approx 0.483$
(iii)	$P(X \le 3) = 0.9293848$ $P(X \ge 2) = 1 - P(X \le 1) = 0.483144$ $P(X \le 3).[P(X \ge 2)]^3.\dfrac{4!}{3!} = 0.41926 \approx 0.419$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>8(i)</td><td>
Cases (Paul, Mary) : (1,2) (2,3) (3,4) (4,5) (5,6)
$\mathrm{P}(X = -1) = \dfrac{5}{36}$ (AG)
<table>
<tr><th>Mary / Paul</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>0</td><td>–1</td><td>–2</td><td>–3</td><td>–4</td><td>–5</td></tr>
<tr><td>2</td><td>1</td><td>0</td><td>–1</td><td>–2</td><td>–3</td><td>–4</td></tr>
<tr><td>3</td><td>2</td><td>1</td><td>0</td><td>–1</td><td>–2</td><td>–3</td></tr>
<tr><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td><td>–1</td><td>–2</td></tr>
<tr><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td><td>–1</td></tr>
<tr><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td></tr>
</table>
Let Paul's winnings per play be X
<table>
<tr><td>X=x</td><td>–5</td><td>–4</td><td>–3</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>P(X=x)</td><td>$\frac{1}{36}$</td><td>$\frac{1}{18}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{9}$</td><td>$\frac{5}{36}$</td><td>$\frac{1}{6}$</td><td>$\frac{5}{36}$</td><td>$\frac{1}{9}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{18}$</td><td>$\frac{1}{36}$</td></tr>
</table>
</td></tr>
<tr><td>(ii)</td><td>
Expectation of Paul's winnings
$\mathrm{E}(X) = (\dfrac{1}{36})[-5-8-9-8-5+0+5+8+9+8+5] = 0$
</td></tr>
<tr><td></td><td>
$\mathrm{E}(X^2) = (\dfrac{1}{36})[25+16(2)+9(3)+4(4)+1(5)+0](2) = \dfrac{210}{36} = \dfrac{35}{6}$
$\mathrm{Var}\ (X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2 = \dfrac{35}{6} - 0 = \dfrac{35}{6}$
</td></tr>
<tr><td>(iii)</td><td>
P(Paul neither wins nor loses after 2 plays of the game)
$= 2\begin{bmatrix} \mathrm{P}(X_1 = -5, X_2 = 5) + \mathrm{P}(X_1 = -4, X_2 = 4) + \mathrm{P}(X_1 = -3, X_2 = 3) \\ + \mathrm{P}(X_1 = -2, X_2 = 2) + \mathrm{P}(X_1 = -1, X_2 = 1) \end{bmatrix}$
$+ \mathrm{P}(X_1 = 0, X_2 = 0) = 2\left[\left(\dfrac{1}{36}\right)^2 + \left(\dfrac{1}{18}\right)^2 + \left(\dfrac{1}{12}\right)^2 + \left(\dfrac{1}{9}\right)^2 + \left(\dfrac{5}{36}\right)^2\right] + \left(\dfrac{1}{6}\right)^2$
$= \dfrac{73}{648}$ or 0.113 (3s.f.)
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Qn	Solution
9 (i)	Unbiased estimate of population mean, $\overline{x}=\dfrac{\sum x}{n}=\dfrac{1615}{30}=\dfrac{323}{6}$ (exact) = 53.8 (3sf)
	Unbiased estimate of population variance, $s^2=\dfrac{1}{n-1}\left[\sum x^2-\dfrac{\left(\sum x\right)^2}{n}\right]$ $=\dfrac{1}{29}\left[87173-\dfrac{(1615)^2}{30}\right]=\dfrac{1393}{174}$ or $8.005747\approx 8.01$ (3 s.f.)
(ii)	Let X be the mass of a randomly chosen egg and μ be the population mean mass of the eggs $H_0: \mu=55$ $H_1: \mu<55$
	Under H_0, since $n=30$ is large, by Central Limit Theorem, $\overline{X}\sim N\left(55, \dfrac{\frac{1393}{174}}{30}\right)$ approximately. Test statistic $Z=\dfrac{\overline{X}-55}{\sqrt{\frac{1393}{174(30)}}}\sim N(0,1)$ Critical region: Reject H_0 if $z<-1.64485$ $z=\dfrac{\frac{323}{6}-55}{\sqrt{\frac{1393}{174(30)}}}=-2.26$ From GC, p-value = $0.011959\approx 0.0120$ (3 s.f.)
	Since p-value < 0.05, we reject H_0 and conclude that at 5% significance level there is sufficient evidence that the farmer's claim is not supported.
(iii)	p-value =0.0120 means there is a probability of 0.0120 of observing a sample mean mass less than or equal to $\frac{323}{6}$ when the population mean mass of the eggs is 55 grams. or p – value = 0.0120 is the smallest level of significance to reject the claim that the mean mass of the eggs is at least 55 grams.

(iv)	$H_0: \mu=55$ $H_1: \mu\neq 55$ Unbiased estimate of the population variance $=\dfrac{60}{59}(9)=\dfrac{540}{59}=9.1525$
	Under H_0, since $n=60$ is large, by Central Limit Theorem, $\overline{X}\sim N\left(55, \dfrac{\frac{540}{59}}{60}\right)$ approximately. Level of significance $\alpha=0.05$ Critical Region: Reject H_0 at 5% significance level if $z\le -1.95996$ or $z\ge 1.95996$
	Reject Ho $\dfrac{m-55}{\sqrt{\frac{9}{59}}}\le -1.95996$ or $\dfrac{m-55}{\sqrt{\frac{9}{59}}}\ge 1.95996$ $m\le 55-1.95996\sqrt{\frac{9}{59}}$ or $m\ge 55+1.95996\sqrt{\frac{9}{59}}$ $m\le 54.2345$ or $m\ge 55.7655$ $m\le 54.23$ or $m\ge 55.77$

Qn	Solution
10(i)	Let X and Y be the duration of Pure Math and Statistics taught a single lesson respectively. Then $X\sim N(55.4, 3.6^2)$ and $Y\sim N(36.2, 2.3^2)$ Let $T=X_1+X_2-3Y\sim N(2(55.4)-3(36.2), 2(3.6^2)+3^2(2.3^2))$ $T\sim N(2.2, 73.53)$ $P(\|T\|\ge 3)=1-P(\|T\|<3)$ $=1-P(-3<T<3)$ $=0.73495\approx 0.735$
(ii)	$\overline{Y}\sim N\left(36.2, \dfrac{2.3^2}{5}\right)=N(36.2, 1.058)$ $P(\overline{Y}>r)\ge 0.3$ $P(\overline{Y}\le r)\le 0.7$ $r\le 36.74$ (to 2 d.p.)
(iii) (a)	Mean = (2.50)(55.4) + (1.50)(36.2) + 24 = 216.80 Variance = $(2.50)^2(3.6^2)+(1.50)^2(2.3^2)=92.9025$

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(iii) **(b)**	Let F be the total cost paid by a student for a randomly chosen lesson. Then $F \sim \mathrm{N}(216.80,\ 92.9025)$ $\mathrm{P}(F < 220) = 0.63005$ Let L be the no. of lessons (out of 15) that cost less than \$220. Then $L \sim \mathrm{B}(15,\ 0.63005)$ $\mathrm{P}(L \le 5) = 0.018997 \approx 0.0190$
(iv)	Let W and V be the duration of Mr Lim and Miss Lee's class in a lesson respectively. $W \sim \mathrm{N}(91.6,\ 18.25)$ and $V \sim \mathrm{N}(88.5,\ 3.8^2)$ Let $M = \dfrac{W_1 + \ldots + W_{20} + V_1 + \ldots + V_{16}}{36} \sim \mathrm{N}\left(\dfrac{812}{9},\ \dfrac{59604}{129600}\right)$ $\mathrm{P}(90 < M < 91) = 0.50271 \approx 0.503$
	Alternatively, Let W and V be the duration of Mr Lim and Miss Lee's class in a lesson respectively. $W \sim \mathrm{N}(91.6,\ 18.25)$ and $V \sim \mathrm{N}(88.5,\ 3.8^2)$ Let $S = W_1 + \ldots + W_{20} + V_1 + \ldots + V_{16} \sim \mathrm{N}(3248,\ 596.04)$ $\mathrm{P}(90 < \dfrac{W_1 + \ldots + W_{20} + V_1 + \ldots + V_{16}}{36} < 91)$ $= \mathrm{P}(90 \times 36 < S < 91 \times 36)$ $= \mathrm{P}(3240 < S < 3276) = 0.50271 \approx 0.503$