A Level H2 Math

Permutations, Combinations and Probability Test 4

Q1

A vehicle insurance company classifies the drivers it insures as class A, B and C according to whether they are of low risk, medium risk or high risk with regard to having an accident. The company estimates that 30% of the drivers who are insured are class A and 50% are class B. The probability that a class A driver will have at least one accident in any 12 month period is 0.01, the corresponding probabilities for class B and C are 0.03 and 0.06 respectively.

(i) Find the probability that a randomly chosen driver will have at least one accident in a 12-month period. [2]

(ii) The company sold a policy to a driver and within 12 months, the driver had at least one accident. Find the probability that the driver is of class C. [2]

(iii) Three drivers insured by the company are chosen randomly. Find the probability that all three drivers are of class C and exactly one of them had at least one accident in a 12-month period. [3]

Q2

Four families arrive at Science Centre together. Mr and Mrs A brought their 2 children while Mr B brought his 2 children. Mr and Mrs C brought their 3 children while Mrs D brought her only child. All these 14 people have to go through a gate one at a time to enter the centre.

(i) In how many different ways can they go through the gate if each family goes in one after another? [2]

There are two experiments at the ***Science Magic Experience*** station.

(ii) In one experiment, participants are to be in groups of twos or threes. In how many different ways can the 8 children from the four families be grouped among themselves? [3]

(iii) In another experiment, the four families have to hold hands to form two separate circles of equal size to experience a science phenomenon. Each circle must have exactly four children and members of the same family must be in the same circle. Find the number of ways of arranging these 14 people in the two circles such that there is no more than one adult between any two children. [3]

Q3

(a) Seven boys and five girls formed a group in a school orientation. During one of the game segments, they are required to arrange themselves in a row. Find the exact probability that

(i) the girls are separated from one another, [2]

(ii) there will be exactly one boy between any two girls. [2]

In another game segment, they are required to sit at a round table with twelve identical chairs. Find the exact probability that one particular boy is seated between two particular girls. [2]

(b) The events A and B are such that $\mathrm{P}(A)=\dfrac{7}{10}$, $\mathrm{P}(B)=\dfrac{2}{5}$ and $\mathrm{P}(A\mid B)=\dfrac{13}{20}$.

(i) Find $\mathrm{P}(A\cup B)$, [3]

(ii) State, with a reason, whether the events A and B are independent. [1]

(c) A man plays a game in which he draws balls, with replacement, from a bag containing 3 yellow balls, 2 red balls and 4 black balls. If he draws a black ball, he loses the game and if he draws a red ball he wins the game. If he draws a yellow ball, the ball is replaced and he draws again. He continues drawing until he either wins or loses the game. Find the probability that he wins the game. [2]

Answers

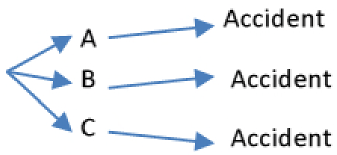
Permutations, Combinations and Probability Test 4

Q1

(i) Required probability

$= (0.3\times0.01)+(0.5\times0.03)+(0.2\times0.06)$

$= 0.03$



(ii) $$\text{P(class C/accident)} = \frac{P(\text{accident} \cap \text{class C})}{P(\text{accident})}$$

$$= \frac{0.2\times0.06}{0.03} = 0.4$$

(iii) P(all three drivers are of class C and exactly one have accident)

$$= (0.2\times0.94)^2 \times 0.2\times0.06\times\frac{3!}{2!}$$

$= 0.00127$ (to 3sf)

Q2

(i)

family	A		B		C		D	
	Adult	kids	Adult	kids	Adult	kids	Adult	kids
	2	2	1	2	1	3	1	1

4 family units, No. of ways $= 4!\times4!\times3!\times5!\times2! = 829,440$

(ii) Case 1: 3,3,2 No. of ways $= \dfrac{{}^8C_3\times{}^5C_3\times{}^2C_2}{2!} = 280$

Case 2: 2,2,2,2 No. of ways $= \dfrac{{}^8C_2\times{}^6C_2\times{}^4C_2\times{}^2C_2}{4!} = 105$

Total no. of ways = 280 + 105 = 385

(iii) There is only 1 way to divide the 8 children and the adults into 2 circles to satisfy all conditions. Family A and B (3 adults & 4 kids) must be in 1 circle and Family C & D are in another circle.

Arrange the children in 1 circle : (4-1)!

Slot in adults in between children : ${}^4C_3\times3!$

No. of ways $= [(4-1)!\times{}^4C_3\times3!]\times[(4-1)!\times{}^4C_3\times3!]$

$= 20736$

Q3

(a)(i)

$$\text{Required probability} = \frac{7! \times {}^{8}C_{5} \times 5!}{12!}$$

$$= \frac{7}{99}$$

(a)(ii)

$$\text{Required probability} = \frac{7! \times 4 \times 5!}{12!}$$

$$= \frac{1}{198}$$

$$\text{Required probability} = \frac{(10-1)! \times 2!}{(12-1)!}$$

$$= \frac{1}{55}$$

(b)(i)

$$\mathrm{P}(A \mid B)=\frac{13}{20}$$

$$\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}=\frac{13}{20}$$

$$\mathrm{P}(A \cap B)=\frac{13}{20}\left(\frac{2}{5}\right)=\frac{13}{50} \quad \text{(or 0.26)}$$

$$\begin{aligned}\mathrm{P}(A \cup B)&=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)\\&=\frac{7}{10}+\frac{2}{5}-\frac{13}{50}\\&=\frac{21}{25} \quad \text{(or 0.84)}\end{aligned}$$

(b)(ii)

Since $\mathrm{P}(A \mid B) \neq \mathrm{P}(A)$, therefore events A and B are not independent.

Alternatively,

Since $\mathrm{P}(A \cap B)=\frac{13}{50}$ and $\mathrm{P}(A) \times \mathrm{P}(B)=\frac{7}{10} \times \frac{2}{5}=\frac{7}{25} \neq \mathrm{P}(A \cap B)$, therefore events A and B are not independent.

(c)

Probability of winning the game

$$\begin{aligned}&=\frac{2}{9}+\frac{2}{9}\left(\frac{3}{9}\right)+\frac{2}{9}\left(\frac{3}{9}\right)^2+\ldots\\&=\frac{2/9}{1-\frac{3}{9}}\\&=\frac{1}{3}\end{aligned}$$