A Level H2 Math

Permutations, Combinations and Probability Test 2

Q1

A planning committee of 12 students consisting of one male and one female student from each of the 6 Arts stream classes (Class A to Class F) in a junior college is to be formed for the Humanities Seminar. There are 10 male and 10 female students in Class A.

(i) How many ways can the representatives from Class A be chosen? [1]

The committee meets for their first planning meeting and is seated at a round table.

(ii) How many ways can the committee be seated if all the members need to be seated together with the member from the same class? [2]

At the seminar, the committee members are to be seated in a row of 14 seats in the theatre together with the Principal and the Guest of Honour. The chairperson and the secretary are selected from the committee and they are both from Class F.

(iii) How many ways can this be done if the Principal and the Guest of Honour occupy the middle seats and the committee members are seated together with the member from the same class except for the chairperson and the secretary? [4]

Q2

A delegation of four students is to be selected from five badminton players, m floorball players, where $m > 3$, and six swimmers to attend the opening ceremony of the 2017 National Games. A pair of twins is among the floorball players. The delegation is to consist of at least one player from each sport.

(i) Show that the number of ways to select the delegation in which neither of the twins is selected is $k(m-2)(m+6)$, where k is an integer to be determined. [3]

(ii) Given that the number of ways to select a delegation in which neither of the twins is selected is more than twice the number of ways to select a delegation which includes exactly one of the twins, find the least value of m. [2]

The pair of twins, one badminton player, one swimmer and two teachers, have been selected to attend a welcome lunch at the opening ceremony. Find the number of ways in which the group can be seated at a round table with distinguishable seats if the pair of twins is to be seated together and the teachers are separated. [3]

Q3

(a) The word DISTRIBUTION has 12 letters.

(i) Find the number of different arrangements of the 12 letters that can be made. [1]

(ii) Find the number of different arrangements which can be made if there are exactly 8 letters between the two Ts. [3]

One of the Is is removed from the word and the remaining letters are arranged randomly.

(iii) Find the probability that no adjacent letters are the same. [4]

(b) The insurance company Adiva classifies 10% of their car policy holders as 'low risk', 60% as 'average risk' and 30% as 'high risk'. Its statistical database has shown that of those classified as 'low risk', 'average risk' and 'high risk', 1%, 15% and 25% are involved in at least one accident respectively.

Find the probability that

(i) a randomly chosen policy holder is not involved in any accident if the holder is classified as 'average risk', [1]

(ii) a randomly chosen policy holder is not involved in any accident, [2]

(iii) a randomly chosen policy holder is classified as 'low risk' if the holder is involved in at least one accident. [2]

It is known that the cost of repairing a car when it meets with an accident has the following probability distribution.

Cost incurred (in thousand dollars)	5	10	50	100
Probability	0.75	0.15	0.08	0.02

It is known that a ‘low risk’ policy holder will not be involved in more than one accident in a year. You may assume that there will be no cost incurred by the company in insuring a holder whose car is not involved in any accident.

(iv) Construct the probability distribution table of the cost incurred by Adiva in insuring a ‘low risk’ policy holder assuming that the cost of repairing a car is independent of a ‘low risk’ policy holder meeting an accident. [1]

(v) In order to have an expected profit of $200 from each policy holder, find the amount that Adiva should charge a ‘low risk’ policy holder when he renews his annual policy. [2]

Answers

Permutations, Combinations and Probability Test 2

Q1

(i)	No. of ways $= {}^{10}C_1 \times {}^{10}C_1 = 100$	Generally well done except for some who did ${}^{10}C_1 + {}^{10}C_1$ instead.
(ii)	No. of ways $= (6-1)! \times (2!)^6 = 7680$	Some students used $(6-1)! \times 2!$ or $(6-1)! \times 6(2)!$ instead. Since there are 6 couples, and for each couple there are 2! ways to arrange them, we do $2! \times 2! \times 2! \times 2! \times 2! \times 2!$, which is different from $6(2)!$.
(iii)	**Method ①:** (1) Arrange P and GoH $= 2!$ (2) Choose side where CP and Sec are on (left or right) $= {}^2C_1$ (3) Choose 2 classes to be seated with CP and Sec $= {}^5C_2$ (4) Arrange the 2 classes and the people within each class $= 2! \times (2!)^2$ (5) Slot in CP and Sec $= {}^3C_2 \times 2!$ (6) Arrange the 3 other classes and the people within each class $= 3! \times (2!)^3$ No. of ways $= 2! \times 2 \times {}^5C_2 \times 2! \times (2!)^2 \times {}^3C_2 \times 2! \times 3! \times (2!)^3 = 92160$ **Method ②: (Arrange the 5 classes at one go)** No. of ways $= 2! \times 2 \times 5! \times (2!)^5 \times {}^3C_2 \times 2! = 92160$ **Method ③: (Complement)** No. of ways = n(CP/S on 1 side but may be tog) − n(CP/S tog) $= \left\{2 \times \left[{}^5C_2 \times 3! \times 2^3\right] \times 2 \times (2)^2 \times 4!\right\} - \left[6! \times (2)^6 \times 2\right] = 92160$	Most students were able to get 1 or 2 marks for this part. Key is for the GoH and P to be seated in the middle, and for the CP and S to be separated, they must be on the same side. Otherwise with 5 students on one side and 7 students on the other side, GoH and P will not be in the middle. So the remaining 5 classes will be split to 3-2, with CP and S joining the side with 2 classes. Hence, using the 2 classes, we will choose 2 out of 3 slots for CP and S. Bear in mind that the 2 classes can either be on the left, or on the right.

Q2

(i)

	5	$(m-2)$	6
	Badminton Players	Floorball Players	Swimmers Players
Case 1	1	1	2
Case 2	1	2	1
Case 3	2	1	1

Case 1: Number of selections is $\binom{5}{1}\binom{m-2}{1}\binom{6}{2}$

Case 2: Number of selections is $\binom{5}{1}\binom{m-2}{2}\binom{6}{1}$

Case 3: Number of selections is $\binom{5}{2}\binom{m-2}{1}\binom{6}{1}$

Total number of selections

$$=\binom{5}{1}\binom{m-2}{1}\binom{6}{2}+\binom{5}{1}\binom{m-2}{2}\binom{6}{1}+\binom{5}{2}\binom{m-2}{1}\binom{6}{1}$$

$$=75(m-2)+30\frac{(m-2)(m-3)}{2!}+60(m-2)$$

$$=135(m-2)+15(m-2)(m-3)$$

$$=15(m-2)(9+m-3)$$

$$=15(m-2)(m+6)$$

$$\therefore k=15$$

Alternative method:

$$\binom{5}{1}\binom{m-2}{1}\binom{6}{1}\binom{m-2+5+6-3}{1}/2!$$

(ii)

Number of ways to select exactly one of the twins

$$=\binom{5}{1}\binom{2}{1}\binom{6}{2}+\binom{5}{1}\binom{m-2}{1}\binom{2}{1}\binom{6}{1}+\binom{5}{2}\binom{2}{1}\binom{6}{1}$$

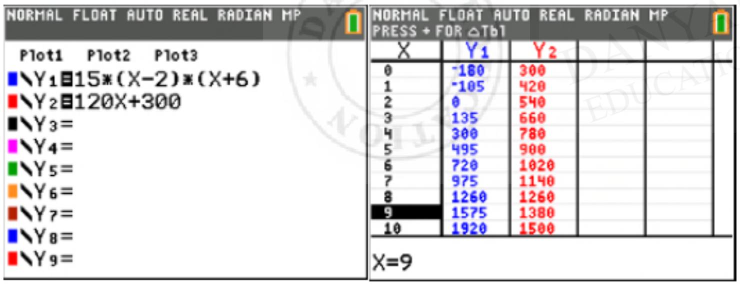
$$=150+60(m-2)+120$$

$$=60m+150$$

Number of ways that the twins are not selected > 2 times the number of ways that exactly one of the twins is selected.

$$15(m-2)(m+6)>2(60m+150)$$

By GC,



least value of m is 9.

Last part

Step 1: Arrange 3 units at the round table = 3!/3

Step 2: Arrange the twins among themselves = 2!

Step 3: Slot in the teachers = $\binom{3}{2}\times 2!$

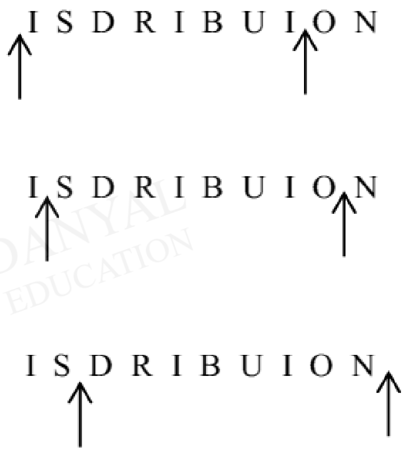
Number of ways for the twins to be seated together and teachers are separated

$$=\frac{3!}{3}\times 2!\times\binom{3}{2}\times 2!\times 6=144$$

Q3

(i) Number of ways = $\dfrac{12!}{3!2!} = 39916800$

(ii) Method 1

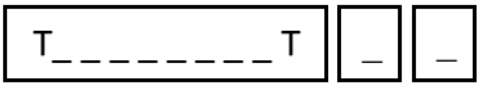


There are 3 ways to slot in the 2 T's

Total number of ways

= total number of ways to arrange the remaining ten letters $\times 3 = \dfrac{10!}{3!} \times 3 = 1814400$

Method 2



Case 1: One I included between the two T's

Number of ways = ${}^{7}C_{6} \times 8! \times \frac{3!}{2!} = 120960$

Case 2: Two I's included between the two T's

Number of ways = ${}^{7}C_{6} \times \frac{8!}{2!} \times 3! = 846720$

Case 3: Three I's included between the two T's

Number of ways = ${}^{7}C_{5} \times \frac{8!}{3!} \times 3! = 846720$

Total number of ways $= 120960 + 2(846720) = 1814400$

(iii) Method 1

Case 1: Both I together but both T separated

7 single letters (excluding Is and Ts) and 1 block of 2Is.

Number of ways = $8! \times {}^{9}C_{2} = 1451520$

Case 2: Both T together but I separated

Number of ways = $8! \times {}^{9}C_{2} = 1451520$ (same approach as case 1)

Case 3: Both I together and both T together

(II)(TT) D S R B U O N

7 single letters (excluding Is and Ts),1 block of Is and 1 block of Ts.

Number of ways = $9! = 362880$

Total number of ways in complement
$= (1451520 \times 2) + 362880 = 3265920$

Method 2

Number of ways in which both T are together = $\frac{10!}{2!}$

Number of ways in which both I are together = $\frac{10!}{2!}$

Number of ways in which both pairs of identical letters are together = 9!

Total number of ways in complement $= 2 \times \frac{10!}{2!} - 9! = 3265920$

Required probability $= 1-\dfrac{3265920}{\dfrac{11!}{2!2!}} = 0.673$

(b)(i) P(holder is not involved in any accident | the holder is classified as ‘average risk’)
$= 100\% - 15\% = 85\% = 0.85$

(ii) Probability of a randomly chosen policy holder not involved in any car accident
$= (0.1)(0.99) + (0.6)(0.85) + (0.3)(0.75)$
$= 0.834$ or $\dfrac{417}{500}$

(iii) P(policy holder is ‘low risk’ | has met at least one car accident)
$= \dfrac{\text{P(holder is classified as 'low risk' and met with at least 1 accident)}}{\text{P(holder meets with at least 1 accident)}}$
$= \dfrac{0.1(0.01)}{1-0.834}$
$= 0.00602$ (to 3sf) or $\dfrac{1}{166}$

(iv) Let C be the cost of insuring a randomly chosen ‘low risk’ policy holder (in thousands).

c	0	5	10	50	100
$P(C=c)$	0.99	(0.01)(0.75) = 0.0075	(0.01)(0.15) = 0.0015	(0.01)(0.08) = 0.0008	(0.01)(0.02) = 0.0002

(v) $E(C) = 100(0.0002)+50(0.0008)+10(0.0015)+5(0.0075)= 0.1125$

Note: “Profit = Premium Charged – Cost Incurred for Repair”

$1000(0.1125) + 200 = 312.5$
The company should charge \$312.50 for a car insurance plan for ‘low risk’.

<u>Alternative (Using “Profit = Premium – Cost Incurred”)</u>

Let P be the premium charged by Adiva for a ‘low risk’ holder.
$P(0.99)+(P-5000)(0.0075)+(P-10000)(0.0015)$
$+(P-50000)(0.0008)+(P-100000)(0.0002)=200$

Solving, $P = 312.5$
The company should charge \$312.50 for a car insurance plan for ‘low risk’.