A Level H2 Math

Normal Distribution Test 5

Q1

A factory manufactures round tables in two sizes: small and large. The radius of a small table, measured in cm, has distribution $N(30, 2^2)$ and the radius of a large table, measured in cm, has distribution $N(50, 5^2)$.

(i) Find the probability that the sum of the radius of 5 randomly chosen small tables is less than 160 cm. [2]

(ii) Find the probability that the sum of the radius of 3 randomly chosen small tables is less than twice the radius of a randomly chosen large table. [2]

(iii) State an assumption needed in your calculation in part **(ii)**. [1]

A shipment of 12 large tables is to be exported. Before shipping, a check is done and the shipment will be rejected if there are at least two tables whose radius is less than 40 cm.

(iv) Find the probability that the shipment is rejected. [3]

The factory decides now to manufacture medium sized tables. The radius of a medium sized table, measured in cm, has distribution $N(\mu, \sigma^2)$. It is known that 20% of the medium sized tables have radius greater than 44 cm and 30% have radius of less than 40 cm.

(v) Find the values of μ and σ. [4]

Q2

In this question, you should state clearly the values of the parameters of any normal distribution you use.

A supermarket sells two types of durians, D25 and Musang Queen. The durians are sold by weight. The masses, in kilograms, of D25 and Musang Queen are modelled as having normal distributions. The means and standard deviations of these distributions, and the selling prices, in $ per kilogram, are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price ($ per kg)
D25	1.5	0.02	9
Musang Queen	1.8	0.035	18

(i) A customer buys 3 D25 durians and 2 Musang Queen durians. Find the probability that the total cost of his purchase is more than $107. [5]

(ii) State an assumption needed for your calculations in part **(i)**. [1]

(iii) The probability that the average weight of n randomly chosen D25 durians exceeding m kg is at least 0.1. Show that n satisfies the inequality

$$(m-1.5)\sqrt{n} \leqslant 0.025631.$$

Hence find the largest possible value of n when $m = 1.51$. [4]

Q3

The diameters of the bolts produced by two manufacturers A and B follow a normal distribution with a standard deviation of 0.16 mm.

The mean diameter of the bolts produced by manufacturer A is 1.56 mm. Of the bolts produced by manufacturer B, 24.2% have a diameter less than 1.52 mm.

(i) Show that the mean diameter of the bolts produced by manufacturer B is 1.632 mm. [3]

(ii) Find the probability that the diameter of a randomly chosen bolt from manufacturer A differs from the diameter of a randomly chosen bolt from manufacturer B by less than 0.1 mm. [3]

(iii) Find the probability that the total diameter of 5 randomly chosen bolt from manufacturer A is more than 5 times the diameter of a randomly chosen bolt from manufacturer B. [3]

(iv) A trading company buys 44% of its stock of bolts from manufacturer A and the rest from manufacturer B. A bolt is chosen at random from the trading company's stock. Show that the probability that the diameter of the bolt is less than 1.52 mm is 0.312. [3]

Answers

Normal Distribution Test 5

Q1

(i)

Let S be the random variable "radius of a small table in cm'.
Let L be the random variable "radius of a large table in cm'.

$S \sim \mathrm{N}(30,\ 2^2)$

$L \sim \mathrm{N}(50,\ 5^2)$

$S_1+S_2+S_3+S_4+S_5 \sim \mathrm{N}(5\times 30,\ 5\times 2^2)$

$S_1+S_2+S_3+S_4+S_5 \sim \mathrm{N}(150,\ 20)$

$\mathrm{P}(S_1+S_2+S_3+S_4+S_5 < 160) = 0.98733 \approx 0.987$

(ii)

$S_1+S_2+S_3-2L \sim \mathrm{N}(3\times 30-2\times 50,\ 3\times 2^2+2^2\times 5^2)$

$S_1+S_2+S_3-2L \sim \mathrm{N}(-10,\ 112)$

$P(S_1+S_2+S_3 < 2L) = P(S_1+S_2+S_3-2L < 0) = 0.82765 \approx 0.828$

(iii)

The radii of the large and small round tables are independent of one another.

(iv)

Let X be the random variable "number of large tables, out of 12, with radius less than 40 cm".

$X \sim \mathrm{B}(12,\ \mathrm{P}(L < 40))$

$X \sim \mathrm{B}(12,\ 0.022750)$

$\mathrm{P}(X \geq 2) = 1-\mathrm{P}(X \leq 1)$

$= 1-0.97064$

$= 0.029357$

≈ 0.0294

(v)

Let Y be the random variable "radius of a medium sized table in cm"

$\mathrm{P}(Y \geq 44) = 0.20$

$\mathrm{P}(Y < 44) = 0.80$

$P\left(Z < \dfrac{44-\mu}{\sigma}\right) = 0.80$

$\dfrac{44-\mu}{\sigma} = 0.84162$

$\mu = 44-0.84162\sigma ----(1)$

$\mathrm{P}(Y < 40) = 0.30$

$P\left(Z < \dfrac{40-\mu}{\sigma}\right) = 0.30$

$\dfrac{40-\mu}{\sigma} = -0.52440$

$\mu = 40+0.52440\sigma ----(2)$

Solving (1) and (2),

$44-0.84162\sigma = 40+0.5244\sigma$

$4 = 1.3660\sigma$

$\sigma = 2.9283 \approx 2.93$

$\mu = 41.535 \approx 41.5$

Q2

8 Let X kg and Y kg be the mass of a randomly chosen D25 durian and Musang Queen durian respectively.

$$X \sim \mathrm{N}(1.5, 0.02^2), \qquad Y \sim \mathrm{N}(1.8, 0.035^2)$$

8ii Let $T = 9(X_1 + X_2 + X_3) + 18(Y_1 + Y_2)$

$$\begin{aligned}\mathrm{E}(T) &= \mathrm{E}\left[9(X_1 + X_2 + X_3) + 18(Y_1 + Y_2)\right] \\ &= 9(3)(1.5) + 18(2)(1.8) \\ &= 105.3\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(T) &= \mathrm{Var}\left[9(X_1 + X_2 + X_3) + 18(Y_1 + Y_2)\right] \\ &= (9)^2(3)(0.02)^2 + (18)^2(2)(0.035)^2 \\ &= 0.891\end{aligned}$$

$$T \sim \mathrm{N}(105.3, 0.891)$$

$$\begin{aligned}\mathrm{P}(T > 107) &= 0.035852 \\ &= 0.0359 \text{ (3 s.f)}\end{aligned}$$

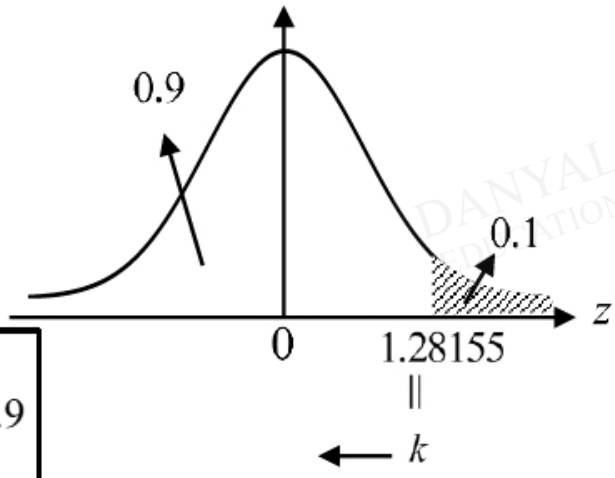
8iii The masses of all the durians are independent of each other.

8iii Let $\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$

$$\overline{X} \sim \mathrm{N}\left(1.5, \frac{0.02^2}{n}\right)$$

Given $\mathrm{P}(\overline{X} > m) \geqslant 0.1$

$$\mathrm{P}\left(Z > \frac{m - 1.5}{0.02/\sqrt{n}}\right) \geqslant 0.1$$



$$\Rightarrow \mathrm{P}\left(Z < \frac{m - 1.5}{0.02/\sqrt{n}}\right) \leqslant 0.9$$

From the GC, $\mathrm{P}(Z < 1.28155) = 0.9$

$$\therefore \frac{m-1.5}{0.02/\sqrt{n}} \leqslant 1.28155$$

$$\Rightarrow (m-1.5)\sqrt{n} \leqslant 0.025631$$

when $m = 1.51$

$$\Rightarrow (1.51-1.5)\sqrt{n} \leqslant 0.025631$$

$$\Rightarrow n \leqslant 6.5695$$

Largest value of n is 6

Q3

(i)	Let X denotes the diameter of bolt from manufacturer A. $X \sim \mathrm{N}(1.56, 0.16^2)$ Let Y denotes the diameter of bolt from manufacturer B. $Y \sim \mathrm{N}(\mu, 0.16^2)$ $\mathrm{P}(Y < 1.52) = 0.242$ $\mathrm{P}(Z < \frac{1.52-\mu}{0.16}) = 0.242$ $\frac{1.52-\mu}{0.16} = -0.6998836 \Rightarrow \mu = 1.63198 = 1.632$	Candidates are expected to show full working for this part as it is a 'show' question. Standardising is the preferred method. Candidates who used graphical method using did not explain the graphs used and how the final answer is attained. Tables should not be used when dealing with a non-integer value.
(ii)	$W = X - Y \sim \mathrm{N}(1.56-1.632, 0.16^2+0.16^2)$ $\mathrm{P}(\lvert W \rvert < 0.1) = \mathrm{P}(-0.1 < W < 0.1) = 0.326$	Quite a few candidates find the distribution of $\lvert W \rvert$ instead of W, which is conceptually incorrect, and hence leading to a wrong answer.
(iii)	$X_1 + X_2 + X_3 + X_4 + X_5 - 5Y \sim \mathrm{N}(5(1.56)\text{-}5(1.632),\ 5(0.16^2) + 5^2(0.16)^2)$ $\mathrm{P}(X_1 + X_2 + X_3 + X_4 + X_5 > 5Y)$ $= \mathrm{P}(X_1 + X_2 + X_3 + X_4 + X_5 - 5Y > 0)$ $= 0.341$	Mistakes on calculating the correct variance were not as common this time round as compared to Midyear exams. However, poor representation of the variables is still commonly seen.
(iv)	$\mathrm{P}(X < 1.52) = 0.40129$ Prob. Req'd $= (0.44)(0.4012) + 0.56(0.242)$ $= 0.3120876$ $= 0.312$	Many candidates were unable to tackle this part. Again, as this is a 'show' question, candidates are expected to work out $\mathrm{P}(X < 1.52) = 0.40129$.