A Level H2 Math

Hypothesis Testing Test 4

Q1

A company manufactures bottles of iced coffee. Machines A and B are used to fill the bottles with iced coffee.

(i) Machine A is set to fill the bottles with 500 ml of iced coffee. A random sample of 50 filled bottles was taken and the volume of iced coffee (x ml) in each bottle was measured. The following data was obtained

$$\sum x = 24965 \quad \sum (x-\bar{x})^2 = 365$$

Calculate unbiased estimates of the population mean and variance. Test at the 2% level of significance, whether the mean volume of iced coffee per bottle is 500 ml. [6]

(ii) The company claims that Machine B filled the bottles with μ_0 ml of iced coffee. A random sample of 70 filled bottles was taken and the mean is 489.1 ml with standard deviation 4 ml. Find the range of values of μ_0 for which there is sufficient evidence for the company to have overstated the mean volume at the 2% level of significance. [5]

Q2

The mass of strawberry jam in a randomly chosen jar follows a normal distribution and has a mean mass of 200 grams. A retailer suspects that the mean mass of the strawberry jam is being overstated. He takes a random sample of 30 jars of strawberry jam and weighs the content, x grams, in each jar. The results are summarized as follows.

$$\sum (x-200) = -66 \quad \text{and} \quad \sum (x-200)^2 = 958$$

(i) Test at 2% significance level, whether the retailer's suspicion is justifiable. [6]

(ii) Explain, in this context, the meaning of 'at 2% significance level'. [1]

(iii) Suppose the retailer now decides to test whether the mean mass differs from 200 grams at 2% significance level. Without carrying out the test, explain whether the conclusion would change in part **(i)**. [1]

The manufacturing process has now been improved and the population standard deviation is 3.5 grams. The retailer selects a new random sample of 20 jars of strawberry jam and the sample mean is found to be k grams. Find the range of possible values of k so that the retailer's suspicion that the mean mass differs from 200 grams is not justified at the 2% significance level. Give your answer correct to one decimal place. [4]

Q3
The power consumption of a randomly chosen Effixion laptop has a normal distribution. The salesman at Elf Superstore claims that the average power consumption of an Effixion laptop is 100 watts. The power consumption, w watts, is measured for a random sample of 50 Effixion laptops. The results are summarised as follows.

$$\sum(w-100)=26 \qquad \sum(w-100)^2=273$$

Test whether this data provides evidence at the 3% level of significance, that the salesman has made an understatement. [6]

The power consumption of another random sample of 50 Effixion laptops is measured. It is found that the sample variance is 6.25. Using this sample only, find the set of values of $\overline{w}$, correct to 2 decimal places, for which the test would result in the rejection of the null hypothesis in favour of the alternative hypothesis at the 1% level of significance. [4]

Answers

Hypothesis Testing Test 4

Q1

(i) Let X be the random variable denoting volume of the randomly chosen iced coffee bottle in ml from Machine A.

$$\bar{x} = \frac{24965}{50} = 499.3$$

Unbiased estimate of population variance

$$s^2 = \frac{1}{n-1}\sum (x-\bar{x})^2 = \frac{50}{49}\left(\frac{365}{50}\right) = \frac{365}{49} = 7.4489 \approx 7.45$$

$H_0 : \mu = 500$

$H_1 : \mu \neq 500$

Two tailed Z test at 2% level of significance

Under H_0, since the sample size of 50 is large, by Central Limit Theorem

$\bar{X} \sim N(500, \frac{7.4489}{50})$ approx.

From GC, p-value = 0.06974> 0.02

Conclusion: Since the p-value is more than the level of significance, we do not reject H_0 and conclude that there is insufficient evidence at 2% that the mean volume is not 500ml.

(ii) Let Y be the random variable denoting the volume of a randomly chosen iced coffee bottle in ml from Machine B.

Unbiased estimate for population variance = $\frac{70}{69}\left(4^2\right) = 16.232$

$H_0 : \mu = \mu_0$

$H_1 : \mu < \mu_0$

One tailed Z test at 2% level of significance

Under H_0, since the sample size of 70 is large, by Central Limit Theorem

$\bar{Y} \sim N\left(\mu_0, \frac{16.232}{70}\right)$ approx.

Value of test statistic, $z_{test} = \dfrac{489.1 - \mu_0}{\sqrt{\dfrac{16.232}{70}}}$

For H_0 to be rejected,

p-value ≤ 0.02

$$\frac{489.1 - \mu_0}{\sqrt{\frac{16.232}{70}}} \leq -2.053748911$$

$\mu_0 \geq 490$ (to 3 s.f.)

Q2

(i)

Let X be the random variable denoting the mass of strawberry jam, in grams, in a randomly chosen jar.

Unbiased estimate of population mean

$$\bar{x} = \frac{-66}{30} + 200 = 197.8$$

Unbiased estimate of population variance

$$s^2 = \frac{1}{29}\left[958 - \frac{(-66)^2}{30}\right] = 28.02759$$

$H_0 : \mu = 200$

$H_1 : \mu < 200$

Test at 2% significance level

Assume H_0 is true. $\bar{X} \sim N\left(200, \frac{28.02759}{30}\right)$

Test statistic: $Z = \frac{\bar{X} - 200}{\sqrt{28.02759/30}} \sim N(0,1)$

Using GC, p-value = 0.011420121 < 0.02

Reject H_0 and conclude that there is sufficient evidence at 2% level of significance that the mean mass of strawberry jam in each jar is overstated. Therefore the retailer's suspicion is justifiable.

(ii)

At 2% significance level means that there is a probability of 0.02 that **the test will indicate** that the mean mass of the strawberry jam in the jar is less than 200 g when in fact it is 200 g.

(iii)

$H_0 : \mu = 200$

$H_1 : \mu \neq 200$

For a two tailed test, the p-value will be twice of 0.0114 which is 0.0228. This value is now more than the 0.02 where we do not reject H_0 at 2% significance level. As such this will result in a different conclusion.

(iv)

$H_0 : \mu = 200$

$H_1 : \mu \neq 200$

Test at 2% significance level

Assume H_0 is true. $\bar{X} \sim N\left(200, \frac{3.5^2}{20}\right)$.

Test statistic: $Z = \frac{\bar{X} - 200}{\sqrt{3.5^2/20}} \sim N(0,1)$

For the retailer's suspicion that the mean mass differs from 200 g to be not justified, **do not reject H_0.**

$\Rightarrow$ z-value falls outside the critical region

$-2.32635 < z\text{-value} < 2.32635$

$$-2.32635 < \frac{k-200}{3.5/\sqrt{20}} < 2.32635$$

$$-1.82066 < k - 200 < 1.82066$$

$$198.17934 < k < 201.82066$$

$\Rightarrow 198.2 < k < 201.8$ (to 1 d.p)

Q3

Let $X = W - 100$. Then we have $\sum x = 26$, $\sum x^2 = 273$

$$\bar{w} = \frac{1}{50}\sum x + 100 = \frac{26}{50} + 100 = 100.52$$

$$s_w{}^2 = s_x{}^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right]$$

$$= \frac{1}{49}\left(273 - \frac{26^2}{50}\right)$$

$$= 5.29551$$

To test $H_0 : \mu = 100$ vs $H_1 : \mu > 100$

Perform a 1–tail test at 3% level of significance.

Under H_0, $\overline{W} \sim N\left(\mu_0, \frac{s^2}{n}\right)$ **approximately** where $\mu_0 = 100$ and $n = 50$

Using a z – test,
p – value $= 0.0550$ (3 s.f.)

Since p – value = 0.0550 > 0.03, we **do not reject** H_0 and conclude that there is **insufficient** evidence, at 3% significance level, that the salesman made an understatement on the average power consumption of the Effixion laptops.

To test $H_0 : \mu = 100$ vs $H_1 : \mu > 100$

Perform a 1–tail test at 1% level of significance.

Sample variance $= 6.25$

$$s^2 = \frac{n}{n-1}[\text{sample variance}]$$

$$= \frac{50}{49}(6.25) = 6.37755$$

In order **to reject** H_0 at 1% level of significance, we need

$P\left(\overline{W} > \bar{w}\right) \le 0.01$ where $\overline{W} \sim N\left(100, \frac{6.37755}{50}\right)$ approximately

$\Rightarrow P\left(\overline{W} < \bar{w}\right) \ge 0.99$

$\Rightarrow \bar{w} \ge 100.8308$

Or $z - value \ge z_{0.99}$

$$\frac{\bar{w} - 100}{\sqrt{\frac{6.37755}{50}}} \ge 2.3263$$

$\bar{w} \ge 100.8308$ (4 d.p.)

Set of values of $\bar{w}$ is $\{\bar{w} \in \mathbb{R} : \bar{w} > 100.83\}$ or $\{\bar{w} \in \mathbb{R} : \bar{w} \ge 100.84\}$