A Level H2 Math

Permutations, Combinations and Probability Test 7

Q1

John and Peter play a game of chess. It is equally likely for either player to make the first move. If John makes the first move, the probability of him winning the game is 0.3 while the probability of Peter winning the game is 0.2. If Peter makes the first move, the probability of him winning the game is 0.5 while the probability of John winning the game is 0.4. If there is no winner, then the game ends in a draw.

(i) Find the probability that Peter made the first move given that he won the game. [3]

(ii) John and Peter played a total of three games. Assuming that the results of the three games are independent, find the probability that John wins exactly one game. [3]

Q2

A teacher wants to randomly form two teams of 5 students from a group of 5 girls and 5 boys for a sports activity. Two of the girls, Ann and Alice, are selected as team leaders. Find the probability that one team has exactly 3 girls. [2]

The ten students are seated at a round table of 10. Find the probability that

(i) Ann and Alice are not seated together, [2]

(ii) no two of the remaining 3 girls are next to each other given that Ann and Alice are not seated together. [4]

Q3

Andy needs two passcodes to open a treasure box. Both passcodes consist of three letters and four digits. Each of the three letters can be any of the twenty-six letters of the alphabet A-Z. Each of the four digits can be any of the ten digits 0-9.

(a) The first passcode consists of three letters followed by four digits. It is also known that no letters and digits are repeated. An example of the code is ABC1234.

(i) Find the total number of possible first passcodes. [2]

(ii) An additional hint is given to Andy to break the first passcode. The four digits of the passcode form a number which is odd and greater than 3000. Find the total number of possible first passcodes. [3]

(b) The second passcode has no fixed arrangement for the letters and digits. Given that the letters and digits can be repeated (i.e. 1AA3C34 can be a possible passcode), find the total number of possible second passcodes. [3]

Answers
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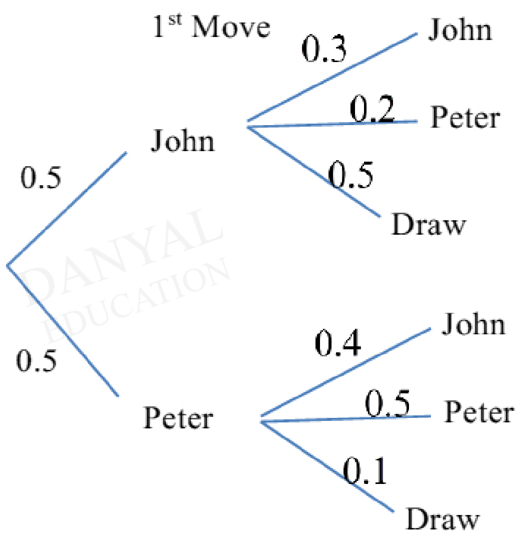
Q1

i



ii

$$P(\text{Peter made first move} \mid \text{Peter won the game})$$

$$= \frac{P(\text{Peter made first move and Peter won the game})}{P(\text{Peter won the game})}$$

$$= \frac{0.5 \times 0.5}{0.5 \times 0.2 + 0.5 \times 0.5}$$

$$= \frac{5}{7}$$

iii

$$P(\text{John wins}) = 0.5 \times 0.3 + 0.5 \times 0.4 = 0.35$$

$$P(\text{John wins in exactly 1 game})$$

$$= (0.35)(0.65)(0.65) \times \frac{3!}{2!}$$

$$= 0.443625 \text{ or } \frac{3549}{8000}$$

$$= 0.444 \text{ (to 3 s.f.)}$$

Alternative

Let X be the number of games won by John out of 3 games.

$X \sim B(3, 0.35)$

$$P(\text{John wins in exactly 1 game})$$

$$= P(X = 1)$$

$$= 0.443625 \text{ or } \frac{3549}{8000}$$

$$= 0.444 \text{ (to 3 s.f.)}$$

Q2

Probability required $= \dfrac{2 \times {}^{3}C_2 \times {}^{5}C_2}{{}^{8}C_4} = \dfrac{6}{7}$

${}^{3}C_2 \times {}^{5}C_2$ – choose 2 girls from the remaining 3 girls and 2 boys from 5 boys for the group with exactly 3 girls. Multiply by 2 because this group can be Ann or Alice's group.
This is a conditional probability as Ann and Alice must be the team leaders, thus ${}^{8}C_4$.

(i) Required probability $= \dfrac{(8-1)!\,{}^{8}C_2\,2!}{(10-1)!} = \dfrac{7}{9}$

OR $\quad 1 - \dfrac{(9-1)!\,2!}{(10-1)!} = \dfrac{7}{9}$

Insertion method or apply Principle of complementation

(ii) Let X be the event that the remaining 3 girls are separated.

Let Y be the event that Ann and Alice are not seated together.

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$$

$$= \frac{P(X) - P(X \cap Y')}{P(Y)}$$

$$= \frac{\dfrac{(7-1)!\,{}^{7}C_3\,3! - (6-1)!\,2!\,{}^{6}C_3\,3!}{(10-1)!}}{7/9}$$

$$= \frac{85/252}{7/9}$$

$$= \frac{85}{196}$$

Not possible to list out all the cases.

Q3

(a)(i)

Total number of possible passcodes $= 26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7 = 78624000$

or

$= {}^{26}C_3 \times 3! \times {}^{10}C_4 \times 4! = 78624000$

or

$= {}^{26}P_3 \times {}^{10}P_4 = 78624000$

(a)(ii)

<u>Case 1:</u> 1st digit 4, 6, 8

4, 6, 8 (3 choices)	8 choices	7 choices	1, 3, 5, 7, 9 (5 choices)

Number of possible passcodes

$= 26 \times 25 \times 24 \times 3 \times 8 \times 7 \times 5 = 13104000$

<u>Case 2:</u> 1st digit 3, 5, 7, 9

3, 5, 7, 9 (4 choices)	8 choices	7 choices	4 choices

Number of possible passcodes

$= 26 \times 25 \times 24 \times 4 \times 8 \times 7 \times 4 = 13977600$

Total number of possible passcodes

$= 13104000 + 13977600 = 27081600$

(b)

Total number of possible passcodes

$= 26^3 \times 10^4 \times \frac{7!}{4!3!} = 6151600000$