A Level H2 Math

Normal Distribution Test 3

Q1

The independent random variables X and Y are normally distributed with the same mean 7 but different variances $\text{Var}(X)$ and $\text{Var}(Y)$, respectively. It is given that $\text{P}(X < 10) = \text{P}(Y > 6)$.

(i) Show that $\text{Var}(X) = 9\text{Var}(Y)$. [3]

(ii) If $\text{Var}(Y) = 1$, find $\text{P}(X < 9)$. [2]

Q2

In this question you should state clearly the values of the parameters of any normal distribution you use.

The mass of a tomato of variety A has normal distribution with mean 80 g and standard deviation 11 g.

(i) Two tomatoes of variety A are randomly chosen. Find the probability that one of the tomatoes has mass more than 90 g and the other has mass less than 90 g. [3]

The mass of a tomato of variety B has normal distribution with mean 70 g. These tomatoes are packed in sixes using packaging that weighs 15 g.

(ii) The probability that a randomly chosen pack of 6 tomatoes of variety B including packaging, weighs less than 450 g is 0.8463. Show that the standard deviation of the mass of a tomato of variety B is 6 g, correct to the nearest gram. [4]

(iii) Tomatoes of variety A are packed in fives using packaging that weighs 25 g. Find the probability that the total mass of a randomly chosen pack of variety A is greater than the total mass of a randomly chosen pack of variety B, using 6 g as the standard deviation of the mass of a tomato of variety B. [5]

Q3

Males and females visiting an amusement park have heights, in centimetres, which are normally distributed with means and standard deviations as shown in the following table:

	Mean (cm)	Standard deviation (cm)
Male	165	12
Female	155	σ

It is found that 38.29% of the females have heights between 150 cm and 160 cm.

(i) Show that $\sigma = 10.0$ cm, correct to 3 significant figures. [2]

(ii) Find the probability that the height of a randomly chosen female is within 20 cm of three-quarter the height of a randomly chosen male. State an assumption that is necessary for the calculation to be valid. [4]

The amount, $\$X$, a visitor has to pay for a popular ride in the park is \$10 if the visitor's height is at least 120 cm but less than 150 cm, and $\$m$ if the visitor's height is 150 cm and above. If the visitor's height is less than 120 cm, he/she does not need to pay for the ride.

(iii) Assuming that a visitor purchasing a ticket for the ride is equally likely to be a male or female, find in terms of m, the probability distribution of X. [3]

Given that the expected amount a visitor will pay for a ride is \$17.93, show that $m = 20.00$, correct to 2 decimal places. [1]

(iv) Three visitors were randomly chosen. Find the probability that the total amount they paid for a ride together is more than \$40. [3]

Answers

Normal Distribution Test 3

Q1

(i)

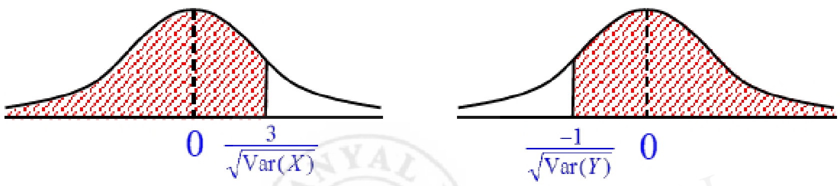
$X \sim \mathrm{N}(7, \mathrm{Var}(X))$

$Y \sim \mathrm{N}(7, \mathrm{Var}(Y))$

$\mathrm{P}(X < 10) = \mathrm{P}(Y > 6)$

$$\mathrm{P}\left(Z < \frac{10-7}{\sqrt{\mathrm{Var}(X)}}\right) = \mathrm{P}\left(Z > \frac{6-7}{\sqrt{\mathrm{Var}(Y)}}\right)$$

$$\mathrm{P}\left(Z < \frac{3}{\sqrt{\mathrm{Var}(X)}}\right) = \mathrm{P}\left(Z > \frac{-1}{\sqrt{\mathrm{Var}(Y)}}\right)$$



$$\therefore \frac{3}{\sqrt{\mathrm{Var}(X)}} = -\left(\frac{-1}{\sqrt{\mathrm{Var}(Y)}}\right)$$

$$3\sqrt{\mathrm{Var}(Y)} = \sqrt{\mathrm{Var}(X)}$$

Hence $\mathrm{Var}(X) = 9\mathrm{Var}(Y)$ (shown)

(ii)

$\mathrm{Var}(X) = 9(1) = 9$

$X \sim \mathrm{N}(7, 9)$

$\therefore \mathrm{P}(X < 9) = 0.748$

Q2

Let A g be the mass of a tomato of variety A and B g be the mass of a tomato of variety B.

$A \sim \mathrm{N}(80, 11^2)$

(i) $\mathrm{P}(A > 90) = 0.18165$

P(one greater than 90 g and one less than 90 g)

$= 2 \times \mathrm{P}(A > 90) \times \mathrm{P}(A < 90)$

$= 2(0.18165)(1 - 0.18165)$

$= \underline{\underline{0.297}}$ (3 sf)

Let $B \sim \mathrm{N}(70, \sigma^2)$.

(ii) Let $S_B = B_1 + B_2 + \ldots + B_6 + 15$

$S_B \sim \mathrm{N}(6 \times 70 + 15, 6\sigma^2)$ i.e., $\mathrm{N}(435, 6\sigma^2)$

$\mathrm{P}(S_B < 450) = 0.8463$

$\mathrm{P}\left(Z < \dfrac{450 - 435}{\sqrt{6}\,\sigma}\right) = 0.8463$

$\dfrac{15}{\sqrt{6}\,\sigma} = 1.0207$

$\sigma = \dfrac{15}{1.0207\sqrt{6}} = 6$ (nearest g) (Shown)

(iii) $S_B \sim \mathrm{N}(435, 216)$

Let $S_A = A_1 + A_2 + \ldots + A_5 + 25$

$S_A \sim \mathrm{N}(5 \times 80 + 25, 5 \times 11^2)$ i.e., $\mathrm{N}(425, 605)$

$S_A - S_B \sim \mathrm{N}(425 - 435, 605 + 216) = \mathrm{N}(-10, 821)$

$\mathrm{P}(S_A > S_B) = \mathrm{P}(S_A - S_B > 0)$

$= \underline{\underline{0.364}}$ (3 sf)

Q3

10(i) Let M denote the random variable: Height of a male visitor in cm. $M \sim N(165, 12^2)$
Let F denote the random variable: Height of a female visitor in cm. $F \sim N(155, \sigma^2)$

$$P(150 < F < 160) = 0.3829$$
$$P(-\frac{5}{\sigma} < Z < \frac{5}{\sigma}) = 0.3829$$
$$P(Z < -\frac{5}{\sigma}) = \frac{1-0.3829}{2} = 0.30855$$

From G.C. $-\frac{5}{\sigma} = -0.4999646$

$\Rightarrow \sigma = 10.0$ cm (3 sig. fig.)

(ii) $\frac{3}{4}M - F \sim N(\frac{3}{4}\times 165 - 155, \frac{9}{16}\times 12^2 + 10^2) = N(-31.25, 181)$

$$P(\left|\frac{3}{4}M - F\right| \le 20) = P(-20 \le \frac{3}{4}M - F \le 20)$$
$$= 0.201 \quad (3 \text{ sig. fig.})$$

Assumption: The heights of all male and female visitors are independent of one another.

(iii) Probability Distribution of X:

x (in \$)	P(X = x)
0	$\frac{1}{2}P(M<120)+\frac{1}{2}P(F<120)=0.00016056$
10	$\frac{1}{2}P(120 \le M < 150)+\frac{1}{2}P(120 \le F < 150)=0.20693$
m	$\frac{1}{2}P(M \ge 150)+\frac{1}{2}P(F \ge 150)=0.79291$

Given $E(X) = 17.93 = 0(0.00016056)+10(0.20693)+m(0.79291)$
$\Rightarrow m = 20.00$ (shown)

(iv) $P(X_1 + X_2 + X_3 > 40) = P(20, 20, 20) + 3.\ P(20, 20, 10)$
$= 0.889$