A Level H2 Math

Permutations, Combinations and Probability Test 1

Q1

A group of 12 students consists of 5 bowlers, 4 canoeists and 3 footballers.

(i) The group sits at a round table with 12 seats. In how many different ways can they sit so that all the players of the same sport sit together? [2]

(ii) The group stands in a line. In how many different ways can they stand so that *either* the bowlers are all next to one another *or* the canoeists are all next to one another *or* both? [2]

(iii) Find the number of ways in which a delegation of 8 can be selected from this group if it must include at least 1 student from each of the 3 sports. [2]

Q2

The number of employees of a statutory board, classified by department and years of working experience, is shown below.

	5 years or less	5 to 10 years	10 years or more	Total
Human Resource Department	20	50	30	100
Legal Department	15	60	45	120
Finance Department	25	30	45	100
Total	60	140	120	320

The Managing Director of the statutory board wishes to select three employees to participate in an overseas conference. The Managing Director selects one employee from each department to participate in the conference.

(i) Find the probability that two of the selected employees have years of working experience '10 years or more' and the remaining one has years of working experience '5 years or less'. [3]

(ii) Given that exactly one of the selected employees has years of working experience '5 years or less', find the probability that one of the selected employees is from the Legal Department and has years of working experience '5 to 10 years'. [3]

Q3

A restaurant is setting up a spinning wheel for its customers to try and win vouchers. The wheel is split into 8 identical segments, comprising of \$0, \$5, \$10, \$15, \$20, \$25, \$30 and \$50.

Find the number of ways the segments can be arranged on the wheel if

(i) there are no restrictions. [1]

(ii) the \$0 segment cannot be next to the \$5 segment [2]

(iii) there must be at least two segments between the \$30 and \$50 segments. [2]

The restaurant decides to replace the \$30 and \$50 segments with another two \$0 segments.

(iv) Find the number of possible arrangements of the 8 segments. [1]

(v) Find the number of possible arrangements if the \$0 segments must be separated. [2]

Answers

Permutations, Combinations and Probability Test 1

Q1

(i)	Number of ways = $(3-1)!\cdot 5!\cdot 4!\cdot 3! = 34\ 560$	Generally well done
(ii)	Number of ways = N(5 bowlers together) + N(4 canoeists together) – N(5 bowlers together & 4 canoeists together) $= 8!\cdot 5! + 9!\cdot 4! - 5!\cdot 5!\cdot 4!$ $= 4\ 838\ 400 + 8\ 709\ 120 - 345\ 600$ $= 13\ 201\ 920$	Most students added the three numbers instead of subtracting the case for intersection: $8!\cdot 5! + 9!\cdot 4! + 5!\cdot 5!\cdot 4!$. If students had drawn a venn diagram, the correct operation would have been clearer.
(iii)	Number of ways = N(Total) – N(0 bowlers) – N(0 canoeists) – N(0 footballers) $= {}^{12}C_8 - 0 - {}^{8}C_8 - {}^{9}C_8 = 485$	Very badly done, although there is a question in Tutorial 20 Q9. Many did ${}^{5}C_1 * {}^{4}C_1 * {}^{3}C_1 * {}^{9}C_1$ which is a gross overcount.

Q2

(i)	Required probability $= \frac{30}{100}\times\frac{45}{120}\times\frac{25}{100}+\frac{30}{100}\times\frac{15}{120}\times\frac{45}{100}+\frac{20}{100}\times\frac{45}{120}\times\frac{45}{100}$ $= \frac{63}{800}$
(ii)	Required probability $= \frac{(0.2)(0.5)(0.75)+(0.8)(0.5)(0.25)}{(0.2)(0.875)(0.75)+(0.8)(0.125)(0.75)+(0.8)(0.875)(0.25)}$ $= \frac{28}{61}$
(iii)	Number of different possible codes $= {}^9C_2 \times 2! \times {}^7C_1$ $= 504$
(iv)	**Method 1: Complementary Method** Number of possible arrangements $= \left[{}^4C_3 \times {}^5C_2 \times 5!\right] - \left[\left({}^4C_3 \times 3!\right)\times {}^5C_2 \times 3!\right]$ $= 3360$ **Method 2: List by Cases** Case 1: All the even digits are separated ${}^4C_3 \times {}^5C_2 \times 2! \times 3! = 480$ Case 2: Exactly two even digits are next to each other (and the third even digit is separated) ${}^4C_3 \times \left({}^3C_2 \times 2!\right)\times {}^5C_2 \times 3! \times {}^2C_1 = 2880$ Number of possible arrangements $= 480 + 2880$ $= 3360$

Q3

(i)

$(8-1)!=5040$

(ii)

No. of ways with $0 and $5 segments adjacent

$=(7-1)!2!$

$=1440$

No. of ways without identical segments adjacent

= total no. of ways − no. of ways with identical segments adjacent

$=5040-1440$

$=3600$

(iii)

<u>Case 1: no segment separating them</u>

$(7-1)!2!=1440$

<u>Case 2: exactly 1 segment separating them</u>

$\binom{6}{1}2!(6-1)!=1440$

Total number of ways $=5040-1440-1440$

$=2160$

ALT

Case 1: exactly 2 segments separating them

$$\binom{6}{2}2!2!(5-1)!=1440$$

Case 2: exactly 3 segments separating them

$$\frac{\binom{6}{3}3!2!(4-1)!}{2}=720$$

Therefore, total number of ways = 2160

(iv)

The segments are \$0, \$0, \$0, \$5, \$10, \$15, \$20, \$25

$$\frac{(8-1)!}{3!}=840$$

(v)

Arrange the other 5 objects in (5 – 1)! = 24 ways
Choose 3 spaces for the \$0 in ${}^5C_3$ = 10 ways
Total = 240 ways