A Level H2 Math

Permutations, Combinations and Probability Test 5

Q1

A group of twelve people consists of six married couples. Each couple consists of a husband and a wife.

(i) The twelve people are to stand in a straight line. Find the number of different arrangements if each husband must stand next to his wife. [2]

(ii) The group of twelve people finds a round table with ten chairs. Assuming only ten people are to be seated, find the probability that five married couples are seated such that each husband sits next to his wife and husbands and wives alternate. [3]

Q2

Seven red counters and two blue counters are placed in a bag. All the counters are indistinguishable except for their colours. Clark and Kara take turns to draw a counter from the bag at random with replacement. The first player to draw a blue counter wins the game and the game ends immediately.

If Clark draws first, find the probability that

(i) Clark wins the game at his third draw, [2]

(ii) Kara wins the game. [3]

Q3

For events X and Y, it is given that $\mathrm{P}(X \mid Y)=\frac{1}{2}$, $\mathrm{P}(Y \mid X)=\frac{2}{3}$ and $\mathrm{P}(X \cup Y)=\frac{5}{6}$.

Find

(i) $\mathrm{P}(X)$, [3]

(ii) $\mathrm{P}(X \cup Y')$. [2]

Answers
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Q1

(i) Number of arrangements $= 6! \times 2^6 = 46080$

(ii)
Required probability

$$= \frac{{}^6C_5 \times (5-1)! \times 2}{{}^{12}C_{10} \times (10-1)!}$$

$$= \frac{288}{23950080}$$

$$= 0.0000120 \text{ (3 sig fig)}$$

Q2

(i)
P(Clark wins in 3rd draw)

$$= \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{2}{9}$$

$$= 0.081322$$

$$= 0.0813$$

(ii)
P(Kara wins)

$$= \frac{7}{9} \times \frac{2}{9} + \left(\frac{7}{9}\right)^3 \times \frac{2}{9} + \left(\frac{7}{9}\right)^5 \times \frac{2}{9} + \ldots.$$

$$= \frac{2}{9}\left[\frac{7}{9} + \left(\frac{7}{9}\right)^3 + \left(\frac{7}{9}\right)^5 + \ldots\right]$$

$$= \frac{2}{9}\left(\frac{\frac{7}{9}}{1-\left(\frac{7}{9}\right)^2}\right)$$

$$= 0.4375 \text{ or } \frac{7}{16}$$

Q3

(i)

$$P(X \mid Y) = \frac{1}{2} \qquad P(Y \mid X) = \frac{2}{3}$$

$$\Rightarrow \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \qquad \Rightarrow \frac{P(X \cap Y)}{P(X)} = \frac{2}{3}$$

$$\Rightarrow P(X \cap Y) = \frac{1}{2}P(Y) \qquad \Rightarrow P(X \cap Y) = \frac{2}{3}P(X)$$

$$P(X \cap Y) = \frac{1}{2}P(Y) = \frac{2}{3}P(X)$$

$$P(X \cup Y) = \frac{5}{6}$$

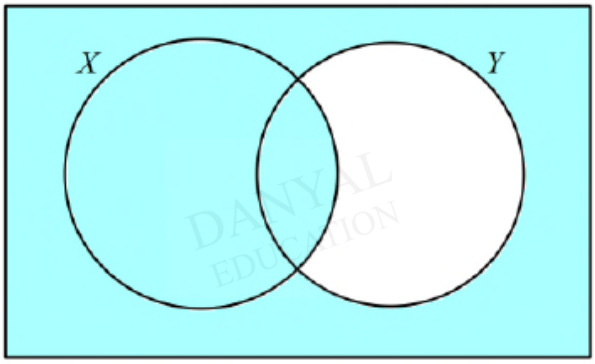
$$\Rightarrow P(X) + P(Y) - P(X \cap Y) = \frac{5}{6}$$

$$\Rightarrow P(X) + \frac{4}{3}P(X) - \frac{2}{3}P(X) = \frac{5}{6}$$

$$\Rightarrow P(X) = \frac{1}{2}$$

(ii)

$$P(X \cap Y) = \frac{2}{3}P(X) = \frac{2}{3}\left(\frac{1}{2}\right) = \frac{1}{3}$$



$X \cup Y'$

$$P(X \cup Y') = 1 - P(Y) + P(X \cap Y)$$

$$= 1 - \frac{4}{3}\left(\frac{1}{2}\right) + \frac{1}{3}$$

$$= \frac{2}{3}$$