H

ANGLO-CHINESE JUNIOR COLLEGE
JC1 PROMOTIONAL EXAMINATION

Higher 2

/100

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS

9758/01

Paper 1

5 October 2020
3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

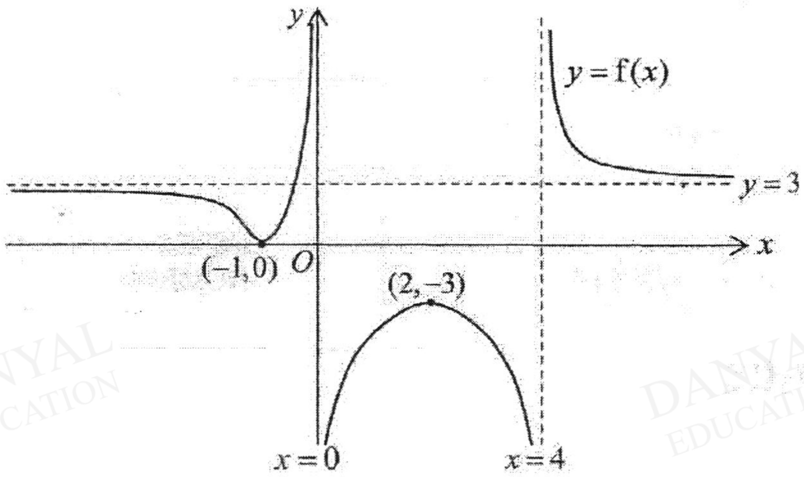
Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Question	Marks
1	/3
2	/4
3	/5
4	/5
5	/7
6	/7
7	/8
8	/8
9	/8
10	/11
11	/12
12	/10
13	/12

1 The diagram below shows the graph of $y = \mathrm{f}(x)$ with asymptotes $x = 0$, $x = 4$ and $y = 3$. The curve has a minimum point at $(-1, 0)$ and a maximum point at $(2, -3)$.



Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$, stating clearly the equations of any asymptotes, the coordinates of any turning points, and the coordinates of points where the curve crosses the axes. [3]

2 Users of a health application are given exercise targets to meet each week. Those who meet their weekly targets get to spin a wheel for a chance to win virtual coins that can be accumulated to exchange for vouchers. A successful spin wins the user either 40, 60, or 100 coins.

In a particular month, 2395 spins were made in total. Of these, 80% were successful spins and a total of 117 640 coins were won. 25% of the spins that won 40 coins, 35% of the spins that won 60 coins, and 40% of the spins that won 100 coins were made in the last week of the month, for a total of 40 255 coins.

Find the number of spins that won 100 coins that month. [4]

3 A curve C has equation $(y-kx)^2+8y-12=0$, where k is a real constant.

(i) Find $\frac{dy}{dx}$ in terms of x, y and k. [2]

(ii) The tangent to C at the point where $x=2$ is parallel to the y-axis. Find the value of k. [3]

4 Sketch the graphs of $y=|x|$ and $y=\dfrac{c}{x-c}$, where $c>0$, on the same diagram, stating clearly the coordinates of any points where the curves cross the axes. [2]

Solve the equation

$$|x|=\frac{c}{x-c},$$

leaving your answer in terms of c. [2]

Hence solve the inequality $|x|>\dfrac{c}{|x|-c}$. [1]

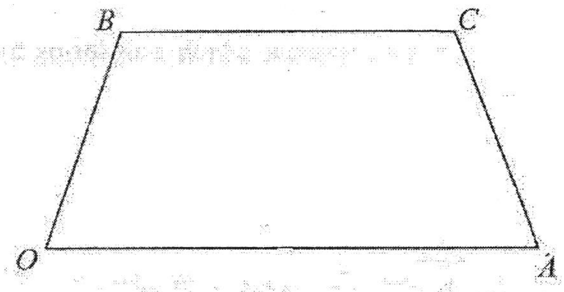
5 Sketch the graph of C_1 given by the equation $y^2-(x+a)^2=1$, where $a>0$, stating clearly the equations of any asymptotes and the coordinates of any turning points. [2]

Hence sketch the graph of $y=\mathrm{f}'(x)$ where $\mathrm{f}(x)=-\sqrt{1+(x+a)^2}$, stating clearly the equations of any asymptotes, and the coordinates of point where the curve crosses the x-axis. [2]

The curve C_1 is transformed onto the curve C_2 with equation $(by+1)^2-(x-1)^2=1$, $b>1$.

Describe a sequence of transformations which transforms the curve C_1 onto the curve C_2. [3]

6 The diagram below shows a trapezium $OACB$, where $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $\overrightarrow{OC}=\mathbf{c}$.



Explain, using the diagram or otherwise, why $\mathbf{c}=\lambda\mathbf{a}+\mathbf{b}$, for some $\lambda\in\mathbb{R}^+$. [1]

By considering area of triangles or otherwise, prove that the area of the trapezium is

$$\frac{1+\lambda}{2}|\mathbf{a}\times\mathbf{b}|.$$ [2]

Show that $|\mathbf{a}.\mathbf{b}|^2+|\mathbf{a}\times\mathbf{b}|^2=(|\mathbf{a}||\mathbf{b}|)^2$. [2]

Given further that $3\mathbf{c}-3\mathbf{b}=\mathbf{a}$, $|\mathbf{a}|=2$, $|\mathbf{b}|=3$, $|\mathbf{a}.\mathbf{b}|=5$, find the area of trapezium $OACB$ exactly. [2]

7 The functions f and g are defined as follows:

$$\mathrm{f}: x \mapsto 7(x-a)^2 - 1,\ x \in \mathbb{R},\ x < 3$$
$$\mathrm{g}: x \mapsto 1 + b - \mathrm{e}^{-x},\ x \geq 0,$$

where a and b are real constants.

(i) State the smallest value of a such that f^{-1} exists. [1]

For the rest of this question, $a = 4$.

(ii) Explain if f^{-1} exists, and if it does, find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(iii) Find the largest value of b such that the composite function fg exists. For this value of b, find $\mathrm{fg}(x)$, state its domain, and find its range. [4]

8 The lines l and m are defined by the equations

$$l : \mathbf{r} = \mathbf{i} - \mathbf{k} + \lambda(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}),$$

$$m : \frac{x-1}{4} = \frac{a-y}{a} = \frac{z+3}{4}.$$

(i) Given that the lines intersect, show that $a = 6$. [2]

(ii) Find the position vector of N, the foot of perpendicular from the point $A(5, 0, 1)$ to the line l. [3]

(iii) Hence or otherwise, find the position vector of the two points on l that are 5 units from A. [3]

9 The plane π passes through the point with position vector $\mathbf{i}-3\mathbf{k}$ and contains the line with equation $\mathbf{r}=\mathbf{i}+\mathbf{j}+\lambda(2\mathbf{i}-\mathbf{j}+\mathbf{k})$.

(i) Show that the cartesian equation of the plane π is $2x+3y-z=5$. [2]

9 **[Continued]**

(ii) Find the shortest distance from $P(-5, 6, -5)$ to the plane π. [2]

The line L that passes through P and is parallel to $3\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ intersects the plane π at the point Q.

(iii) Find the coordinates of Q. [2]

(iv) Hence or otherwise, find the length of projection of $\overrightarrow{PQ}$ on the plane π. [2]

10 By considering partial fractions, show that

$$\sum_{r=1}^{n} \frac{2r+3}{r(r+1)(r+2)} = A - \frac{3}{2(n+1)} - \frac{1}{2(n+2)},$$

where A is a constant to be determined. [4]

10 [Continued]

(i) Explain why $\sum_{r=1}^{\infty} \frac{2r+3}{r(r+1)(r+2)}$ converges, and state the convergence limit. [2]

(ii) Find the least value of n such that $\sum_{r=1}^{n} \frac{2r+3}{r(r+1)(r+2)} > \frac{8}{5}$. [2]

(iii) Evaluate

$$\frac{9}{3\times4\times5}+\frac{11}{4\times5\times6}+\frac{13}{5\times6\times7}+\ldots\ldots+\frac{2N+1}{N\left(N^2-1\right)},$$

leaving your answer in terms of N. [3]

11 A curve C has parametric equations $x=\frac{t}{\sqrt{1-4t^2}}$ and $y=\sin^{-1}2t$, for $-\frac{1}{2}<t<\frac{1}{2}$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}=2(1-4t^2)$ and explain why the gradient is always positive for all points on the curve. [4]

(ii) Describe the behaviour of C as $x \to \pm\infty$. Sketch C, stating clearly the equations of any asymptotes and the coordinates of any points of intersection with the axes. [3]

(iii) Given that A is the point on C with parameter $\frac{1}{2\sqrt{2}}$, find the equation of l, the tangent to C at the point A, leaving your answer in exact form. [3]

11 **[Continued]**

Show that l meets the curve C again at another point B and find the coordinates of B. [2]

12 [For this question, leave all numerical answers to the nearest cent.]

Ann is buying an apartment that is projected to be valued at \$500 000 on 1 January 2021. She intends to pay \$200 000 upfront, and take a loan of \$300 000 from a bank that charges interest at 2.5% per year, compounded on the outstanding loan amount at the end of each year.

The loan is disbursed on 1 January 2021 and she starts her yearly instalment payment of \$$x$ on that date.

(i) Show that the outstanding loan amount at the end of n years after interest is charged is

$$\$\left[1.025^n(300\,000)-41x\left(1.025^n-1\right)\right].$$ [3]

(ii) Ann intends to pay $\$x$ at the beginning of every year so that her last payment is on 1 January 2040, after which she would have fully paid for the apartment. Calculate x. [2]

For the rest of this question, Ann's yearly instalment is $\$x$, as calculated in **(ii)**.

(iii) Calculate the total amount she paid the bank in interest. [2]

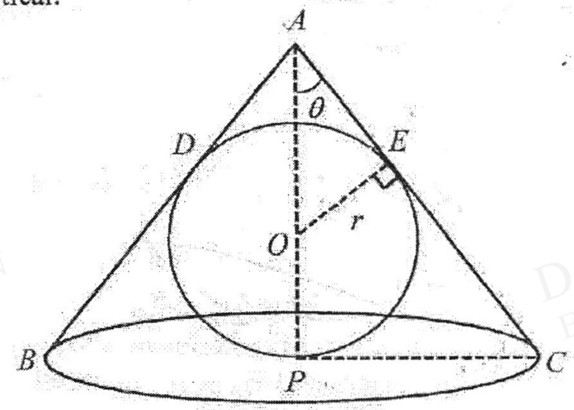
The apartment that Ann is buying typically appreciates in value by 3% per year.

(iv) Find the value of Ann's apartment after 10 years. [1]

(v) If Ann sells the apartment on 1 January 2031 before paying her usual instalment, by first calculating the outstanding loan amount that she needs to repay the bank, find the amount she earns from the sale. [2]

13 [A cone of radius r and height h has volume $V = \frac{1}{3}\pi r^2 h$.]

(a) An ornament consists of a sphere of fixed radius r cm inscribed in a right circular cone. The sphere is in contact with the base of the cone at the point P, and with the inner surfaces of the cone at the points D and E respectively, as shown in the diagram. Each of the lengths AB and AC makes an angle of θ radians with the downward vertical.



(i) Show that the volume of the cone, V cm^3, is given by

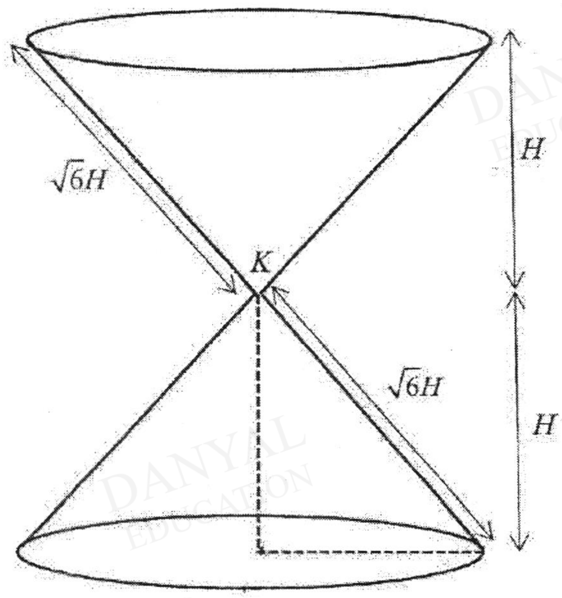
$$V = \frac{1}{3}\pi r^3 (1 + \operatorname{cosec}\theta)^3 \tan^2\theta .$$ [2]

(ii) Using differentiation, show that the volume of the cone is minimum when $\sin\theta = \dfrac{1}{3}$. [5]

13 [Continued]

(b) Another ornament in the shape of an hour-glass is made up of two identical right circular cones of fixed height H cm and slant height $\sqrt{6}H$ cm as shown in the diagram below. The hour-glass is filled with some luminous liquid such that when inverted, the liquid in the upper cone flows into the lower cone at a rate of $4\,\text{cm}^3\text{s}^{-1}$ through a hole of negligible size at K. At time t s, the depth of the liquid in the lower cone is h cm. Show that W cm^3, the volume of liquid in the lower cone at t s, is given by

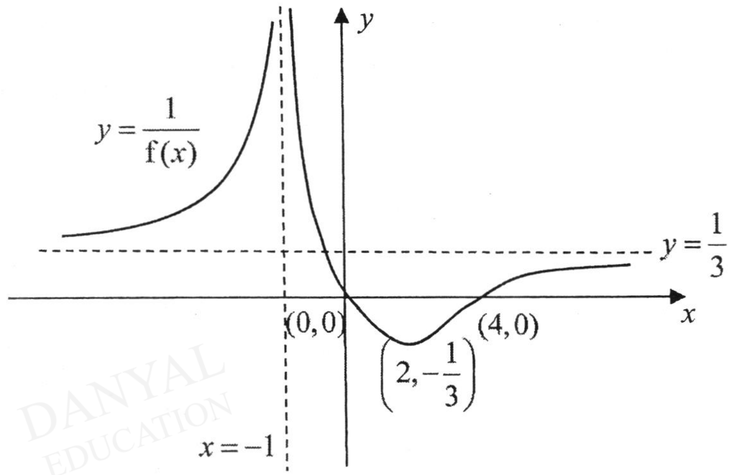
$$W = \frac{5}{3}\pi\left[H^3 - (H-h)^3\right].$$ [2]



Find the rate of increase of the depth of the liquid in the lower cone when the depth of the liquid in the lower cone is $\frac{1}{25}H$ cm. Leave your answer in exact form in terms of π and H. [3]

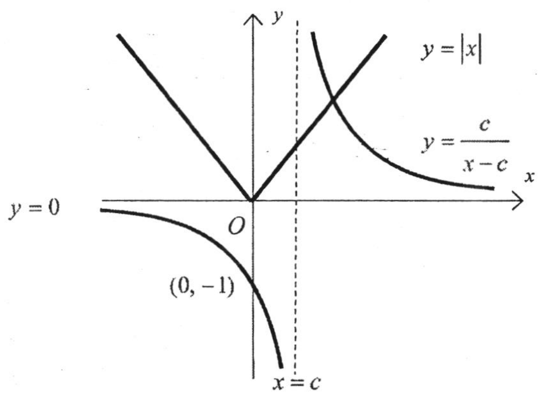
BLANK PAGE

2020 ACJC H2 Math Promo Solutions & Marker's Report

Qn	Solution	Comments
1		This question was generally well-done. A small number of students excluded the points at the two x-intercepts, eg (0, 0) and (4, 0) which is not supposed to. Some students did not label the x-intercepts as coordinates.
2	Let x, y and z be the number of successful spins that won 40, 60 or 100 coins respectively. $x+y+z=2395\times0.8$ $40x+60y+100z=117\ 640$ $(0.25\times40x)+(0.35\times60y)+(0.40\times100z)=40\ 255$ From G.C, $x=836,\quad y=595,\quad z=485$. Hence 485 spins won 100 coins that month.	x, y and z should be clearly defined. Some students were not clear with the definitions and ended up forming equations which were inconsistent with each other or the definitions. No or little credit was given for such cases. Quite a number of students missed one equation and only formed two equations. They then incorrectly tried to use some limitations to solve the question. Students should also be aware that something is wrong if negative values or values greater than 2395 (the total number of spins made) were obtained. It is good practice to write a short statement to answer the question.

[FOR INTERNAL USE ONLY]

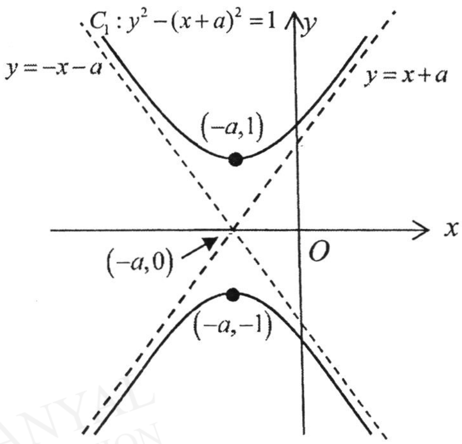
Qn	Solution	Comments
3	$(y-kx)^2+8y-12=0$ ----- (1) Differentiating with respect to x, $2(y-kx)\left(\frac{\mathrm{d}y}{\mathrm{d}x}-k\right)+8\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $[2(y-kx)+8]\frac{\mathrm{d}y}{\mathrm{d}x}-2k(y-kx)=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2k(y-kx)}{2(y-kx)+8}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{k(y-kx)}{y-kx+4}$ ALTERNATIVELY, $(y-kx)^2+8y-12=0$ $y^2-2kxy+k^2x^2+8y-12=0$ Differentiating with respect to x, $2y\frac{\mathrm{d}y}{\mathrm{d}x}-2k\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)+2k^2x+8\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $(2y-2kx+8)\frac{\mathrm{d}y}{\mathrm{d}x}=2ky-2k^2x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{k(y-kx)}{y-kx+4}$	This question was generally well-done. Common mistakes include the wrong sign when expanding $2y\frac{\mathrm{d}y}{\mathrm{d}x}-2k\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)+2k^2x+8\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into $2y\frac{\mathrm{d}y}{\mathrm{d}x}-2kx\frac{\mathrm{d}y}{\mathrm{d}x}+2ky+2k^2x+8\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Students should also note that in A level exam, they are expected to simplify the final answer, instead of leaving it as $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2k(y-kx)}{2y-2kx+8}$
	Tangent is parallel to y-axis at $x=2 \Rightarrow y-2k+4=0$ $\Rightarrow y=2k-4$ Substitute into (1) at $x=2$: $(-4)^2+8(kx-4)=12$ $8(2k-4)=-4$ $k=\frac{7}{4}$ ALTERNATIVELY, Sub $x=2$ into (1): $(y-2k)^2+8y-12=0$ $y^2+(8-4k)y+4k^2-12=0$ Since $x=2$ is tangent to C, discriminant = 0 $(8-4k)^2-4(4k^2-12)=0$ $64-64k+16k^2-16k^2+48=0$ $k=\frac{7}{4}$	A minority group wrongly interpreted 'the tangent to C at $x=2$ is parallel to y-axis' as $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ or $y=0$. Some students obtained $y=2k-4$ but did not know how to carry on. Many students also wasted time in rewriting equation of C as $y^2-4ky++4k^2+8y-12=0$ and then substituting $y=2k-4$ to find k, instead of using the original equation $(y-2k)^2+8y-12=0$ to find k.

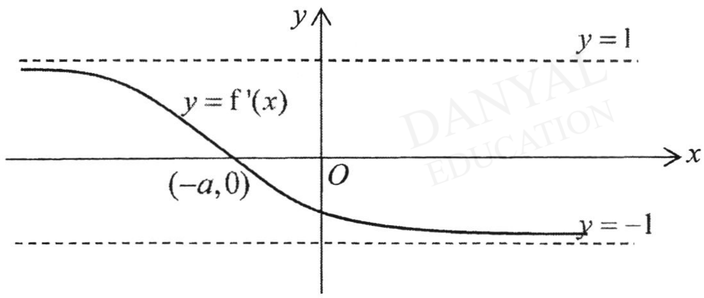
Qn	Solution	Comments
4		Almost all students were able to sketch both graphs correctly. Some students misread the " $c > 0$ " condition as the domain to sketch the graphs for. Students should remember to label the graphs that they sketch. **$y = \|x\|$:** • A number of students omitted labelling the origin or $(0,0)$ as an axial intercept. • A few students reflected the $y = x$ graph in the x-axis instead of the y-axis. **$y = \frac{c}{c-x}$:** • The y-intercept was often wrongly calculated to be $(0, c)$ or $(0, -c)$, perhaps due to students making assumptions from the equation of the graph. • Students are reminded to label axial intercepts with **full** coordinates, to show clearly what the x- and y-coordinates of the point are, instead of just the non-zero value (in this case -1). There were students who mistakenly labelled the point $(-1, 0)$. • Some students sketched the graph without considering the vertical asymptote (perhaps due to relying on the GC for graph sketching). • Students should also be reminded to label the axes with their equations if they happen to be asymptotes (in this case, $y = 0$).

[FOR INTERNAL USE ONLY]

Qn	Solution	Comments
4 **(cont'd)**	Curves intersect on $x>0$, therefore $\lvert x\rvert = x$ $\Rightarrow x=\dfrac{c}{x-c}$ $\Rightarrow x^2-cx-c=0$ $\Rightarrow x=\dfrac{c\pm\sqrt{c^2+4c}}{2}$ $\Rightarrow x=\dfrac{c+\sqrt{c^2+4c}}{2}\ (\because x>c)$	Responses to this part were varied. A good proportion of students were able to solve the quadratic equation using the formula or completing the square, with some carelessness in signs in the formula or the constants in completing the square (e.g. $\left(x-\frac{c}{2}\right)^2-\frac{c}{4}-c=0$). However, many students did not consider the graph drawn earlier in the question, and solved two quadratic equations and/or listed down more than one solution at the end, without rejecting invalid answers. Some solutions were also not very clear with how they deal with the modulus on the left hand side, even though they eventually got the same answer. Of the remaining population of the students, some students did not seem to know how to solve quadratic equations. Some seemed to be making the age-old mistake carried over from O Levels, concluding from $x(x-c)=c$ that $x=c$ or $x-c=c$ (or equivalent). A good number of students also solved for the constant *c* instead of the variable *x*, ignoring the question's instructions to leave answers "in terms of *c*".
4 **(cont'd)**	$y=\dfrac{c}{x-c}\xrightarrow{\text{replace } x \text{ by } \lvert x\rvert} y=\dfrac{c}{\lvert x\rvert-c}$ Hence for $\lvert x\rvert>\dfrac{c}{\lvert x\rvert-c}$, $x<\dfrac{-c-\sqrt{c^2+4c}}{2}$ or $-c<x<c$ or $x>\dfrac{c+\sqrt{c^2+4c}}{2}$	Only a grand total of 4 students had gotten this fully correct. Students who were half correct largely fell into two overlapping categories: • The vast majority of students overlooked a common blind spot for solving inequalities with graphs like these. Students are quick to spot the correct interval where the graphs intersect, but forget to check that the interval $x<c$, where the graphs do not intersect, also satisfies the inequality. Students should be reminded that when solving inequalities using graphs, they should inspect each interval of *x*-values separated by intersection points **or** vertical asymptotes. • A good number of students also wrote down the correct solution for $\lvert x\rvert>\dfrac{c}{x-c}$, but missed out on the solutions in the negative *x* region. It might be that some students overlooked the $\lvert x\rvert$ in the denominator of the right hand side, which required students to reflect the graphs for $x>0$ in the *y*-axis. Alternatively, to complete their solution, they simply needed to replace *x* with $\lvert x\rvert$ in their solution, then express the full solution without the modulus. Several students also mistakenly wrote down $\dfrac{c-\sqrt{c^2+4c}}{2}$ instead of $\dfrac{-c-\sqrt{c^2+4c}}{2}$ as the bound for the negative *x* solution. They might have assumed that the other root of the equation $x^2-cx-c=0$ was the correct bound after applying the $\lvert x\rvert$ substitution.

[FOR INTERNAL USE ONLY]

<table>
<tr><th>Qn</th><th>Solution</th><th>Comments</th></tr>
<tr><td>5</td><td>




</td><td>
General tips to score for curve sketching
1) The curve should approach the asymptotes as $x \to \pm\infty$.
2) Equations of asymptotes and coordinates of intercepts/turning points must be labelled on the diagram.

Generally well done except for some students who got the wrong turning points and asymptotes because of carelessness in the negative signs. Students should be aware that the two oblique asymptotes will cut through $(-a,0)$. Some very confused students labelled the asymptotes with equations of the curve instead i.e. $y = \pm\sqrt{(x+a)^2+1}$.

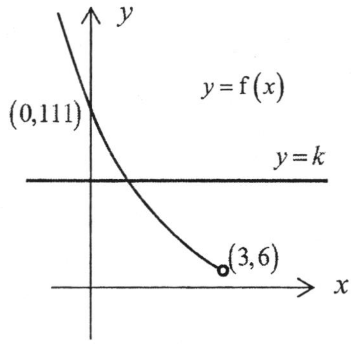
Most students are able to label the correct coordinate of x-intercept and equations of asymptotes, but have difficulty with the shape of the curve.
</td></tr>
<tr><td></td><td>
Sequence of transformations
(1) Translation of $a+1$ units in the positive x-direction.
(2) Translation of 1 unit in the negative y-direction.
(3) Scaling by factor $\frac{1}{b}$ parallel to y-axis.

ALTERNATIVELY,
(1) Scaling by factor $\frac{1}{b}$ parallel to y-axis.
(2) Translation of $\frac{1}{b}$ units in the negative y-direction.
(3) Translation of $a+1$ units in the positive x-direction.
</td><td>
Poor description of transformation seen e.g. "flip", "stretch", "shift left", students ought to be clear and precise with the words used for description. Note that for scaling, there should **not** be "units" written in the description.

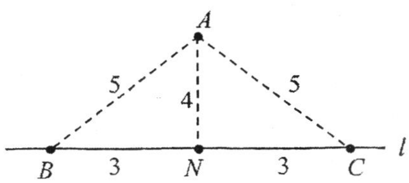
Only some students are aware that the order of transformation matters. Need to relearn on this concept.
</td></tr>
</table>

Qn	Solution	Comments
6	$\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}=\mathbf{b}+\lambda\mathbf{a}$ for some $\lambda\in\mathbb{R}^+$. Since $\overrightarrow{BC}$ is in the direction of $\overrightarrow{OA}$.	Some students who explained by merely rewriting as $\overrightarrow{OC}=\lambda\overrightarrow{OA}+\overrightarrow{OB}$ to give $\mathbf{c}=\lambda\mathbf{a}+\mathbf{b}$, earned no marks. There were several neat explanations like $\overrightarrow{BC}=\lambda\overrightarrow{OA}$ to give $\mathbf{c}-\mathbf{b}=\lambda\mathbf{a}$. A few students used vector equation of a straight line. A few others mentioned that **c** can be expressed as a linear combination of **a** and **b** with $\mu=1$.
	Area of trapezium = Area of OAB + Area of ABC $=\frac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OB}\right\|+\frac{1}{2}\left\|\overrightarrow{BA}\times\overrightarrow{BC}\right\|$ $=\frac{1}{2}\left\|\mathbf{a}\times\mathbf{b}\right\|+\frac{1}{2}\left\|(\mathbf{a}-\mathbf{b})\times(\lambda\mathbf{a})\right\|$ $=\frac{1}{2}\left\|\mathbf{a}\times\mathbf{b}\right\|+\frac{1}{2}\left\|-\lambda\mathbf{b}\times\mathbf{a}\right\|$ $=\frac{1}{2}(1+\lambda)\left\|\mathbf{a}\times\mathbf{b}\right\|$	Solutions to find the area of trapezium were varied. Some students considered ΔAOC + ΔCOB giving $\frac{1}{2}\left\|\mathbf{a}\times\mathbf{c}\right\|+\frac{1}{2}\left\|\mathbf{c}\times\mathbf{b}\right\|$ $=\frac{1}{2}\left\|\mathbf{a}\times(\mathbf{b}+\lambda\mathbf{a})\right\|+\frac{1}{2}\left\|(\lambda\mathbf{a}+\mathbf{b})\times\mathbf{b}\right\|$. $=\frac{1}{2}\left\|\mathbf{a}\times\mathbf{b}\right\|+\frac{1}{2}\lambda\left\|\mathbf{a}\times\mathbf{b}\right\|$ A few students mixed up cross product with dot product. They had extra terms because $\left\|\mathbf{a}\times\mathbf{a}\right\|=\left\|\mathbf{a}\right\|^2$ which was WRONG. Some students considered parallelogram + triangle, a few of them ended up with $\lambda\left\|\mathbf{a}\times\mathbf{b}\right\|+\frac{1}{2}(1-\lambda)\left\|\mathbf{a}\times\mathbf{b}\right\|$, unable to factor out and proceed. Some students quoted the area of trapezium, missing out the modulus signs and giving wrong height. Area of trapezium is $\frac{1}{2}\left(\left\|\lambda\mathbf{a}\right\|+\left\|\mathbf{a}\right\|\right)\left(\left\|\mathbf{b}\times\hat{\mathbf{a}}\right\|\right)$.
	$\left\|\mathbf{a}.\mathbf{b}\right\|^2+\left\|\mathbf{a}\times\mathbf{b}\right\|^2=\left(\left\|\mathbf{a}\right\|\left\|\mathbf{b}\right\|\cos\theta\right)^2+\left(\left\|\mathbf{a}\right\|\left\|\mathbf{b}\right\|\sin\theta\right)^2$ $=\left(\left\|\mathbf{a}\right\|\left\|\mathbf{b}\right\|\right)^2\left(\sin^2\theta+\cos^2\theta\right)$ $=\left(\left\|\mathbf{a}\right\|\left\|\mathbf{b}\right\|\right)^2$	A lot of students wrote $\left\|\mathbf{a}.\mathbf{b}\right\|^2=\left\|\mathbf{a}\right\|^2+2\mathbf{a}.\mathbf{b}+\left\|\mathbf{b}\right\|^2$ and $\left\|\mathbf{a}\times\mathbf{b}\right\|^2=\left\|\mathbf{a}\right\|^2+2\mathbf{a}\times\mathbf{b}+\left\|\mathbf{b}\right\|^2$ which were WRONG, when it should be $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})=\left\|\mathbf{a}\right\|^2+2\mathbf{a}.\mathbf{b}+\left\|\mathbf{b}\right\|^2$ and $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{b})=\mathbf{0}$ Note:

		$\|\mathbf{a}.\mathbf{b}\|^2 \neq (\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})$ $\|\mathbf{a}\times\mathbf{b}\|^2 \neq (\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{b})$ $\|\mathbf{a}.\mathbf{b}\|^2 \neq \|\mathbf{a}\|^2.\|\mathbf{b}\|^2$ unless $\cos\theta = 1$ $\|\mathbf{a}\times\mathbf{b}\|^2 \neq \|\mathbf{a}\|^2\times\|\mathbf{b}\|^2$ unless $\sin\theta = 1$
	$3\mathbf{c}-3\mathbf{b}=\mathbf{a} \Rightarrow \mathbf{c}=\frac{1}{3}\mathbf{a}+\mathbf{b} \Rightarrow \lambda=\frac{1}{3}$ $\|\mathbf{a}\times\mathbf{b}\|^2 = (\|\mathbf{a}\|\|\mathbf{b}\|)^2 - \|\mathbf{a}.\mathbf{b}\|^2 = 36-25 = 11$ $\|\mathbf{a}\times\mathbf{b}\| = \sqrt{11}$ Therefore area of trapezium $=\frac{1+\frac{1}{3}}{2}\sqrt{11} = \frac{2}{3}\sqrt{11}$	A few students found λ by writing $\lambda = \frac{\mathbf{c}-\mathbf{b}}{\mathbf{a}}$ which was WRONG. Some students were unable to connect and apply the 2 proven results fully when they went the extra step to find θ in $\|\mathbf{a}\times\mathbf{b}\|$ from $\|\mathbf{a}.\mathbf{b}\|$. Many common mistakes in not getting $\|\mathbf{a}\times\mathbf{b}\| = \sqrt{11}$ using $\|\mathbf{a}\times\mathbf{b}\|^2 = (\|\mathbf{a}\|\|\mathbf{b}\|)^2 - \|\mathbf{a}.\mathbf{b}\|^2$: • $\|\mathbf{a}\times\mathbf{b}\| = \sqrt{36-25} = \sqrt{9} = 3$ • $\|\mathbf{a}\times\mathbf{b}\| = 36-25 = 11$ • $\|\mathbf{a}\times\mathbf{b}\| = \sqrt{36}-\sqrt{25} = 1$ • $\|\mathbf{a}\times\mathbf{b}\| = \frac{\sqrt{36}}{\sqrt{25}} = \frac{6}{5}$

Qn	Solution	Comments
7(i)	3	
(ii)	$a=4$  The line $y=k$ where $k\in\mathbb{R}$ cuts the graph of $y=\mathrm{f}(x)$ at most once. Hence f is one-one and f^{-1} exists. To find f^{-1}, let $y=\mathrm{f}(x)$, i.e. $y=7(x-4)^2-1$ $\Rightarrow\ (x-4)^2=\frac{y+1}{7}$ $\Rightarrow\ x=4\pm\sqrt{\frac{y+1}{7}}$. Since $x<3$, $x=4-\sqrt{\frac{y+1}{7}}$. Hence $\mathrm{f}^{-1}(x)=4-\sqrt{\frac{x+1}{7}},\quad x>6$.	Many many many many students did not draw graph of $y=\mathrm{f}(x)$ and the horizontal line to show f is one-one. Those who did, drew the whole quadratic graph completely disregarding the domain of f. It is obvious from this simple part which student revised the topic of functions. Many students also only quoted one specific horizontal line, e.g. $y=7$ for the horizontal line test. This is not enough because we need ALL horizontal lines to cut the graph of $y=\mathrm{f}(x)$ at most once. A good number of students did not choose the negative square root. Because the graph was not properly sketched, many students thought $R_f=(3,\infty)$
(iii)	For fg to exist, $R_g\subseteq D_f$. Now $D_f=(-\infty,3)$ and $R_g=[b,1+b)$. Hence for $R_g\subseteq D_f$, $1+b\leq 3$. Hence $b\leq 2$. Therefore largest value of b for fg to exist is 2. $\mathrm{fg}(x)=\mathrm{f}(3-\mathrm{e}^{-x})$ $=7\left[(3-\mathrm{e}^{-x})-4\right]^2-1$ $=7(-1-\mathrm{e}^{-x})^2-1$ $=7(1+\mathrm{e}^{-x})^2-1$ $D_{fg}=D_g=[0,\infty)$. $R_{fg}:\quad [0,\infty)\xrightarrow{g}[2,3)\xrightarrow{f}(6,27]$ Hence $R_{fg}=(6,27]$.	Many students struggle with the notations for this chapter. Domains and Ranges were all over the place: $[27,6)$, $D_g=x\geq 0$ etc. wrong notations were all penalized. Students need to know that Functions is a highly technical topic, and care needs to be taken to understand all the notations and definitions required of them. Note that $1+b\leq 3$, not $1+b-\mathrm{e}^{-x}<3$. If condition for existence is $R_g\subseteq D_f$, find both R_g and D_f, and then determine how to make sure the condition is satisfied.

Qn	Solution	Comments
8(i)	$l:\mathbf{r}=\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\-6\\3\end{pmatrix}$ $\quad m:\mathbf{r}=\begin{pmatrix}1\\a\\-3\end{pmatrix}+\mu\begin{pmatrix}4\\-a\\4\end{pmatrix}$ For intersection, $\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\-6\\3\end{pmatrix}=\begin{pmatrix}1\\a\\-3\end{pmatrix}+\mu\begin{pmatrix}4\\-a\\4\end{pmatrix}$ $\Rightarrow \left.\begin{matrix}1+2\lambda=1+4\mu\\-6\lambda=a-a\mu\\-1+3\lambda=-3+4\mu\end{matrix}\right\}\Rightarrow\left.\begin{matrix}2\lambda-4\mu=0\\-6\lambda=a-a\mu\\3\lambda-4\mu=-2\end{matrix}\right\}$ Solving the first and third equation gives $\lambda=-2,\ \mu=-1$ Because the lines intersect, therefore $\lambda=-2,\ \mu=-1$ satisfies the second equation $-6(-2)=a-a(-1)\Rightarrow 2a=12\Rightarrow a=6$ (shown)	Students generally make mistakes here when converting the equation of *m* into vector form, especially the **j** and **k** component. There are a handful of students who use the same letter for the parameter for both lines, i.e. λ. Hence resulting in inconsistent answers. There are others who also try to solve equations 1 and 2 and 3 together, resulting in long and tedious working.
(ii)	$l:\mathbf{r}=\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\-6\\3\end{pmatrix}$ $\quad \overrightarrow{OA}=\begin{pmatrix}5\\0\\1\end{pmatrix}$ Since *N* is on *l*, $\overrightarrow{ON}=\begin{pmatrix}1+2\lambda\\-6\lambda\\-1+3\lambda\end{pmatrix}\Rightarrow\overrightarrow{AN}=\begin{pmatrix}-4+2\lambda\\-6\lambda\\-2+3\lambda\end{pmatrix}$ $\overrightarrow{AN}$ is perpendicular to *l*, therefore $\begin{pmatrix}-4+2\lambda\\-6\lambda\\-2+3\lambda\end{pmatrix}.\begin{pmatrix}2\\-6\\3\end{pmatrix}=0$ $\Rightarrow(-8-6)+\lambda(4+36+9)=0$ $\Rightarrow\lambda=\frac{2}{7}$ $\overrightarrow{ON}=\begin{pmatrix}1\\0\\-1\end{pmatrix}+\frac{2}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}=\frac{1}{7}\left(\begin{pmatrix}7\\0\\-7\end{pmatrix}+\begin{pmatrix}4\\-12\\6\end{pmatrix}\right)=\frac{1}{7}\begin{pmatrix}11\\-12\\-1\end{pmatrix}.$	Those who studied well will get this right if they use the first method. There is an equal number of people using the first and second method with the latter making more conceptual mistakes. Those who used the first method usually will get full marks except for careless mistakes such as $49\lambda=14\Rightarrow\lambda=\frac{7}{2}$.

Qn	Solution	Comments
8(ii) **(cont'd)**	ALTERNATIVE (Projection method) Let Q denote the point $(1, 0, -1)$ $\overrightarrow{QN}=\dfrac{\left(\overrightarrow{QA}.\begin{pmatrix}2\\-6\\3\end{pmatrix}\right)}{\sqrt{2^2+6^2+3^2}}\dfrac{\begin{pmatrix}2\\-6\\3\end{pmatrix}}{\sqrt{2^2+6^2+3^2}}$ $=\dfrac{\left(\begin{pmatrix}4\\0\\2\end{pmatrix}.\begin{pmatrix}2\\-6\\3\end{pmatrix}\right)}{7}\dfrac{\begin{pmatrix}2\\-6\\3\end{pmatrix}}{7}=\dfrac{2}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}$ $\overrightarrow{ON}=\overrightarrow{OQ}+\overrightarrow{QN}$ $=\begin{pmatrix}1\\0\\-1\end{pmatrix}+\dfrac{2}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}=\dfrac{1}{7}\left(\begin{pmatrix}7\\0\\-7\end{pmatrix}+\begin{pmatrix}4\\-12\\6\end{pmatrix}\right)=\dfrac{1}{7}\begin{pmatrix}11\\-12\\-1\end{pmatrix}$	Many students who used the projection vector method do not know what they are doing. Common mistakes includes - $\overrightarrow{QN}=\lvert\overrightarrow{QA}.\mathbf{b}\rvert\mathbf{b}$ - $\overrightarrow{AN}=\left(\overrightarrow{QA}.\mathbf{b}\right)\mathbf{b}$ - $\overrightarrow{NQ}=\left(\overrightarrow{QA}.\mathbf{b}\right)\mathbf{b}$ Others find the length of projection then multiplied by the unit vector. But this method might not get the answer due to the fact that the direction of the projection vector could be opposite to **b**. **However there are quite a lot of students who tried to use cross product to find the vector perpendicular to the line.** This must be made clear to them that the perpendicular distance from point to line can be found using the length of a cross product. But removing the length does not give the vector. $\lvert\overrightarrow{AN}\rvert=\lvert\overrightarrow{AQ}\times\mathbf{b}\rvert$ but $\overrightarrow{AN}\neq\overrightarrow{AQ}\times\mathbf{b}$.
(iii)	Observe that from (ii), $\overrightarrow{AN}=\begin{pmatrix}-4+2\left(\frac{2}{7}\right)\\-6\left(\frac{2}{7}\right)\\-2+3\left(\frac{2}{7}\right)\end{pmatrix}=-\dfrac{4}{7}\begin{pmatrix}6\\3\\2\end{pmatrix}$  Hence $AN=\dfrac{4}{7}\sqrt{36+9+4}=4$. Let the two points be B and C. Then $AB=AC=5$. Therefore $BN=NC=3$ Check that the direction vector of l, $\begin{pmatrix}2\\-6\\3\end{pmatrix}$ has magnitude 7 units.	A lot of students just leave this part blank. Of those who attempted it, there are equal number of people using each of the two methods. The common mistake with the first method is assuming $\lvert\overrightarrow{NC}\rvert$ is 5 or the students just memorized the question in the tutorial.

[FOR INTERNAL USE ONLY]

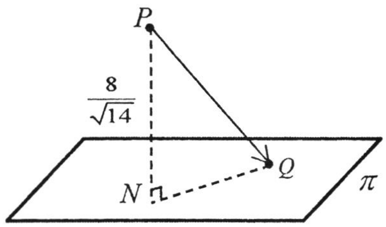
	Hence $\overrightarrow{NC}=\frac{3}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}$ and $\overrightarrow{NB}=-\frac{3}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}$. Therefore, $\overrightarrow{OC}=\overrightarrow{ON}+\overrightarrow{NC}=\frac{1}{7}\begin{pmatrix}11\\-12\\-1\end{pmatrix}+\frac{3}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}=\frac{1}{7}\begin{pmatrix}17\\-30\\8\end{pmatrix}$ $\overrightarrow{OB}=\overrightarrow{ON}+\overrightarrow{NC}=\frac{1}{7}\begin{pmatrix}11\\-12\\-1\end{pmatrix}-\frac{3}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}=\frac{1}{7}\begin{pmatrix}5\\6\\-10\end{pmatrix}$	Others made careless mistakes in calculating the lengths involved in Pythagoras theorem.

Qn	Solution	Comments
8(iii) **(cont'd)**	ALTERNATIVELY, Let B be a point on l such that $AB=5$. Then $\overrightarrow{OB}=\begin{pmatrix}1+2\lambda\\-6\lambda\\-1+3\lambda\end{pmatrix}$ for some λ $\Rightarrow \overrightarrow{AB}=\begin{pmatrix}1+2\lambda\\-6\lambda\\-1+3\lambda\end{pmatrix}-\begin{pmatrix}5\\0\\1\end{pmatrix}=\begin{pmatrix}-4+2\lambda\\-6\lambda\\-2+3\lambda\end{pmatrix}$ $\therefore AB=\left\lVert\begin{pmatrix}-4+2\lambda\\-6\lambda\\-2+3\lambda\end{pmatrix}\right\rVert=5$ $\Rightarrow \sqrt{(2\lambda-4)^2+(-6\lambda)^2+(3\lambda-2)^2}=5$ $\Rightarrow (4+36+9)\lambda^2+(-16-12)\lambda+16+4=25$ $\Rightarrow 49\lambda^2-28\lambda-5=0$ $\Rightarrow (7\lambda-5)(7\lambda+1)=0$ $\Rightarrow \lambda=\frac{5}{7}$ or $\lambda=-\frac{1}{7}$ $\therefore \overrightarrow{OB}=\begin{pmatrix}1\\0\\-1\end{pmatrix}+\frac{5}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}$ or $\overrightarrow{OB}=\begin{pmatrix}1\\0\\-1\end{pmatrix}-\frac{1}{7}\begin{pmatrix}2\\-6\\3\end{pmatrix}$ $=\frac{1}{7}\begin{pmatrix}17\\-30\\8\end{pmatrix}$ $\quad$ $=\frac{1}{7}\begin{pmatrix}5\\6\\-10\end{pmatrix}$	Quite a number of students who use the second method are careless with the forming of the vector $\overrightarrow{AB}$. Hence they were unable to obtain the correct quadratic equation.

Qn	Solution	Comments

9(i)	Another vector parallel to the plane is
	$\begin{pmatrix}1\\0\\-3\end{pmatrix}-\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}0\\-1\\-3\end{pmatrix}//\begin{pmatrix}0\\1\\3\end{pmatrix}$
	Therefore normal $=\begin{pmatrix}0\\1\\3\end{pmatrix}\times\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}4\\6\\-2\end{pmatrix}//\begin{pmatrix}2\\3\\-1\end{pmatrix}$
	Hence $\mathbf{r}.\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}1\\0\\-3\end{pmatrix}.\begin{pmatrix}2\\3\\-1\end{pmatrix}=2+3=5\Rightarrow 2x+3y-z=5$.
		In this question, we see a lot of copy errors, missing negative signs from the vectors or negative signs changing positions or numbers becoming another number due to bad handwriting.. like A LOT of them.
		
		Those who studied are usually able to do (i) effortlessly. Common errors are copy errors such as missing negative signs etc.

(ii)	$\pi:\mathbf{r}.\begin{pmatrix}2\\3\\-1\end{pmatrix}=5 \qquad L:\mathbf{r}=\begin{pmatrix}-5\\6\\-5\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\2\end{pmatrix}$ Let a point on the plane be $A(1,1,0)$. Distance from P to the plane $\pi=\lvert\overrightarrow{AP}.\mathbf{n}\rvert$ $=\dfrac{\left\lvert\begin{pmatrix}-6\\5\\-5\end{pmatrix}.\begin{pmatrix}2\\3\\-1\end{pmatrix}\right\rvert}{\sqrt{2^2+3^2+1^2}}=\dfrac{8}{\sqrt{14}}$.	Apart from the the projection method, there are 2 other commonly seen methods. 1. finding foot of perpendicular and then finding length. 2. finding parallel plane that contains A and use the formula for distance between 2 parallel planes. Some common mistakes are not using the unit vector, missing negative signs, careless simple calculation errors. There are a number of students who use cross product which is a conceptual error that needs to be clarified.
(iii)	For intersection of L and π, $\left(\begin{pmatrix}-5\\6\\-5\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\2\end{pmatrix}\right).\begin{pmatrix}2\\3\\-1\end{pmatrix}=5$ $(-10+18+5)+\lambda(6-6-2)=5$ $\Rightarrow\lambda=4$ $\Rightarrow\overrightarrow{OQ}=\begin{pmatrix}-5\\6\\-5\end{pmatrix}+4\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}7\\-2\\3\end{pmatrix}$	Most students can do this part correctly. Common errors are simple calculation mistakes.

(iv)	<u>Method 1</u> $\overrightarrow{PQ}=\begin{pmatrix}7\\-2\\3\end{pmatrix}-\begin{pmatrix}-5\\6\\-5\end{pmatrix}=\begin{pmatrix}12\\-8\\8\end{pmatrix}$ $\left\lvert\overrightarrow{PQ}\right\rvert=\left\lvert\begin{pmatrix}12\\-8\\8\end{pmatrix}\right\rvert=4\sqrt{17}$  By Pythagoras theorem, Length of projection of PQ on plane $=\sqrt{\left(4\sqrt{17}\right)^2-\left(\frac{8}{\sqrt{14}}\right)^2}=\sqrt{\frac{1872}{7}}=4\sqrt{\frac{117}{7}}=16.4\text{ (3 s.f.)}$.	Most students who got this correct used the cross product method. Those who got it wrong is usually because (iii) was wrong. A good number of students tried to project the vector $\overrightarrow{PQ}$ onto a vector parallel to the plane, namely $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ which was given in the question. However this vector is not the direction of the projection of $\overrightarrow{PQ}$ onto the plane.
Qn	**Solution**	**Comments**
9(iv) **(cont'd)**	<u>Method 2</u> Length of projection of PQ on plane $=\left\lvert\overrightarrow{PQ}\times\mathbf{n}\right\rvert$ $=\dfrac{\left\lvert\begin{pmatrix}12\\-8\\8\end{pmatrix}\times\begin{pmatrix}2\\3\\-1\end{pmatrix}\right\rvert}{\sqrt{2^2+3^2+1^2}}=\frac{4}{\sqrt{14}}\left\lvert\begin{pmatrix}3\\-2\\2\end{pmatrix}\times\begin{pmatrix}2\\3\\-1\end{pmatrix}\right\rvert=\frac{4}{\sqrt{14}}\left\lvert\begin{pmatrix}-4\\7\\13\end{pmatrix}\right\rvert$ $=\frac{4}{\sqrt{14}}\sqrt{234}=\frac{4\sqrt{117}}{\sqrt{7}}=16.4\text{ (3 s.f.)}$.	Quite a number of students were able to use the cross product method correctly. There were a few who factorized '4' from the vector $\begin{pmatrix}12\\-8\\8\end{pmatrix}$ and then reduce it to $\begin{pmatrix}3\\-2\\2\end{pmatrix}$ by 'throwing' 4 away. This is wrong because that changes the length of PQ and hence the length of projection.

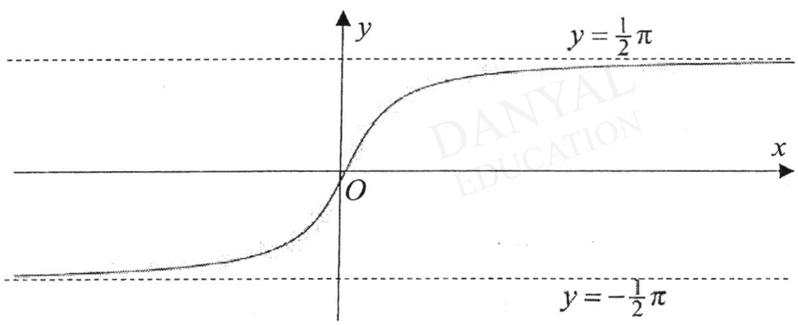
Qn	Solution	Comments
10	By partial fractions, $\dfrac{2r+3}{r(r+1)(r+2)}=\dfrac{1}{2}\left(\dfrac{3}{r}-\dfrac{2}{r+1}-\dfrac{1}{r+2}\right)$. Hence, $\displaystyle\sum_{r=1}^{n}\frac{2r+3}{r(r+1)(r+2)}=\frac{1}{2}\sum_{r=1}^{n}\left(\frac{3}{r}-\frac{2}{r+1}-\frac{1}{r+2}\right)$ $=\dfrac{1}{2}\left[\begin{array}{l}\frac{3}{1}-\frac{2}{2}-\cancel{\frac{1}{3}}\\ +\frac{3}{2}-\cancel{\frac{2}{3}}-\cancel{\frac{1}{4}}\\ +\cancel{\frac{3}{3}}-\cancel{\frac{2}{4}}-\cancel{\frac{1}{5}}\\ +\ldots\\ +\frac{3}{n-2}-\frac{2}{n-1}-\cancel{\frac{1}{n}}\\ +\frac{3}{n-1}-\cancel{\frac{2}{n}}-\cancel{\frac{1}{n+1}}\\ +\cancel{\frac{3}{n}}-\cancel{\frac{2}{n+1}}-\cancel{\frac{1}{n+2}}\end{array}\right]$ $=\dfrac{1}{2}\left(\dfrac{7}{2}-\dfrac{3}{n+1}-\dfrac{1}{n+2}\right)$ $=\dfrac{7}{4}-\dfrac{3}{2(n+1)}-\dfrac{1}{2(n+2)}$. Hence $A=\dfrac{7}{4}$	Too many students forgot how to do partial fractions. Too many made the same silly careless mistake when finding the coefficient for $\dfrac{1}{r}$. Many students had trouble seeing which terms in the expansion cancelled out, while some innovative ones simply substituted a value of n into $\displaystyle\sum_{r=1}^{n}\frac{2r+3}{r(r+1)(r+2)}=A-\frac{3}{2(n+1)}-\frac{1}{2(n+2)}$ to solve for A. Since this is a "show" question, credit is given for the value of A, but not for the method of difference required to find A.
(i)	As $n\to\infty$, $\dfrac{3}{2(n+1)}\to 0$ and $\dfrac{1}{2(n+2)}\to 0$. Hence $\dfrac{7}{4}-\dfrac{3}{2(n+1)}-\dfrac{1}{2(n+2)}\to\dfrac{7}{4}$. Therefore convergence limit is $\dfrac{7}{4}$.	Even if students could not do the MOD part above, if they wrote down $\dfrac{3}{2(n+1)}\to 0$ and $\dfrac{1}{2(n+2)}\to 0$, to get convergence limit as A, they would have been given full credit.
(ii)	$\dfrac{7}{4}-\dfrac{3}{2(n+1)}-\dfrac{1}{2(n+2)}>\dfrac{8}{5}$ $\Rightarrow\dfrac{3}{2(n+1)}+\dfrac{1}{2(n+2)}<\dfrac{3}{20}=0.15$. From G.C., When $n=12$, $\dfrac{3}{2(12+1)}+\dfrac{1}{2(12+2)}=0.1511$ When $n=13$, $\dfrac{3}{2(13+1)}+\dfrac{1}{2(13+2)}=0.1405$ Hence least value of n is 13.	Method mark for solving the inequality is awarded only if there is evidence student attempted to solve using G.C. Though many students did use the GC pretty accurately here, many did not present the GC table to show how they got the answer. Students need to know that this working step is worth mark should their answer from the first part is wrong.

[FOR INTERNAL USE ONLY]

Qn	Solution	Comments
(iii)	$\dfrac{9}{3\times4\times5}+\dfrac{11}{4\times5\times6}+\dfrac{13}{5\times6\times7}+\ldots\ldots+\dfrac{2N+1}{N\left(N^2-1\right)}$ $=\displaystyle\sum_{r=3}^{N-1}\frac{2r+3}{r(r+1)(r+2)}$ $=\dfrac{7}{4}-\dfrac{3}{2N}-\dfrac{1}{2(N+1)}-\dfrac{5}{6}-\dfrac{7}{24}$ $=\dfrac{5}{8}-\dfrac{3}{2N}-\dfrac{1}{2(N+1)}.$	Students who expressed the sum as $\displaystyle\sum_{r=4}^{N}\frac{2r+1}{r\left(r^2-1\right)}$ were given credit, though many could not continue to find its relationship with $\displaystyle\sum_{r=3}^{N-1}\frac{2r+3}{r(r+1)(r+2)}$. In fact even if they used the result given in the question without finding A, $A-\dfrac{3}{2(n+1)}-\dfrac{1}{2(n+2)}$, they would still have gotten the correct answer (since A cancels out), and full credit would have been earned.

Qn	Solution	Comments
11(i)	$x=\dfrac{t}{\sqrt{1-4t^2}}, \quad y=\sin^{-1}2t, \quad -\dfrac{1}{2}<t<\dfrac{1}{2}$ $\dfrac{dx}{dt}=\dfrac{(1-4t^2)^{\frac{1}{2}}-t(\frac{1}{2})(1-4t^2)^{-\frac{1}{2}}(-8t)}{(1-4t^2)}$ $=\dfrac{(1-4t^2)^{-\frac{1}{2}}\left[(1-4t^2)+4t^2\right]}{(1-4t^2)}$ $=(1-4t^2)^{-\frac{3}{2}}$ $\dfrac{dy}{dt}=\dfrac{2}{\sqrt{1-4t^2}}$ $\dfrac{dy}{dx}=\dfrac{2}{\sqrt{1-4t^2}}\div(1-4t^2)^{-\frac{3}{2}}$ $=\dfrac{2}{\sqrt{1-4t^2}}\times(1-4t^2)^{\frac{3}{2}}$ $=2(1-4t^2)$ (shown) Since $-\dfrac{1}{2}<t<\dfrac{1}{2}, \quad t^2<\dfrac{1}{4} \quad \therefore 4t^2<1$ So $\dfrac{dy}{dx}=2(1-4t^2)>0$, thus gradient is positive for all points on the curve.	Most students were able to correctly differentiate the expressions and perform the parametric differentiation. Some careless mistakes were made such as multiplying by $8t$ instead of $-8t$. Some students also had problems with the algebraic manipulations to arrive at the required expression. No credit was awarded if too many "intermediate" mistakes were made. Common error: $(1-4t^2)^{\frac{3}{2}}=\left(\sqrt{1-4t^2}\right)^3$ $\neq\sqrt[3]{1-4t^2}$ (cube root!!) Few did not use the quotient or product rule or used it wrongly. No credit was given for such cases. This is a "Show" question. Steps should not be skipped – it should be clear how the next expression is obtained. Little or no credit was given if steps were not clearly shown, even if the answer was reached because the "answer" is given for this question. In showing that the gradient is positive for all points on the curve, it is essential to state that $-\dfrac{1}{2}<t<\dfrac{1}{2}$, which then results in $t^2<\dfrac{1}{4}$ (or equivalent). Starting with just $t^2<\dfrac{1}{4}$ (or equivalent), giving the incorrect

[FOR INTERNAL USE ONLY]

		reason for why $2(1-4t^2)>0$ (or equivalent), such as $\because t^2 \geq 0$, or using just test points (e.g. substituting $t=\pm\frac{1}{2}$ or $t=\pm\frac{1}{3}$ earned no credit. Some students had some issues with the inequality signs also. Some students tried using the second derivative test or discriminant which is conceptually not right.
11 (ii)	As $x \to \pm\infty$, $t \to \pm\frac{1}{2}$, $\therefore y \to \pm\frac{\pi}{2}$ the curve approaches $y=\pm\frac{\pi}{2}$. 	The graph was quite badly drawn. The behavior of the curve was also often not stated or analysed. The analysis of the behavior of the curve was intended to aid in the sketching of the graph since the GC had its limitations in showing the extreme behavior of the curve. Often the behavior stated and the actual graph drawn were also not consistent. The origin and the correct equations of the asymptotes of the curve should be clearly shown as required by the question. As shown in part (i), no parts of the curve should have negative or 0 gradient (no stationary points).
11 (iii)	At point A, $p=\frac{1}{2\sqrt{2}}$, $x=\frac{\frac{1}{2\sqrt{2}}}{\sqrt{1-4(\frac{1}{8})}}=\frac{\frac{1}{2\sqrt{2}}}{\sqrt{\frac{1}{2}}}=\frac{1}{2}$, and $y=\sin^{-1}2\left(\frac{1}{2\sqrt{2}}\right)=\frac{\pi}{4}$ Equation of tangent l is $y-\frac{\pi}{4}=2(1-4\left(\frac{1}{8}\right))\left(x-\frac{1}{2}\right)$ $y=x-\frac{1}{2}+\frac{\pi}{4}$	This question is a rather basic one of finding the tangent equation of a parametrically defined curve. For those who attempted this part, it was generally well done. Some careless mistakes were made in computing the gradient and coordinates at A. Credit was not given if any value was incorrect and no working was shown. A few students used the parameter value of $\frac{1}{2\sqrt{2}}$ given

<table>
<tr><td></td><td></td><td>as the x-value when clearly it was the t-value (the parameter is t).</td></tr>
</table>

<table>
<tr><th>Qn</th><th>Solution</th><th>Comments</th></tr>
<tr><td>11 (iii)</td><td>
Curve C: $x=\dfrac{t}{\sqrt{1-4t^2}}$, $y=\sin^{-1}2t$
Line l: $y=x-\dfrac{1}{2}+\dfrac{\pi}{4}$
At the point of intersection B,
Substitute $x=\dfrac{t}{\sqrt{1-4t^2}}$, $y=\sin^{-1}2t$ into $y=x-\dfrac{1}{2}+\dfrac{\pi}{4}$,
$\therefore \sin^{-1}2t=\dfrac{t}{\sqrt{1-4t^2}}-\dfrac{1}{2}+\dfrac{\pi}{4}$
From GC, $t=0.35355$ or $t=-0.47564$
Now, $t=0.35355$ corresponds to point A
Thus $t=-0.47564$, and $x=\dfrac{-0.47564}{\sqrt{1-4(-0.47564)^2}}=-1.54278$,
$y=\sin^{-1}2(-0.47564)=-1.25737$
Coordinates of B are $(-1.54,-1.26)$.
</td><td>No credit was awarded if many mistakes were made in trying to obtain the Cartesian equation of C.
Some students correctly substituted in the parametric equations of C into the tangent equation, but could not continue from there.
The key was to use the GC to solve for the t-value and subsequently substitute the t-value back into the parametric equations of C to obtain the coordinates.
Very few students obtained the correct coordinates of B.</td></tr>
</table>

<table>
<tr><th>Qn</th><th>Solution</th><th>Comments</th></tr>
<tr><td>12(i)</td><td>
<table>
<tr><th>Year</th><th>Beginning</th><th>End</th></tr>
<tr><td>1</td><td>$300\,000-x$</td><td>$1.025(300\,000-x)$</td></tr>
<tr><td>2</td><td>$1.025(300\,000-x)-x$</td><td>$1.025^2(300\,000-x)-1.025x$</td></tr>
</table>
By the end of the n^{th} year, the outstanding loan amount is
$1.025^n(300\,000-x)-1.025^{n-1}x-\ldots-1.025x$
$=1.025^n(300\,000)-1.025^n x-1.025^{n-1}x-\ldots-1.025x$
$=1.025^n(300\,000)-1.025x\underbrace{\left(1.025^{n-1}+1.025^{n-2}+\ldots+1\right)}$
GP: $a=1$, $r=1.025$, n terms
$=1.025^n(300\,000)-1.025x\dfrac{\left(1.025^n-1\right)}{1.025-1}$
$=1.025^n(300\,000)-41x\left(1.025^n-1\right)$. (shown)
</td><td>Question stated that all numerical answers must be in nearest cent, unfortunately many students did not notice this. To nearest cent = round off to 2 decimal places.
Important tip to assure accuracy of answers, use 2 more decimal places for workings. Eg if question wants answers in nearest cent, students should be using 4 decimal places for workings, alternatively, students can make use of the STO function in the GC, to store the values.

Part (i) is a proving question, hence students must give more details and workings instead of stating the answer. Note that the question is to calculate the outstanding loan at the end of the year.</td></tr>
<tr><td>(ii)</td><td>
On 1 January 2040, $n=20$.
At the end of 31 December 2039, outstanding loan amount is
$1.025^{19}(300\,000)-41x\left(1.025^{19}-1\right)$.
For the last instalment of \$$x$ to be on 1 January 2040,
$1.025^{19}(300\,000)-41x\left(1.025^{19}-1\right)=x$
$\Rightarrow 479\,595.0557-24.5447x=x$
$\Rightarrow x=18774.77$ (nearest cent)
</td><td>Question stated that the last payment is on 1 January 2040, it means that it will be her 20th payment = \$$x$.

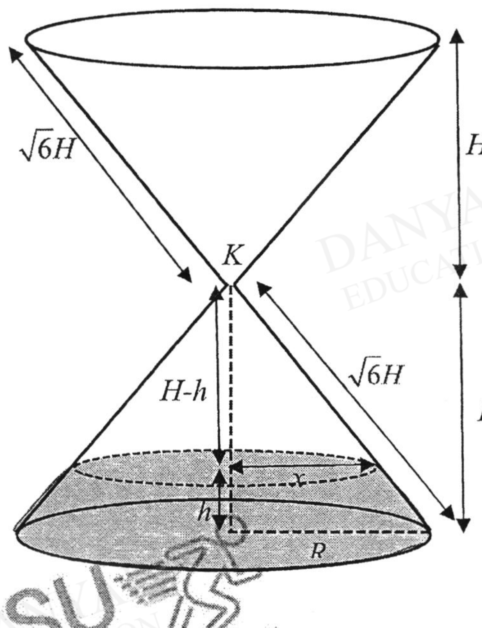
Students should relate to part (i) answer and solve for x.</td></tr>
</table>

	ALTERNATIVELY, Outstanding amount at the end of 20 years is 0: $1.025^{20}(300\,000)-41x\left(1.025^{20}-1\right)=0$ Solving, $x=18774.77$ (nearest cent).	
(iii)	$(18\,774.7694\times 20)-300\,000$ $=75495.39$ (nearest cent).	Note that she borrowed \$300000, her last payment is the 20th payment, hence the total amount paid including interest = $20x-300000$. Some students forgot to relate part (iii) with part (ii) and went to calculate the interest paid only using another GP formula.
(iv)	$500\,000\times 1.03^{10}=671\,958.19$ (nearest cent).	This part was well done, except for some students who used the $U_n=ar^{n-1}$ formula for GP and took it for 9 years instead of 10 years and got the wrong answer. Some students calculate depreciation instead of appreciation in the value of the apartment, and took $r=0.97$ instead of $r=1.03$.
(v)	Outstanding loan amount on 31 December 2030 $=1.025^{10}(300\,000)-41(18\,774.7694)\left(1.025^{10}-1\right)$ $=168\,425.93$ (nearest cent) Amount Ann earns = value on 1 Jan 2031 – upfront payment – outstanding loan – all instalments $=671\,958.1897-200\,000-168\,425.9313-(10\times 18\,774.7694)$ $=115\,784.56$ (nearest cent).	Very few students interpreted the question correctly, common errors in counting the months wrongly and forgotten to consider the upfront payment of \$200000 and instalments paid \$10x. Only one student obtained full marks for Q12.

Qn	Solution	Comments
13(a) (i)	$V=\frac{1}{3}\pi(PC)^2(AP)$ In ΔAOE, $\sin\theta=\frac{r}{AO}\Rightarrow AO=r\operatorname{cosec}\theta$ $\therefore AP=r+r\operatorname{cosec}\theta$ In ΔAPC, $\tan\theta=\frac{PC}{AP}\Rightarrow PC=AP\tan\theta$ $=r(1+\operatorname{cosec}\theta)\tan\theta$ $V=\frac{1}{3}\pi\left[r(1+\operatorname{cosec}\theta)\tan\theta\right]^2 r(1+\operatorname{cosec}\theta)$ $V=\frac{1}{3}\pi r^3(1+\operatorname{cosec}\theta)^3\tan^2\theta$ (shown)	Quite a number of students wrote $\sin\theta=\frac{r}{AO}$ $\Rightarrow AO=r\sin\theta$, which is very careless. Some students mistaken O as the midpoint of AP and wrote $AP=2AO=\frac{2r}{\sin\theta}$, which leads to a wrong expression in the show part.

		Many students mistaken r as the radius of the cone, however, r is given as the radius of sphere in the question. eg $V=\frac{1}{3}\pi r^2\left[r(1+\operatorname{cosec}\theta)\right]$ $=\frac{1}{3}\pi r^3(1+\operatorname{cosec}\theta)$ Some students have gotten the correct expression for the radius and height, however, they expanded the expression when finding the volume and this made the expression very complicated. A better strategy is to factorize the common factors and the correct expression can be derived easily.
13(a) (ii)	$\frac{\mathrm{d}V}{\mathrm{d}\theta}=\frac{1}{3}\pi r^3[3(1+\operatorname{cosec}\theta)^2\left(-\operatorname{cosec}\theta\cot\theta\right)\tan^2\theta$ $+(1+\operatorname{cosec}\theta)^3\,2\tan\theta\sec^2\theta]$ $=\frac{1}{3}\pi r^3(1+\operatorname{cosec}\theta)^2[-3\operatorname{cosec}\theta\tan\theta+(1+\operatorname{cosec}\theta)2\tan\theta\sec^2\theta]$ $\frac{\mathrm{d}V}{\mathrm{d}\theta}=0$ $\Rightarrow\frac{1}{3}\pi r^3(1+\operatorname{cosec}\theta)^2[-3\operatorname{cosec}\theta\tan\theta+(1+\operatorname{cosec}\theta)2\tan\theta\sec^2\theta]=0$ $\Rightarrow(1+\operatorname{cosec}\theta)^2=0$ or $-3\left(\frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)+\left(1+\frac{1}{\sin\theta}\right)\left(\frac{2\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos^2\theta}\right)=0$ $\Rightarrow\operatorname{cosec}\theta=-1$ or $-\frac{3}{\cos\theta}+\left(\frac{\sin\theta+1}{\sin\theta}\right)\left(\frac{2\sin\theta}{\cos^3\theta}\right)=0$ $\Rightarrow\sin\theta=-1$ or $-\frac{3}{\cos\theta}+\frac{2\sin\theta}{\cos^3\theta}+\frac{2}{\cos^3\theta}=0$ (N.A.) or $-3\cos^2\theta+2\sin\theta+2=0$ or $-3(1-\sin^2\theta)+2\sin\theta+2=0$ or $3\sin^2\theta+2\sin\theta-1=0$ or $(\sin\theta+1)(3\sin\theta-1)=0$ or $\sin\theta=-1$ or $\sin\theta=\frac{1}{3}$ (N.A.)	Many students did not realise that r is a constant and θ is the variable. As a result, they differentiated V with respected to r, which is totally wrong. Some students did not understand the question. They substituted $\sin\theta=\frac{1}{3}$ into $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ and did not know how to continue from there. To find the min value, you should always let $\frac{\mathrm{d}V}{\mathrm{d}\theta}=0$ and show that $\sin\theta=\frac{1}{3}$ after solving the equation.

<table>
<tr><td></td><td>When $\sin\theta = \frac{1}{3}$, from GC, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 75.4r^3 > 0$ since $r > 0$
$\therefore V$ is minimum when $\sin\theta = \frac{1}{3}$.</td><td></td></tr>
</table>

<table>
<tr><th>Qn</th><th>Solution</th><th>Comments</th></tr>
<tr><td>13(a)
(ii)
(cont'd)</td><td>Alternatively
<table>
<tr><td>θ</td><td>$\left(\sin^{-1}\frac{1}{3}\right)^-$
$= 0.339$</td><td>$\sin^{-1}\frac{1}{3}$
$= 0.340$</td><td>$\left(\sin^{-1}\frac{1}{3}\right)^+$
$= 0.341$</td></tr>
<tr><td>$\frac{\mathrm{d}V}{\mathrm{d}\theta}$</td><td>$-0.0633r^3$
< 0</td><td>0</td><td>$0.0874r^3$
> 0</td></tr>
</table>
</td><td>Some students used first derivative test to show the volume is minimum. However, specific values of θ and $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ used for the test were not stated in the table and marks are not awarded for such case.</td></tr>
<tr><td>13
(b)</td><td>
</td><td></td></tr>
<tr><td></td><td>Let R be radius of cone and x be radius of water level when depth of water is h cm.
Radius of cone, $R = \sqrt{(\sqrt{6}H)^2 - H^2} = \sqrt{5}H$
By similar triangles, $\frac{x}{R} = \frac{H-h}{H}$
$\Rightarrow \frac{x}{\sqrt{5}H} = \frac{H-h}{H}$
$\Rightarrow x = \sqrt{5}(H-h)$</td><td>Many students did not show clear working when using similar triangles to find the radius of the smaller cone or the volume of the smaller cone.</td></tr>
</table>

	Volume of liquid in lower cone at time t, $W = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi x^2 (H-h)$ $= \frac{1}{3}\pi\left(\sqrt{5}H\right)^2 H - \frac{1}{3}\pi\left(\sqrt{5}(H-h)\right)^2 (H-h)$ $= \frac{5}{3}\pi H^3 - \frac{5}{3}\pi (H-h)^3$ (shown)	

[FOR INTERNAL USE ONLY]

Qn	Solution	Comments
13 (b) (cont'd)	$\frac{dW}{dh} = 5\pi(H-h)^2$. Given $\frac{dW}{dt} = 4$ cms^{-1} Now, $\frac{dW}{dt} = \frac{dW}{dh} \times \frac{dh}{dt}$ $4 = 5\pi\left(H-h\right)^2 \times \frac{dh}{dt}$ When $h = \frac{1}{25}H$, $4 = 5\pi\left(\frac{24}{25}H\right)^2 \times \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{125}{144\pi H^2}$ cms^{-1} Alternatively, $W = \frac{5}{3}\pi H^3 - \frac{5}{3}\pi(H-h)^3$ Differentiating with respect to t, $\frac{dW}{dt} = 5\pi\left(H-h\right)^2 \frac{dh}{dt}$ Sub. $\frac{dW}{dt} = 4, \quad h = \frac{1}{25}H$, $\therefore 4 = 5\pi\left(H - \frac{1}{25}H\right)^2 \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{125}{144\pi H^2}$ cms^{-1}	Many students did not realise that H is a constant and h is the variable. As a result, they differentiated W with respected to H, which is totally wrong. Common mistakes: • Differentiated W with respect to H. • Sub $h = \frac{1}{25}H$ into W and differentiated W with respect to H. • Sub $H = 25h$ into W and differentiated W with respect to h. A few students made a careless mistake by keeping the term with H^3 when differentiating W with respect to h. A few students have gotten the correct answer but forgot to simplify.