A Level H2 Math

Normal Distribution Test 4

Q1

An online survey revealed that 34.1% of junior college students spent between 3 to 3.8 hours on their mobile phones daily. Assuming that the amount of time a randomly chosen junior college student spends on mobile phones daily follows a normal distribution with mean 3.4 hours and standard deviation σ hours, show that $\sigma = 0.906$, correct to 3 decimal places. [3]

Find the probability that

(i) four randomly chosen students each spend between 3 to 3.8 hours daily on their mobile phones. [1]

(ii) the total time spent on their mobile phones daily by the three randomly chosen junior college students is less than twice that of another randomly chosen junior college student. [3]

(iii) State an assumption required for your calculations in **(i)** and **(ii)** to be valid. [1]

N samples, each consisting of 50 randomly selected junior college students, are selected. It is expected that 15 of these samples will have a mean daily time spent on mobile phones greater than 3.5 hours.

(iv) Estimate the value of N. [4]

Q2

In order to recruit the best possible employees, a large corporation has designed an entrance test that consists of three components, namely Logical Reasoning, Personality and Communication. The scores obtained by candidates in each of the three components are independent random variables L, P and C which are normally distributed with means and standard deviations as shown in the table.

	Mean	Standard deviation
Logical Reasoning, L	35.2	5.2
Personality, P	24.6	3.8
Communication, C	29.3	4.3

(i) For a particular role in the corporation, the Logical Reasoning and the Personality scores of a candidate is valued and hence a special score of $3L+2P$ is computed.

(a) Find the special score that is exceeded by only 1% of candidates taking the test. Leave your answer in 1 decimal place. [4]

(b) Five candidates are selected randomly. Find the probability that three of them obtained a special score of more than 150, and the other two obtained less than 140. [3]

(ii) For another role in the corporation, a candidate must achieve a result such that his special score of $3L+2P$ differs from $5C$ by less than 25. Find the percentage of candidates who will be able to achieve this. [4]

Q3

(a) In Country S, each household's monthly income per capita is calculated by taking the gross household income divided by the total number of members in the household. It is assumed that this amount for a randomly chosen household consisting of 3 members follows a normal distribution with mean \$2601 and standard deviation \$768.

(i) The Ministry of Education offers financial aid to students from households consisting of 3 members each and with a household monthly income per capita lower than \$1800. Find the probability that a randomly chosen household with 3 members does not qualify for financial aid. [1]

(ii) It is found that there is a 50% chance that a randomly chosen household with 3 members has a gross household income between \$5000 and \$$a$, where $a > 5000$. Find the value of a, correct to the nearest dollar. [3]

(b) Mr Tan is self-employed and his monthly income follows a normal distribution with mean \$6000 and standard deviation \$1000 whereas Mrs Tan works part-time and earns a fixed amount of \$1500 a month. Their family's monthly expenditure follows a normal distribution with mean $\propto$ dollars and standard deviation 650 dollars.

(i) It was found that 10% of the time they spend more than \$5900 in a month. Find the value of $\propto$, correct to the nearest dollar. [2]

(ii) Mr and Mrs Tan save the remaining amount of their income after deducting their expenditure every month. Find the probability that their monthly savings in August and in September differ by more than \$1000. [4]

(iii) State an assumption needed for your calculation in part **(b)(ii)**. [1]

Answers

Normal Distribution Test 4

Q1

Let X denote the random variable representing the amount of time a randomly chosen junior college student spends on mobile phones each day.

$\therefore X \sim N(3.4, \sigma^2)$

$P(3 < X < 3.8) = 0.341$

$P\left(\frac{3-3.4}{\sigma} < Z < \frac{3.8-3.4}{\sigma}\right) = 0.341$

$P\left(\frac{-0.4}{\sigma} < Z < \frac{0.4}{\sigma}\right) = 0.341$

$\Rightarrow P\left(Z < \frac{-0.4}{\sigma}\right) = \frac{1-0.341}{2} = 0.3295$

From GC, $\frac{-0.4}{\sigma} = -0.4412942379$

$\Rightarrow \sigma = 0.90642 = 0.906$ (3 dp)

(i) Probability required $= (0.341)^4$
$= 0.0135$ (3 sf)

(ii) Probability required $= P(X_1 + X_2 + X_3 < 2X_4)$
$= P(X_1 + X_2 + X_3 - 2X_4 < 0)$

$X_1 + X_2 + X_3 - 2X_4 \sim N(3.4\times3 - 2\times3.4, 0.90642^2\times3 + 2^2\times0.90642^2)$

i.e. $X_1 + X_2 + X_3 - 2X_4 \sim N(3.4, 5.75118)$

$\therefore$ From GC, $(X_1 + X_2 + X_3 - 2X_4 < 0) = 0.0781$ (3 sf)

(iii) Assumption: The amount of time spent by a randomly chosen student on mobile phones is independent of the amount of time spent by another randomly chosen student.

(iv) $\overline{X} \sim N\left(3.4, \frac{0.90642^2}{50}\right)$

From GC, $P\left(\overline{X} > 3.5\right) = 0.217663$

Since expected number of samples with mean time exceeding 3.5 hours = 15, then $0.217663 \times N = 15$

$\Rightarrow N = 68.9 \approx 69$

Q2

(i)(a)

Given: $L \sim N\left(35.2, 5.2^2\right)$ $P \sim N\left(24.6, 3.8^2\right)$ $C \sim N\left(29.3, 4.3^2\right)$

Let $T = 3L + 2P$.

$E(T) = 3 \times 35.2 + 2 \times 24.6 = 154.8$

$Var(T) = 3^2 \times 5.2^2 + 2^2 \times 3.8^2 = 301.12$

$\therefore T \sim N(154.8, 301.12)$

Let a be the required score exceed by 1% of the candidates.

$P(T > a) = 0.01$

$\Rightarrow P(T \le a) = 0.99$

Using GC, $a = 195.2$ (1 dec pl)

(i)(b)

Required probability

$= [P(T > 150)]^3 [P(T < 140)]^2 \times \left(\frac{5!}{2!3!}\right)$

$= 0.0875$ (3 sig fig)

(ii)

Consider $A = 3L + 2P - 5C$

$E(A) = 154.8 - 5(29.3) = 8.3$

$Var(A) = 301.12 + 5^2\left(4.3^2\right) = 763.37$

$\therefore A \sim N(8.3, 763.37)$

Required probability

$= P(|A| < 25)$

$= P(-25 < A < 25)$

$= 0.613$ (3 sig fig)

Required percentage = 61.3%

Q3

(ai)

Let X be the random variable denoting the household income per capita in dollars of a randomly chosen family in Country S.

Then $X \sim \mathrm{N}(2601,\ 768^2)$.

$\mathrm{P}(X > 1800) = 0.852$ (3s.f.)

(aii)

Let Y be the random variable denoting the gross income in dollars of a randomly chosen family with 3 family members.

$Y = 3X \sim N(3\times 2601, 9\times 768^2)$

$\Rightarrow Y \sim \mathrm{N}(7803, 9\times 768^2)$

$\mathrm{P}(5000 < Y < a) = 0.5$

$\Rightarrow \mathrm{P}(Y < a) - \mathrm{P}(Y < 5000) = 0.5$

$\Rightarrow \mathrm{P}(Y < a) = 0.5 + \mathrm{P}(Y < 5000) = 0.61188$

$\Rightarrow \mathrm{P}(Y < a) = 0.61188$

$\Rightarrow a = 8458$ (to nearest dollars)

<u>Alternative:</u>

$\mathrm{P}(5000 < Y < a) = 0.5$

$\mathrm{P}\left(\frac{5000}{3} < X < \frac{a}{3}\right) = 0.5$ since $Y = 3X$

$\Rightarrow \mathrm{P}\left(X < \frac{a}{3}\right) - \mathrm{P}\left(X < \frac{5000}{3}\right) = 0.5$

$\Rightarrow \mathrm{P}\left(X < \frac{a}{3}\right) = 0.5 + \mathrm{P}\left(X < \frac{5000}{3}\right) = 0.61188$

$\Rightarrow \frac{a}{3} = 2819.28$ (2dp)

$\Rightarrow a = 8458$ (to nearest dollar)

(bi)

Let V be the random variable denoting the family's monthly expenditure in dollars.

Then $V \sim \mathrm{N}(\mu,\ 650^2)$

$\mathrm{P}(V > 5900) = 0.1$

$\Rightarrow \mathrm{P}(V < 5900) = 0.9$

$\Rightarrow \mathrm{P}(Z < \frac{5900-\mu}{650}) = 0.9$

From GC: $\mathrm{P}(Z < 1.28155) = 0.9$

$\Rightarrow \frac{5900-\mu}{650} = 1.28155$

$\Rightarrow \mu = 5067$ (to the nearest dollar)

(bii)

Let W be the random variable denoting the family's monthly saving in dollars.
$W = T + 1500 - V$, where T denotes Mr Tan's monthly income.

Then $W \sim N(7500 - 5067, 1000^2 + 0 + 650^2)$

i.e. $W \sim N(2433, 1422500)$

and $W_1 - W_2 \sim N(0, 2845000)$

$\Rightarrow \mathrm{P}(|W_1 - W_2| > 1000)$

$= 2\mathrm{P}(W_1 - W_2 < -1000)$

$= 0.553$ (3sf)

(biii)

It is assumed that Mr Tan's income and the family's expenditure in a particular month are independent. Alternatively, Mr Tan's income and the family's expenditure in a month are independent of what he earned and how much the family spent in another month.