A Level H2 Math

Normal Distribution Test 6

Q1

(i) Factory A produces nuts whose mass may be assumed to be normally distributed with mean μ grams and standard deviation σ grams. A random sample of 50 nuts is taken. It is given that the probability that the mean mass is less than 247 grams is 0.018079, correct to 5 significant figures. It is also given that the probability that the total mass exceeds 12600 grams is 0.78397, correct to 5 significant figures. Find the values of μ and σ, giving your answers to the nearest grams. [5]

(ii) (For this question, you should state clearly the values of the parameters of any normal distribution you use.)

Factory B produces bolts and nuts. The masses, in grams, of bolts and nuts produced are modelled as having independent normal distributions with means and standard deviations as shown in the table:

	Mean Mass (in grams)	Standard Deviation (in grams)
Bolts	745	7.3
Nuts	250	5

(a) Find the probability that the mass of a randomly chosen bolt differs from 3 times the mass of a randomly chosen nut by at least 40 grams. [4]

(b) This factory introduces a new process which is able to reduce the mass of each nut by 10%. Find the probability that the total mass, after the introduction of this process, of 10 randomly chosen nuts is less than 2.24 kg. [3]

Q2

A flange beam is a steel beam with a "H"-shaped cross section, and is used as a supporting structure in construction and civil engineering. A factory manufactures both Grade X and Grade Y flange beams. The load that can be supported by a Grade X flange beam follows a normal distribution with mean 2.43×10^5 kN and standard deviation 4.5×10^4 kN. The load that can be supported by a Grade Y flange beam is 1.5 times of the load that can be supported by a Grade X flange beam.

(i) Find the probability that the combined load that can be supported by two randomly chosen Grade Y flange beams is within 1×10^4 kN of the combined load that can be supported by three randomly chosen Grade X flange beams. [4]

(ii) A construction company wants to buy 100 sets of three Grade X flange beams. Find the probability that fewer than 95 of these sets can support more than 6×10^5 kN. [3]

The company decides to place an order with the factory for a custom-made flange beam such that the probability of being able to support a load of at least 6×10^5 kN must be at least 0.999. It is also assumed that the load that can be supported by the custom-made flange beam also follows a normal distribution.

(iii) By taking the standard deviation of a custom-made flange beam to be 3×10^4 kN, find the smallest possible mean load in kN, giving your answer correct to the nearest thousand, for the factory to meet the company's requirements for the custom-made flange beam. [5]

Q3

In the manufacture of child car seats, a resin made up of three ingredients is used. The ingredients are polymer A, polymer B and an impact modifier. The resin is prepared in batches and each ingredient is supplied by a separate feeder. The masses, in kg, of polymer A, polymer B and the impact modifier in each batch of resin are assumed to be normally distributed with means and standard deviations as shown in the table. The three feeders are also assumed to operate independently of each other.

	Mean	Standard deviation
Polymer A	2030	44.8
Polymer B	1563	22.7
Impact modifier	$\propto$	σ

It is known that 3% of the batches of resin have less than 1350 kg of impact modifier and 30% of the batches of resin have more than 1414 kg of impact modifier.

(i) Show that $\propto \approx 1400$ and $\sigma \approx 26.6$. [3]

(ii) Given that polymer A costs \$2.20 per kg, polymer B costs \$2.80 per kg and the impact modifier costs \$1.50 per kg, find the probability that the total cost of 2 batches of resin exceeds \$22,000. [4]

(iii) A random sample of n batches of resin is chosen. If the probability that at most 6 batches of resin has more than 1414 kg of impact modifier is less than 0.001, find the least value of n. [3]

(iv) Each batch of resin is used to make a large number of car seats. It is found that the tensile strength (N/m^2) of resin for a car seat has mean 125 and standard deviation 17. A random sample of 50 car seats is selected. Find the probability that the average tensile strength of resin for these 50 car seats is less than 130 N/m^2. [3]

Answers

Normal Distribution Test 6

Q1

(i)

Let X be the mass of a randomly chosen nut in grams.

$X \sim \mathrm{N}\left(\mu, \sigma^2\right)$

$\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{50}\right)$ and $X_1 + \ldots + X_{50} \sim \mathrm{N}\left(50\mu, 50\sigma^2\right)$

Given $\mathrm{P}\left(\overline{X} < 247\right) = 0.018079$ and $\mathrm{P}\left(X_1 + \ldots + X_{50} > 12600\right) = 0.78397$

Standardizing, $Z \sim \mathrm{N}(0,1)$

$$\mathrm{P}\left(Z < \frac{247-\mu}{\sigma/\sqrt{50}}\right) = 0.018079$$

$$\frac{247-\mu}{\sigma/\sqrt{50}} = -2.095146$$

$$\mu - 0.2962984\sigma = 247 \ldots(1)$$

$$\mathrm{P}\left(Z > \frac{12600-50\mu}{\sqrt{50}\sigma}\right) = 0.78397$$

$$\mathrm{P}\left(Z < \frac{12600-50\mu}{\sqrt{50}\sigma}\right) = 0.21603$$

$$\frac{12600-50\mu}{\sqrt{50}\sigma} = -0.7856714$$

$$50\mu - 0.7856714\left(\sqrt{50}\sigma\right) = 12600 \ldots(2)$$

Solving equation (1) and (2), using GC,

$\mu = 255$ (nearest gram)

$\sigma = 27$ (nearest gram)

(ii)(a)

Let Y be the mass of a randomly chosen nut in grams.

$Y \sim \mathrm{N}\left(250, 5^2\right)$

Let W be the mass of a randomly chosen bolt in grams.

$W \sim \mathrm{N}\left(745, 7.3^2\right)$

$W - 3Y \sim \mathrm{N}\left(745 - 3\times 250, 7.3^2 + 3^2 \times 5^2\right)$

i.e. $W - 3Y \sim \mathrm{N}\left(-5, 278.29\right)$

$$\mathrm{P}\left(|W-3Y| \geq 40\right) = \mathrm{P}\left(W-3Y < -40\right) + \mathrm{P}\left(W-3Y > 40\right)$$
$$= 0.0214 \text{ (3s.f.)}$$

(ii)(b)

Let T be total mass of 10 randomly chosen nut, made using new material, in grams.

$T = 0.9Y_1 + 0.9Y_2 + \ldots + 0.9Y_{10} \sim \mathrm{N}\left(10\times 0.9 \times 250, 10 \times 0.9^2 \times 5^2\right)$

$T \sim \mathrm{N}\left(2250, 202.5\right)$

$\mathrm{P}\left(T < 2240\right) = 0.241$ (3s.f.)

Q2

Let A and B be the load that can be supported (in kN) by a Grade X and Grade Y flange beam respectively. Then, $A \sim N\left(2.43\times10^5, \left(4.5\times10^4\right)^2\right)$.	Many students did not define the random variables. Some defined it wrongly and just wrote it as "Let A be the Grade X and B be the Grade Y." There are some who took $\left(4.5\times10^4\right)$ as the variance of A.
Since $B=1.5A$, then $B \sim N\left(1.5\left(2.43\times10^5\right), 1.5^2\left(4.5\times10^4\right)^2\right)$ i.e., $B \sim N\left(3.645\times10^5, \left(6.75\times10^4\right)^2\right)$ leave in exact decimal!	Common mistakes for Var(B): 1. Var(B) = $\left(4.5\times10^4\right)^2$ 2. Var(B) = $1.5\left(4.5\times10^4\right)^2$
(i) Want to find $\mathrm{P}\left(\left\|\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)\right\|<1\times10^4\right)$ → $\mathrm{E}\left(\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)\right)$ $=2\times3.645\times10^5-3\times2.43\times10^5=0$ $\mathrm{Var}\left(\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)\right)$ $=2\times\left(6.75\times10^4\right)^2+3\left(4.5\times10^4\right)^2=1.51875\times10^{10}$	There are still students who wrote $2B-3A$ instead of $\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)$. Students are advised not to correct their working answer to 3 s.f especially if the exact decimal answer is obtained. For e.g. in this case, if students round of their answer for E(B) to 3.65×10^5, their answer for $\mathrm{E}\left(\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)\right)$ is 1000 instead of 0.
i.e. $\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)\sim N\left(0,1.51875\times10^{10}\right)$ →	Must always write down the distribution after finding expectation and variance!
Required probability $=\mathrm{P}\left(\left\|\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)\right\|<1\times10^4\right)$ $=\mathrm{P}\left(-1\times10^4<\left(B_1+B_2\right)-\left(A_1+A_2+A_3\right)<1\times10^4\right)$ $=0.0647$ (to 3 s.f.)	Some students do not understand what is meant by A within 1×10^4 kN of B. *Note:* In general, $\mathrm{P}\left(\left\|T\right\|<1\times10^4\right)\neq\mathrm{P}\left(T<1\times10^4\right)+\mathrm{P}\left(T>-1\times10^4\right)$

(ii) $A_1 + A_2 + A_3 \sim N\left(3\times 2.43\times 10^5, 3\left(4.5\times 10^4\right)^2\right)$ $P\left(A_1 + A_2 + A_3 > 6\times 10^5\right) \approx 0.951045$ Let T be the number of sets (out of 100 sets) of three Grade X flange beams that can support more than 6×10^5 kN. Then, $T \sim B(100,\ 0.951045)$ Required probability $= P(T < 95)$ $= P(T \le 94)$ $= 0.365$ (to 3 s.f.)	Some students missed out on the three in the "100 sets of three Grade X flange beams" and went to find $P\left(A > 6\times 10^5\right)$ Did not change $P(T<95)$ to $P(T \le 94)$ when using binomcdf
(iii) Let W be the load that can be supported (in kN) by a custom-made flange beam. Given: $W \sim N\left(\mu,\ \left(3\times 10^4\right)^2\right)$ $P(W \ge 6\times 10^5) \ge 0.999$ $\Rightarrow\ 1 - P(W < 6\times 10^5) \ge 0.999$ $\Rightarrow\ P(W < 6\times 10^5) \le 0.001$ $\Rightarrow\ P\left(Z \le \dfrac{6\times 10^5 - \mu}{3\times 10^4}\right) \le 0.001$ By GC, $\dfrac{6\times 10^5 - \mu}{3\times 10^4} \le -3.0902$ $6\times 10^5 - \mu \le -92760$ $-\mu \le -92760 - 6\times 10^5$ $\mu \ge 692760$ Thus, smallest mean = 693 kN (to nearest thousand)	Some denote the smallest possible mean as $\bar{x}$ which is incorrect as $\bar{x}$ always denote sample mean. A few standardised wrongly. Wrote $\dfrac{\mu - 6\times 10^5}{3\times 10^4}$. *Common mistake:* Took invNorm of the right area which is 0.999 instead of left area, 0.001 Some started off with inequality but ended up with an equation after taking invNorm.

Marker's comments

- Students are advised not to use Z to denote the random variable as Z denotes the Standard Normal Variable, i.e $Z \sim N(0, 1)$
- Students are advised to use exact decimal workings answer or working answers with more decimal places to avoid loss of accuracy in their final answer.
- There are at least a few in each class who do not know how to correct their answer to the nearest thousand.
- For part(iii), students are advised not to use GC Table although the unknown is to be corrected to the nearest thousand. Students who use GC table but did not show their workings clearly do not get the full marks.

Q3

(i) Let X be the random variable 'amount (in kg) of impact modifier in a batch of resin'

$X \sim N(\mu, \sigma^2)$

$\mathrm{P}(X < 1350) = 0.03$

$\mathrm{P}(Z < \frac{1350-\mu}{\sigma}) = 0.03$

$\frac{1350-\mu}{\sigma} = -1.88079361$

$\mu - 1.88079361\sigma = 1350 - -(1)$

$\mathrm{P}(X > 1414) = 0.3$

$\mathrm{P}(Z < \frac{1414-\mu}{\sigma}) = 0.7$

$\frac{1414-\mu}{\sigma} = 0.5244005101$

$\mu + 0.5244005101\sigma = 1414 - - - (2)$

Solve (1) and (2),

$\mu = 1400.046 = 1400$ (shown)

$\sigma = 26.609 = 26.6$ (shown)

(ii) Let Y be the random variable 'amount (in kg) of Polymer A in a batch of resin'

Let W be the random variable 'amount (in kg) of Polymer B in a batch of resin'

$Y \sim N(2030, 44.8^2)$, $W \sim N(1563, 22.7^2)$

Total cost of a batch,

$T = 2.20Y + 2.80W + 1.50X \sim N(10942.4, 15345.9572)$

Total cost of 2 batches, $T_1 + T_2 \sim N(21884.8, 30715.9144)$

$P(T_1 + T_2 > 22000) = 0.255$ (3.s.f.)

(iii) Let H be the r.v.' number of batches of resin with more than 1414 kg of impact modifier out of n batches.'

$H \sim B(n, 0.3)$

$\mathrm{P}(H \le 6) < 0.001$

Using GC.

When $n = 53$,

$\mathrm{P}(H \le 6) = 0.00120 > 0.001$

When $n = 54$,

$\mathrm{P}(H \le 6) = 9.44 \times 10^{-4} < 0.001$

Therefore, least $n = 54$

(iv) Let S be the r.v.' tensile strength (in N/m^2) of resin in a car seat'

$\mathrm{E}(S) = 125$, $\mathrm{Var}(S) = 17^2$

$\overline{S} = \frac{S_1 + S_2 + \ldots + S_{50}}{50}$

$\overline{S} \sim N\left(125, \frac{17^2}{50}\right)$ approx by Central Limit Thm

$\mathrm{P}(\overline{S} < 130) = 0.981$ (3 s.f.)