A Level H2 Math

Permutations, Combinations and Probability Test 6

Q1

(a) It is given that the probability that 21 randomly chosen people were all born on different days of the year is 0.55631, correct to 5 decimal places.

Find the probability that in a random sample of 22 people, there are at least 2 people with the same date of birth. [3]

[You may assume there are 365 days in a year and the probability that a person is born on any of the 365 days is the same.]

(b) A soccer team consists of 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards.

Country N has a squad of 3 goalkeepers, 6 defenders, 9 midfielders and 4 forwards.

(i) How many different soccer teams can be formed by country N? [2]

One of the defenders and one of the midfielders in the squad are twin brothers.

(ii) How many different teams can be formed which include at most one of the twin brothers? [3]

The following table shows the dates of birth of the 22 players in the squad of country N:

Jersey Number	Position	Date of birth
1	Goalkeeper	29 October
2	Defender	3 May
3	Defender	17 July
4	Defender	15 May
5	Defender	14 December
6	Midfielder	12 October
7	Midfielder	15 May
8	Forward	10 May
9	Forward	1 July
10	Midfielder	1 April
11	Midfielder	29 October

Jersey Number	Position	Date of birth
12	Midfielder	15 August
13	Defender	11 July
14	Midfielder	29 March
15	Defender	22 October
16	Midfielder	13 March
17	Forward	29 November
18	Goalkeeper	20 December
19	Midfielder	5 February
20	Midfielder	1 March
21	Forward	27 March
22	Goalkeeper	31 October

(iii) Find the probability that the team formed by country N contains no players with the same date of birth. [4]

Q2

(a) There are three yellow balls, three red balls and three blue balls. Balls of each colour are numbered 1, 2, and 3. Find the number of ways of arranging the balls in a row such that adjacent balls do not sum up to two. **[2]**

(b) In a restaurant, there were two round tables available, a table for five and a table for six. Find the number of ways eleven friends can be seated if two particular friends are not seated next to each other. **[4]**

Q3

For the events A and B, it is given that

$$P(A \cap B') = 0.6, \quad P(A \cup B') = 0.83 \quad \text{and} \quad P(A \mid B') = 0.83$$

Find,

(i) $P(B)$ [2]

(ii) $P(A \cap B)$ [2]

(iii) $P(B \mid A')$ [2]

Hence determine whether A and B are independent. [1]

Answers
Permutations, Combinations and Probability Test 6

Q1

(a)
Complement Method:
Probability
$=1 - \text{P(22 people were all born on different days)}$
$=1 - \text{P(21 people were all born on different days)} \times \frac{365-21}{365}$
$\approx 1 - 0.55631 \times \frac{344}{365}$
$= 0.476$ (3sf)

Alternative method:
P(at least 2 with same date of birth in 21 people)
+ P(21 people all born on different days and 22nd person shares same date of birth with someone else)
$= (1 - 0.55631) + (0.55631) \times \frac{21}{365}$

(bi)
Number of ways $= {}^3C_1 \times {}^6C_4 \times {}^9C_4 \times {}^4C_2 = 34020$

(bii)

Complement Method:
Number of ways
= n(teams formed without restriction)
− n(teams which include both twins)
$= 34020 - {}^3C_1 \times {}^5C_3 \times {}^8C_3 \times {}^4C_2$
$= 34020 - 10080 = 23940$

Alternative method:
Number of ways
= n(teams which include the twin defender and not the twin midfielder) + n(teams which include the twin midfielder and not the twin defender) + n(teams which do not include both twins)
$= {}^3C_1 \times {}^5C_4 \times {}^8C_4 \times {}^4C_2 + {}^3C_1 \times {}^5C_3 \times {}^8C_4 \times {}^4C_2$
$+ {}^3C_1 \times {}^5C_4 \times {}^8C_3 \times {}^4C_2$
$= 6300 + 12600 + 5040$

(biii)
Let A denote the event that player 1 and player 11 are both in the team.
Let B denote the event that player 4 and player 7 are both in the team.
$n(A) = {}^6C_4 \times {}^8C_3 \times {}^4C_2 = 5040$
$n(B) = {}^3C_1 \times {}^5C_3 \times {}^8C_3 \times {}^4C_2 = 10080$
$n(A \cap B) = {}^5C_3 \times {}^7C_2 \times {}^4C_2 = 1260$

$\therefore n(A \cup B) = 5040 + 10080 - 1260 = 13860$

Hence required probability $= 1 - \frac{13860}{34020} = \frac{16}{27}$

Q2

(a)

Since adjacent balls do not sum up to two, balls numbered '1' needs be separated.

Number of ways of arranging the other balls with no restriction = 6!

Slotting in the balls numbered '1', permutation is done as balls are of different colour = ${}^{7}C_{3} \times 3!$

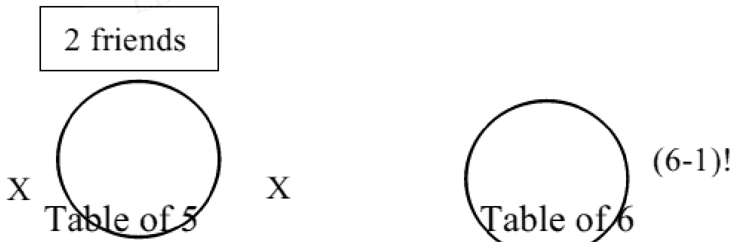
No of ways

$= 6! \times {}^{7}C_{3} \times 3!$

$= 151200$

(b)

Method 1



Case 1 – 2 friends are seated together at table of 5

X

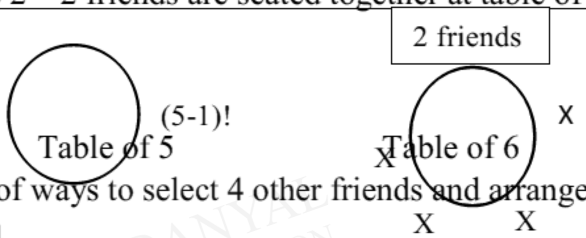
No. of ways to select 3 other friends and arrange them at the table of 5 = $^{9}C_{3}\times(4-1)!$

No. of ways to arrange the 2 friends = 2!

No. of ways to sit the remaining friends at the table of 6
= (6-1)! = 5! = 120

Total no. of ways = $^{9}C_{3}\times(4-1)!\times 2!\times 5! = 120960$

<u>Case 2 – 2 friends are seated together at table of 6</u>



No. of ways to select 4 other friends and arrange them at the table of 6 = $^{9}C_{4}\times(5-1)! =$ 3024

No. of ways to sit the 2 friends at the table of 6 = 2!

No. of ways to sit the remaining friends at the table of 5
= (5-1)! = 4! = 24

Total no. of ways = $^{9}C_{4}\times(5-1)!\times 2!\times 4! = 145152$

No of ways to arrange 11 friends without restrictions
= $^{11}C_{5}\times(5-1)!\times(6-1)! = 1330560$

Total no. of ways of arranging 11 people such that 2 particular friends are not seated together
= 1330560 – 120960 – 145152 = 1064448

<u>Method 2</u>

<u>Alternative Method</u>

Case 1: Two particular friends seated at table of 5

No of ways
$= {}^{9}C_{3}\times 2!\times 3\times 2\times 5!$
$= 120960$

$^{9}C_{3}$: Selection of friends to be seated at table of 5. This automatically selects friends to be seated at table of 6.

(3-1)!: Arranging the 3 other friends in table of 5.

$^{3}P_{2}$: Slotting in the 2 particular friends

5!: Arranging the 6 other friends in table of 6.

Case 2: Two particular friends seated at table of 6
No of ways
$= {}^{9}C_{4}\times 4!\times 3!\times 4\times 3$
$= 217728$

$^{9}C_{4}$: Selection of friends to be seated at table of 5. This automatically selects friends to be seated at table of 6.

(5-1)!: Arranging the 5 friends in table of 5.

4!: Arranging the 5 friends in table of 6.

${}^{4}P_{2}$: Slotting in the 2 particular friends

Case 3: Two particular friends seated at separate tables
No of ways
$= {}^{9}C_{4} \times 4! \times 5! \times 2$
$= 725760$

${}^{9}C_{4}$: Selection of friends to be seated at table of 5. This automatically selects friends to be seated at table of 6.

(5-1)!: Arranging the 5 friends in table of 5.

(6-1)!: Arranging the 6 friends in table of 6.

x2: The 2 particular friends can switch tables

Total no. of ways
$= 120960 + 217728 + 725760$
$= 1064448$

Q3

(i)

Given $P(A|B') = 0.83$

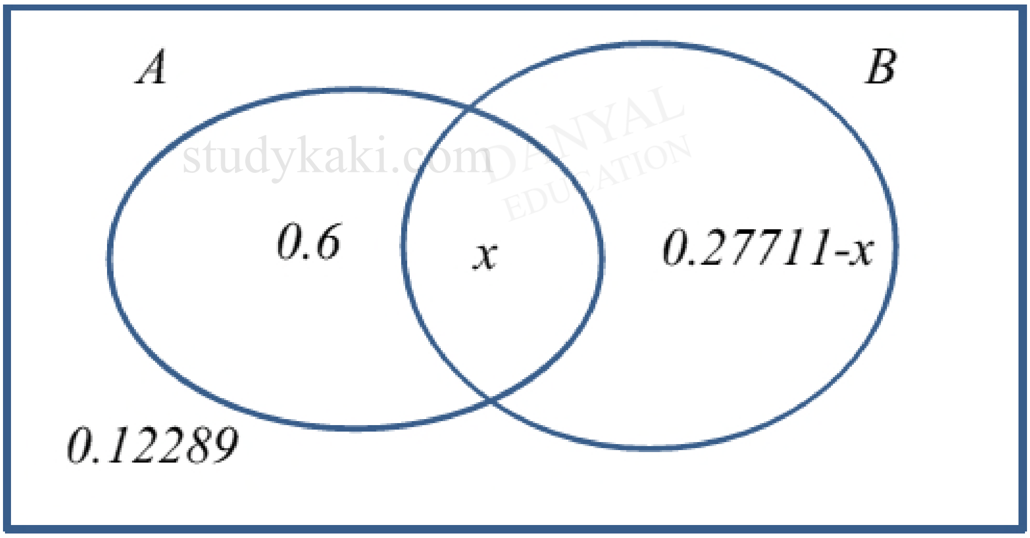
$$\Rightarrow \frac{P(A \cap B')}{P(B')} = 0.83$$

$$\Rightarrow \frac{0.6}{1 - P(B)} = 0.83$$

$$\Rightarrow P(B) = 1 - 0.72289 = 0.27711 = 0.277$$

(ii)

Let P($A \cap B$) = x



$$P(A \cup B) = P(A \cap B') + P(B)$$
$$= 0.6 + x + 0.27711 - x$$
$$= 0.87711$$

$P(A \cup B)' = 1 - 0.87711 = 0.12289$

Since $P(A \cup B') = 0.83$

$\therefore 0.6 + x + 0.12289 = 0.83$

$\Rightarrow x = 0.10711$

$\therefore P(A \cap B) = 0.107$.

(iii)

$$P(B \mid A') = \frac{P(B \cap A')}{P(A')}$$
$$= \frac{0.27711 - 0.10711}{1 - (0.6 + 0.10711)}$$
$$= \frac{0.17}{0.29289}$$
$$= 0.58042$$
$$= 0.580$$

Since $P(B \mid A') \neq P(B) \Rightarrow B$ is not independent of A'

$\therefore$ A and B are not independent.