A Level H2 Math

Hypothesis Testing Test 2

Q1

A computer hard drive manufacturer claims that the mean usage hours before failure of their R series hard drives is 50 thousand hours. A technology columnist wishes to investigate this claim and collected the usage hours, t thousand hours for each of the 50 randomly chosen hard drives which were submitted to the local service centre for drive failures. The data is summarized as follows.

$$n = 50 \qquad \Sigma t = 2384.5 \qquad \Sigma t^2 = 115885.23$$

The technology columnist wants to use hypothesis testing to test whether the mean usage hours before failure of a hard drive is different from what the manufacturer has stated.

(i) Explain whether it is necessary for the columnist to know about the distribution of the usage hours before failure of the drives in order to carry out a hypothesis test. [1]

(ii) Find the unbiased estimates of the population mean and variance and carry out the test at 1% level of significance for the columnist. [6]

The columnist published the data and the results of the hypothesis testing in an online article.

(iii) Suggest a reason why the test result might not be useful to a reader of the article who is deciding whether to buy an R series hard drive from the manufacturer. [1]

(iv) State an alternative hypothesis that is more relevant to the decision making process and explain whether the result will differ from the earlier test carried out by the columnist at 1% level of significance. [2]

(v) State a necessary assumption that was made for all the tests carried out. [1]

Q2

Heart rate, also known as pulse, is the number of times a person's heart beats per minute. The normal heart rate of teenagers has a mean of 75 at the resting state.

Obesity is a leading preventable cause of death worldwide. It is most commonly caused by a combination of excessive food intake, lack of physical activity and genetic susceptibility. To examine the effect of obesity on heart rate, 70 obese teenagers are randomly selected and their heart rates h are measured in a resting state. The results are summarised as follows.

$$n = 70 \qquad \sum h = 5411 \qquad \sum h^2 = 426433$$

The Health Promotion Board (HPB) wishes to test whether the mean heart rate for obese teenagers differs from the normal heart rate by carrying out a hypothesis test.

(i) Explain whether HPB should use a 1-tail test or a 2-tail test. [1]

(ii) Explain why HPB is able to carry out a hypothesis test without knowing anything about the distribution and variance of the heart rates. [2]

(iii) Find the unbiased estimates of the population mean and variance, and carry out the test at the 10% level of significance for the HPB. [6]

A researcher wishes to test whether obese teenagers have a **higher** mean heart rate. He finds that the mean heart rate for 80 randomly obese teenagers is 79.4, then carries out a hypothesis test at the 10% level of significance.

(iv) Explain, with justification, how the population variance of the heart rates will affect the conclusion made by the researcher. [3]

(v) Show that the probability of any normal variable lying within one standard deviation from its mean is approximately 0.683. [1]

By considering **(iv)** and **(v)**, explain why it is likely for the researcher to reject the null hypothesis in this test if it is assumed that heart rates follow a normal distribution at the resting state. [1]

Q3

The company Snatch provides a ride-hailing service comprising taxis and private cars in Singapore. Snatch claims that the mean waiting time for a passenger from the booking time to the time of the vehicle’s arrival is 7 minutes.

To test whether the claim is true, a random sample of 30 passengers’ waiting times is obtained. The standard deviation of the sample is 2 minutes. A hypothesis test conducted concludes that there is sufficient evidence at the 1% significance level to reject the claim.

(i) State appropriate hypotheses and the distribution of the test statistic used. [3]

(ii) Find the range of values of the sample mean waiting time, $\overline{t}$. [3]

(iii) A hypothesis test is conducted at the 1% significance level whether the mean waiting time of passengers is more than 7 minutes. Using the existing sample, deduce the conclusion of this test if the sample mean waiting time is more than 7 minutes. [2]

Answers

Hypothesis Testing Test 2

Q1

	Solution	Comments
(i)	It is not necessary as the sample size is sufficiently large for Central Limit Theorem to apply.	This part is poorly attempted with many students discussing about the PARAMETERS of the distribution rather than the fact of whether it is a normal distribution. Some students simply stated that it is necessary as the hypothesis testing requires the use of a normal distribution, showing clearly their lack of understanding for the Central Limit Theorem and Sampling Distributions in general. There are also quite a number of students who either left out the fact that the sample size is large or that Central Limit Theorem is applicable, resulting in an incomplete explanation.
(ii)	$\bar{t} = 2384.5/50 = 47.69$ thousand hours $s^2 = \frac{1}{50-1}\left(115885.23 - \frac{2384.5^2}{50}\right) = 44.25357143 = 44.3$ $H_0: \mu = 50$ $H_1: \mu \neq 50$ Under H_0, since n is large, by C.L.T. $\bar{T} \sim N\left(50, \frac{44.25357143}{50}\right)$ appx p-value = 0.01407 OR test statistic = – 2.4554	Most candidates are successful with the unbiased estimates, but some left the answer as 47.7 for population mean, not realizing that it is an exact decimal. Quite a number of students also quoted the wrong formula for population variance or left their answers as the value before dividing by 49. While most students with the correct estimates were successful

	Since p-value = 0.01407 > 0.01, we do not reject H_0 and conclude that there is insufficient evidence at 1% level of significance to claim that the mean number of hours before failure is not 50 thousand hour.	with the testing, there is also a significant number of students who lost all marks by simply stating an incorrect p-value based on their wrong parameters. P-values calculated based on 3 s.f. values of the parameters or a combination of 5 s.f. and 3 s.f. values were accepted. Correct p-values based on erroneous presentation of the sampling distributions were not penalized due to benefit of doubt given. Students who attempted to use critical values were less successful as they applied modulus to the the test statistic without doing so for the critical value resulting in erroneous comparisons. Students should state clearly the rejection region when using critical values. Most conclusions were not given in context, did not mention level of significance, or were not phrase in terms of the alternate hypothesis. Many students phrased the conclusion wrongly as "having sufficient evidence to claim that the mean is 50 thousand hours".
(iii)	The reader would be more interested to test whether the mean is actually lower than the stated value which is not beneficial to them.	Most students simply stated that the test did not indicate whether the mean is more of less, but did not make any reference to why these

		cases would matter to the reader. Students who manage to make reference to higher mean being beneficial and/or lower mean being not beneficial, were given credit based on benefit of doubt.
(iv)	$H_1 : \mu < 50$ Yes, the result will differ as the p-value will be halved when switching to a one-tail test.	Most students were successful with stating the correct alternate hypothesis, except some who used the left tail test in (ii). However, not all were able to provide an explanation to support the change in conclusion, especially those who used critical values in (ii). Students who did not identify that the p-value is exactly half of the value found in (ii) will have to state the actual value, simply mentioning that the p-value is smaller is in sufficient as 0.011 is also a smaller p-value, but it will not result in a change in the conclusion. Most students who re-did the test were most successful for this part, but many went on to write the full conclusion, which is not required by the question.
(v)	We need to assume that the usage hours before failure for hard drives are independent for all hard drives.	Many students were able to state the assumption needed, but some did not exhibit any understanding of the situation.

Q2

(i)
2-tail as HPB is looking for a change in either way.

(ii)
Central Limit Theorem states that the sample <u>mean</u> heart rate will follow a normal distribution approximately when the sample is large (in this case, 70 > 20).

An unbiased estimate for the unknown population variance can be found obtained from the sample.

(iii)
Unbiased estimate of population mean $\bar{h} = \frac{5411}{70} = 77.3$.

Unbiased estimate of population variance,

$$s^2 = \frac{1}{69}\left(426433 - \frac{5411^2}{70}\right) = 118.3.$$

Let μ denote the mean heart rate of the teenagers in the obesity group.

To test at 10% significance level:

H_0: $\mu = 75$

H_1: $\mu \neq 75$

Under H_0, since n is large, by CLT, $\overline{H} \sim \mathrm{N}\left(75, \frac{118.3}{70}\right)$ approximately,

(AND/OR $\frac{\overline{H} - 75}{\sqrt{\frac{118.3}{70}}} \sim \mathrm{N}(0,1)$)

By GC, p-value = 0.0768 < 0.10.

(Alternatively, CR: $|z| > 1.645$, $z = 1.769$ is in CR)

Hence we <u>reject H_0</u> at the <u>10% level of significance</u> and conclude there is <u>sufficient evidence</u> that obesity will cause change in the mean heart rate.

(iv)

An one-tail test is used instead:

H_0: $\mu = 75$

H_1: $\mu > 75$

CR: $z > z_{0.9} = 1.28155$

To reject H_0,

$$\frac{79.4-75}{\sqrt{\frac{\sigma^2}{80}}} \geq 1.28155$$

$$\sigma^2 \leq \left(\frac{4.4\sqrt{80}}{1.28155}\right)^2 = 943 \text{ (3 sf)}$$

The researcher should conclude that obese teenagers evidentially has a higher mean heart rate if and only if the variance is not more (less) than 943.

(v)
$\mathrm{P}(\mu-\sigma < X < \mu+\sigma) = \mathrm{P}(-1 < Z < 1) = 0.68268 \approx 0.683$.
Since heart rates follow a normal distribution,

$\mathrm{P}(\mu-\sigma < H < \mu+\sigma) \approx 0.683$

We know that from (iv), null hypothesis will be rejected whenever $\sigma \leq 30.7$.

Taking $\sigma = 30.7$, under H_0, $\mathrm{P}(75-30.7 < H < 75+30.7) \approx 0.683$

$\Rightarrow \mathrm{P}(44.3 < H < 105.7) \approx 0.683$

and null hypothesis will be rejected.

We can say that for $\sigma \leq 30.7$ and when null hypothesis is rejected ,
$\mathrm{P}(44.3 < H < 105.7) \geq 0.683$ or $\mathrm{P}(H < 44.3) + \mathrm{P}(H > 105.7) < 0.317$

We know that the teenager's heart rate is rarely below 44.3 or above 105.7 in a resting state, so it is likely for the researcher to reject the null hypothesis.

In reality, it is unlikely for sigma to be as large as 30.7 such that the probability for H to be within one standard deviation from mean to be 0.683.

Q3

(i) Let μ be the mean of X.

$H_0: \ \mu = 7$

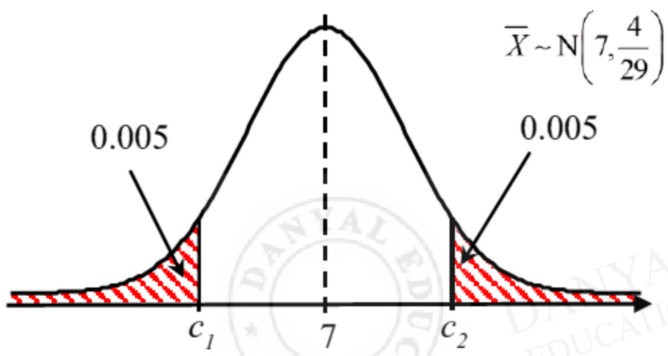
$H_1: \ \mu \neq 7$

$s^2 = \frac{30}{29}(\text{sample variance}) = \frac{30}{29}(4) = \frac{120}{29}$

Under H_0, since the sample size is large, the test statistic is

$\overline{T} \sim N\left(7, \frac{4}{29}\right)$ approximately by Central Limit Theorem.

(ii) Since the claim is rejected i.e. to reject H_0 at 1% significance level.



From GC, $c_1 = 6.04$ and $c_2 = 7.96$.

$\bar{t} \leq 6.04$ or $\bar{t} \geq 7.96$

(iii) From the two tail test, we know that p-value (two tail) ≤ 0.01. For a one-tail test, $\text{p-value(one tail)} = \frac{\text{p-value (two tail)}}{2} \leq 0.005 < 0.01$,

therefore we reject H_0 and conclude that there is sufficient evidence at 1% significance level to say that mean waiting time is more than 7 minutes.

Alternatively,

From the two tail test and $\bar{t} > 7$, $P(\overline{T} > \bar{t}) < 0.005$.

Thus, $P(\overline{T} > \bar{t}) < 0.005 < 0.01$.

p-value for one-tail test = $P(\overline{T} > \bar{t}) < 0.01$. Therefore we reject H_0 and conclude that there is sufficient evidence at 1% significance level to say that mean waiting time is more than 7 minutes.