A Level H2 Math

Hypothesis Testing Test 6

Q1

In an assembly line, a machine is programmed to dispense shampoo into empty bottles and the volume of shampoo dispensed into each bottle is a normally distributed continuous random variable X. Under ordinary conditions, the expected value of X is 325 ml.

(i) After a routine servicing of the machine, the assembly manager suspects that the machine is dispensing more shampoo than expected. A random sample of 60 bottles is taken and the data is as follows:

Volume of shampoo (correct to nearest ml)	324	325	326	327	328	329	330
Number of bottles	16	20	9	8	4	1	2

Find unbiased estimates of the population mean and variance, giving your answers to 2 decimal places. [2]

Test, at the 5% significance level, whether the assembly manager's suspicion is valid. [4]

Explain what it meant by the phrase 'at 5% significance level' in the context of the question. [1]

(ii) Due to the assembly manager's suspicion, the machine is being recalibrated to dispense 325 ml of shampoo. Another random sample of 50 is taken and a two-tailed test, at the 5% significance level, concluded that the recalibration is done accurately. Given that the volume of shampoo dispensed into each bottle is normally distributed with standard deviation 1.2 ml, find the set of values the mean volume of the 50 bottles can take, giving your answers to 2 decimal places. [4]

Q2

(a) College students intending to further their studies overseas have to sit for a mandatory Overseas Universities Test (OUT). Researcher Mr Anand wishes to find out if male college students tend to score higher for OUT compared to female college students. Mr Anand's colleague randomly selects 150 male and 150 female students from the combined student population of three particular colleges near his home to form a sample of 300 college students for the research. Explain whether this sample is a random sample. [2]

(b) The mean OUT score for all college students in 2016 is 66.
Mr Anand randomly selects 240 college students taking OUT in 2017 and their scores, x, are summarised in the following table:

Score, x	60	65	68	70	75	80
Frequency, f	40	90	63	27	18	2

(i) Write down the unbiased estimates of the population mean and variance of the OUT scores for the college students in 2017. [1]

(ii) Test, at the 10% level of significance, whether the mean OUT score for all college students in 2017 is higher than the mean score attained in 2016. [4]

(iii) Explain what is meant by the phrase "10% level of significance" in this context. [1]

(iv) Mr Anand draws a new sample of 240 male college students. Using the unbiased estimate for the population variance computed in **(i)**, find the range of values for the sample mean $\overline{x}$ that is required for this new sample to achieve a different conclusion from that in **(ii)**. [4]

(c) The 2017 OUT scores of the male and female college students are independent and assumed to be normally distributed with means and standard deviations as shown in the following table:

	Mean	Standard deviation
Male College Students	64	5.5
Female College Students	66	3.5

Mr Beng and Miss Charlene both scored 70. Explain who performed better relative to their respective gender cohort. [2]

Q3

In a factory, the average time taken by a machine to assemble a smartphone is 53 minutes. A new assembly process is trialled and the time taken to assemble a smartphone, x minutes, is recorded for a random sample of 60 smartphones. The total time taken was found to be 3129 minutes and the variance of the time was 18.35 minutes^2.

The engineer wants to test whether the average time taken by a machine to assemble a smartphone has decreased, by carrying out a hypothesis test.

(i) Explain why the engineer is able to carry out a hypothesis test without assuming anything about the distribution of the times taken to assemble a smartphone. [1]

(ii) Find unbiased estimates of the population mean and variance and carry out the test at the 10% level of significance. [6]

(iii) Explain, in the context of the question, the meaning of 'at 10% level of significance'. [1]

After several trials, the engineer claims that the average time taken by a machine to assemble a smartphone is 45 minutes using the new assembly process. The internal control manager wishes to test whether the engineer's claim is valid. The population variance of the time taken to assemble a smartphone using the new assembly process may be assumed to be 9 minutes^2. A random sample of 50 smartphones is taken.

(iv) Find the range of values of the mean time of this sample for which the engineer's claim would be rejected at the 10% significance level. [4]

Answers

Hypothesis Testing Test 6

Q1

(i) Using GC,
Unbiased estimate of population mean is $\bar{x} = 325.58$ (2 d.p.)
Unbiased estimate of population variance is $s^2 = 1.5326^2 = 2.35$ (2 d.p.)

Let μ denote the population mean volume of shampoo dispensed by the machine.

Given $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $\therefore \bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$

H_0: $\mu = 325$
H_1: $\mu > 325$

Test statistic: $Z = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$

Level of significance: 5%

Alternatively,

Reject H_0 if z-value > 1.6449

Reject H_0 if $p -$ value < 0.05

Under H_0, using GC,

$p -$ value $= 0.00160$ (3 s.f) or 0.00169 (3 s.f)
Conclusion:
Since $p -$ value $= 0.00169 < 0.05$, we **reject H_0** and conclude that there is **sufficient evidence**, at the 5% significance level, that the mean volume dispensed is more than 325 ml.

Thus, the assembly manager's suspicion is valid at 5% level of significance.

There is a probability of 0.05 of concluding that the mean volume of shampoo dispensed is more than 325 ml when in fact, it is 325 ml.

(ii) H_0: $\mu = 325$
H_1: $\mu \neq 325$

Given $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $\therefore \bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$

Test statistic: $Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

Level of significance: 5%
Since H_0 is not rejected,

$$-1.9600 < z - \text{value} < 1.9600$$

$$-1.9600 < \frac{\bar{x} - 325}{\frac{1.2}{\sqrt{50}}} < 1.9600$$

$$324.67 < \bar{x} < 325.33 \quad (2 \text{ d.p.})$$

$\left\{\bar{x} \in \mathbb{R} : 324.67 < \bar{x} < 325.33\right\}$

Q2

(a) Sample is non-random/biased since students from other colleges do not have any chance of being selected.	Need to mention that the probability of a student being selected into the sample is not the same for every student taking OUT in 2017 since students from other colleges do not have any chance of being selected.
(b)(i) Using GC, unbiased estimate of population mean, $\bar{x} = 66.391$ $= 66.4$ (to 3 s.f.) and unbiased estimate of population variance, $s^2 = 4.1048^2 = 16.8$ (to 3 s.f.)	A number of students forgot to square the value 4.1048.
(b)(ii) Let μ be the population mean OUT score of students in 2017 . $H_0 : \mu = 66$ $H_1 : \mu > 66$ Level of significance: 10% Test Statistic: $\dfrac{\overline{X} - \mu}{s/\sqrt{n}} \sim N(0,1)$ by Central Limit Theorem since $n = 240$ is large. Under H_0, with $\bar{x} = 66.391$, $s = 4.1048$, $n = 240$, we have $p = 0.0697$ Since p-value < 0.1, we reject H_0 There is sufficient evidence at the 10% level of significance to conclude that the mean OUT score of male college students is higher than 66.	
(iii) There is a probability of 0.1 of wrongly concluding that the mean OUT score of male college students is higher than 66.	10% chance or probability of 0.1

(iv) $H_0 : \mu = 66$ $H_1 : \mu > 66$ Level of significance: 10% Do not reject H_0, $p > 0.10$ $$\frac{\bar{x}-66}{4.1048/\sqrt{240}} < 1.28155$$ $\Rightarrow \quad \bar{x} > 66.3396$ $\therefore \quad \bar{x} > 66.3$ (to 3 s.f.)	A number of students wrote the critical value as −1.28155 (invnorm(0.10)), without paying attention to H_1.
(c) Let M and F be the OUT scores of male and female college students respectively Given: $M \sim N(64, 5.5^2)$ and $F \sim N(66, 3.5^2)$ $P(M \le 70) = 0.86234$ $\Rightarrow$ Mr Beng is in the 86th percentile of male students (or Mr Beng scored higher than 86% of the male cohort) $P(F \le 70) = 0.87345$ $\Rightarrow$ Miss Charlene is in the 87th percentile of female students $\therefore$ Miss Charlene performed better relative to her gender cohort.	

Marker's comments

(a) Many students were able to recognise that the sample is not random. However, many of them were not able to give precise explanation. Wrong responses included mentioning the proportion of males and females, abilities of students in colleges which were not apparent in the question.

(b) (i) This part requires students to write out the unbiased estimates of the population mean and variance upon entering the data into the GC. Many students were not able to retrieve the correct unbiased estimate of population variance, they wrote down the sample variance instead. A number of students applied the formulas to find the unbiased estimates using the statistics, some with more success obtaining the values, some used the wrong statistics or wrong formula and did not obtain the correct values.

(b) (ii) Most students gained full marks here. Those who did not get (b)(i) correct would have lost some marks but not all if they have written the correct hypotheses, and conclusion given in context.

(b) (iii) This part was badly done. Many students were not able to explain precisely the phrase "10% level of significance", some students seemed to have problem remembering the definition.

(b) (iv) Students have some grasp of what was required, there were many varied errors in setting up the inequality to achieve a different conclusion from (b)(ii).

(c) Many students attempted to answer this part with lengthy paragraphs about standard deviations of the distribution of OUT scores of the male and female students. Many failed to explain using percentiles or probabilities of Beng and Charlene scoring 70 marks and above/or below. A large number of students thought they were computing the probability of Beng/Charlene scoring 70 marks using the normalpdf function. They did not understand that the probability is defined as area under the normal curve and hence the value they obtained were not able to explain who performed better in their cohort.

Q3

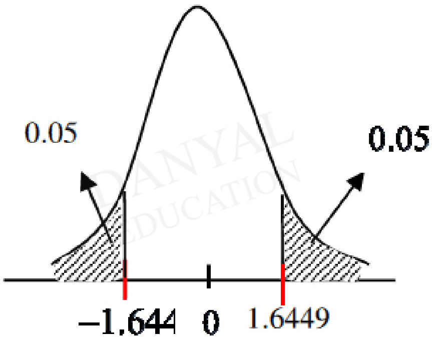
10(i)	Since n is large, by Central Limit Theorem, the sample mean time for 60 smartphones is approximately normal. Hence the assumption that the time taken by a machine to assembly a smartphone is not necessary.
(ii)	Unbiased estimate for population mean μ is $\bar{x}$ $=\frac{3129}{60}=52.15$ Unbiased estimate for population variance σ^2 is s^2 $=\frac{60}{59}(18.35)$ $=18.661$ $=18.7$ (3sf) $H_0: \mu = 53$ $H_1: \mu < 53$ Under H_0, the test statistic $Z=\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N(0,1)$ approx. by CLT, where $\mu = 53, s=\sqrt{18.661}, \bar{x}=52.15,\ n=60$. By GC, p-value $=0.0637(3\text{ s.f.})$. Since p-value < 0.1, we reject H_0 and conclude at 10% level that there is sufficient evidence that average time taken by a machine to assembly a smartphone has reduced.
(iii)	There is a probability of 0.1 of concluding that the average time taken by a machine to assembly a smartphone has decreased when the average time taken by a machine to assembly a smartphone is 53 minutes.

(iv) $H_0 : \mu = 45$

$H_1 : \mu \neq 45$

Under H_0, the test statistic $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$ approx. by CLT, where

$\mu = 45, \sigma = \sqrt{9},\ n = 50$.



Since H_0 is rejected,

$\dfrac{\overline{x} - 45}{\sqrt{9} / \sqrt{50}} < -1.6449$ or $\dfrac{\overline{x} - 45}{\sqrt{9} / \sqrt{50}} > 1.6449$

$\overline{x} < 44.3021$ $\quad$ $\overline{x} > 45.698$

$\overline{x} < 44.3 \text{ (3 s.f.)}$ $\quad$ $\overline{x} > 45.7 \text{ (3 s.f.)}$

Range of values of $\overline{x}$:

$\overline{x} < 44.3 \text{ (3 s.f.)}$ or $\overline{x} > 45.7 \text{ (3 s.f.)}$