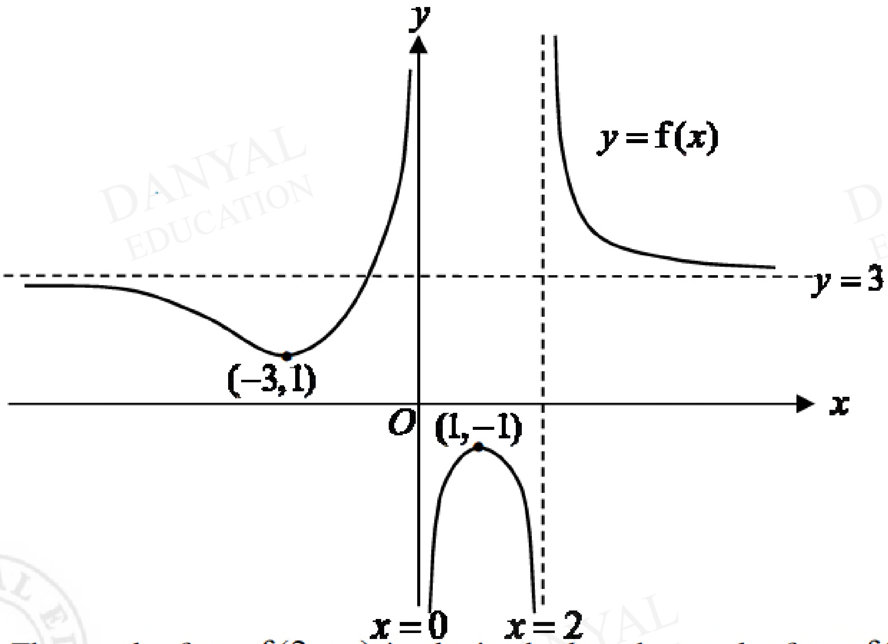
A Level H2 Math

Graphs and Transformations Test 8

Q1

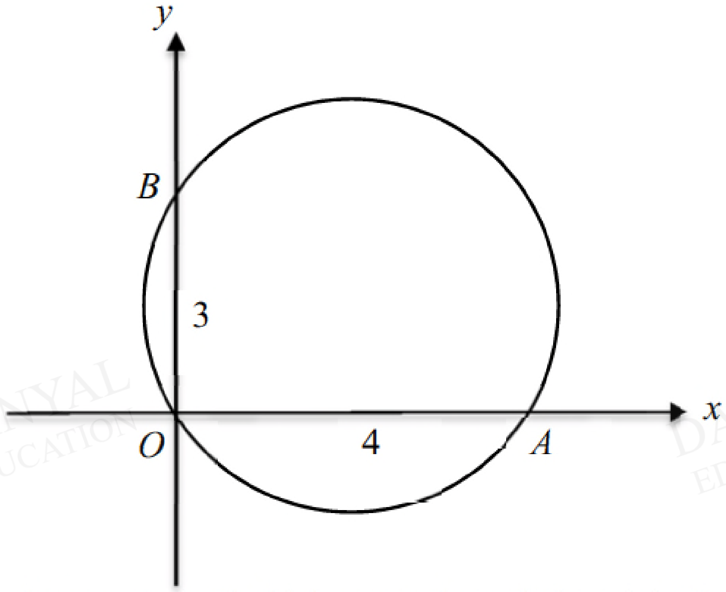
The graph of $y = \mathrm{f}(x)$ is shown below.



(a) The graph of $y = \mathrm{f}(2 - x)$ is obtained when the graph of $y = \mathrm{f}(x)$ undergoes a sequence of transformations. Describe the sequence of transformations. [2]

(b) Sketch the graph of $y = \mathrm{f}'(x)$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [3]

Q2

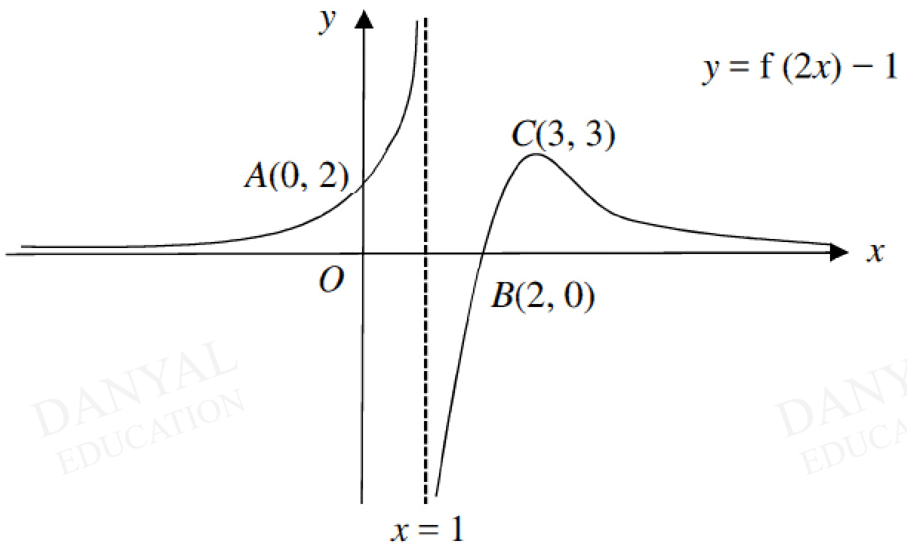


The diagram above shows a circle C which passes through the origin O and the points A and B. It is given that $OA = 4$ units and $OB = 3$ units.

(i) Show that the coordinates of the centre of C is $\left(2, \frac{3}{2}\right)$. Hence write down the equation of C in the form $(x-2)^2 + \left(y - \frac{3}{2}\right)^2 = r^2$, where r is a constant to be determined. [2]

(ii) By adding a suitable line to the diagram above, find the range of values of m for which the equation $mx - \frac{3}{2} = \sqrt{\frac{25}{4} - (x-2)^2}$ has a solution. [4]

Q3



The diagram shows the curve $y = \mathrm{f}(2x) - 1$ with a maximum point at $C(3, 3)$. The curve crosses the axes at the points $A(0, 2)$ and $B(2, 0)$. The line $x = 1$ and the x-axis are the asymptotes of the curve.

On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}(x)$, [2]

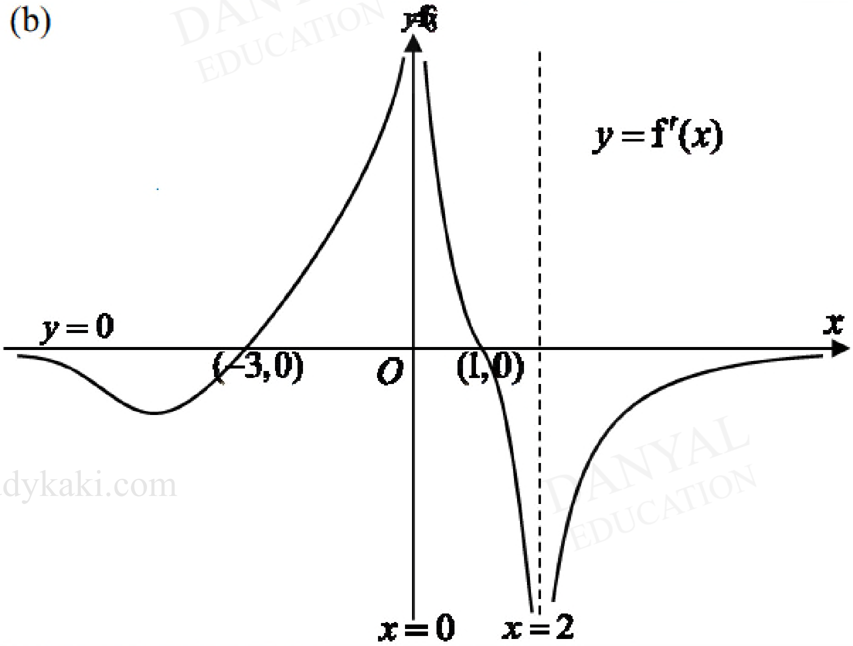
(ii) $y = \mathrm{f}'(x)$, [2]

stating clearly the equations of the asymptotes and the coordinates of the points corresponding to A, B and C where appropriate.

Answers

Graphs and Transformations Test 8

Q1

(a) **Translation** of 2 **units** in the negative x-**direction**, followed by **reflection about** the y-axis. Alternative solution Reflection about the y-axis followed by translation of 2 units in the positive x-direction.	Need to use the correct words (**in bold**) when describing the transformations.
(b) 	All (3) asymptotes must be labelled, and intersections with axes written in coordinate form as instructed by the question.
Marker's comments This question is generally well-attempted.	

Q2

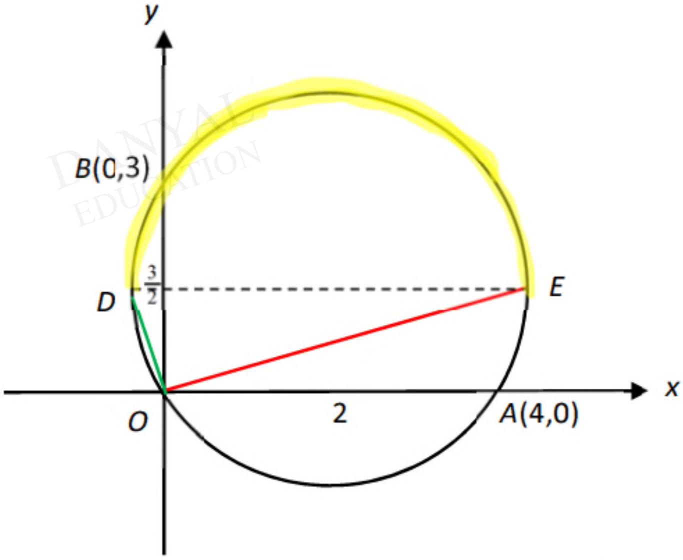
(i)	Since ΔAOB is a right-angle in a semi-circle, AB forms the diameter of the circle. Hence, centre of circle is at the mid point of AB, i.e., $\left(2,\ \frac{3}{2}\right)$. $AB=\sqrt{3^2+4^2}=5 \Rightarrow$ radius, $r=\frac{5}{2}$ units Therefore, equation of C is $(x-2)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{5}{2}\right)^2$.	Since this is a "Show" question, marks are awarded only if a clear explanation of how the centre coordinates are derived. Inefficient methods such as substituting coordinates into the circle equation and solving them simultaneously were used.

(ii) $$(x-2)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{5}{2}\right)^2$$

$$\Rightarrow \quad y-\frac{3}{2}=\pm\sqrt{\frac{25}{4}-(x-2)^2}$$

Most students did not realise that the question only involves the top half of the circle (positive root).

Suitable line to add: $y=mx$



Realise that all possible $y=mx$ will lie between the green and red line. This motivates us to find the gradient of the green and red line.

$D=\left(-\frac{1}{2},\ \frac{3}{2}\right)$ and $E=\left(\frac{9}{2},\ \frac{3}{2}\right)$

Gradient of line $OD=-\frac{3/2}{1/2}=-3$

Gradient of line $OE=\frac{3/2}{9/2}=\frac{1}{3}$

$\therefore\ m\le -3$ or $m\ge\frac{1}{3}$

Since points D and E are the two (left/right) ends of the semicircle, their coordinates can be easily deduced using the centre coordinates and radius.

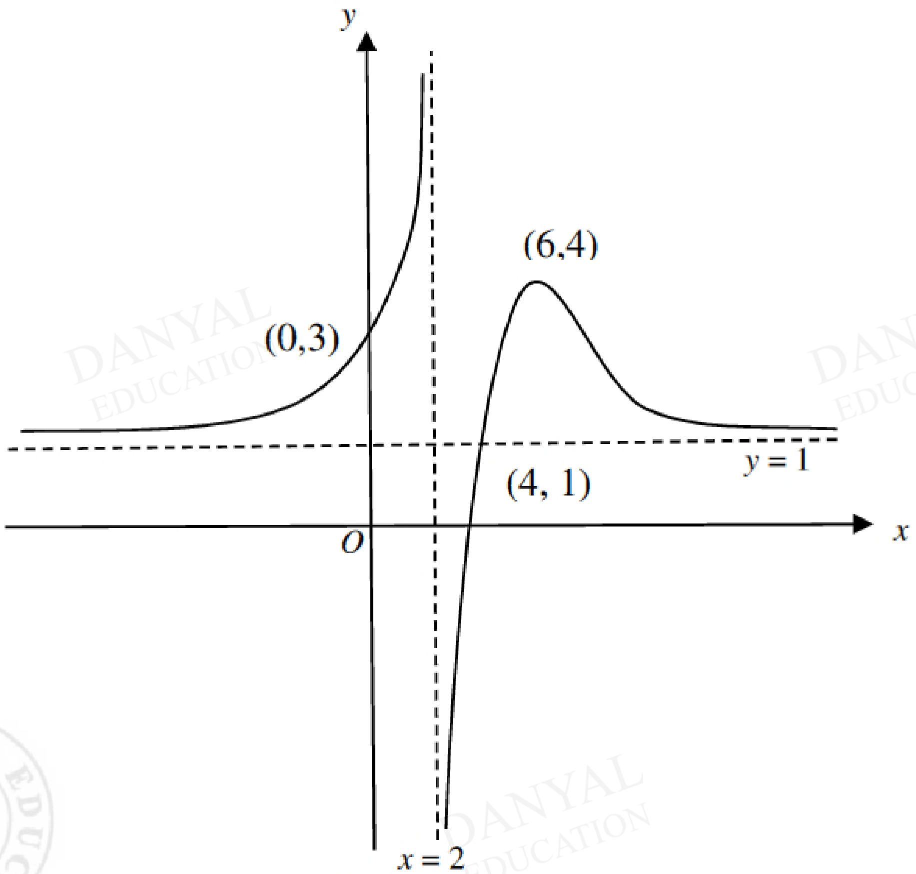
Marker's comments

Common mistakes:

1. *Differentiating the equation of the circle to find the gradient of the tangent*:
   From the diagram, it is clear that $y=mx$ can intersect the semicircle even if it is not a tangent to the circle.
2. *Setting the discriminant to be 0*:
   The quadratic equation represents all the intersection points of $y=mx$ with the whole circle (instead of the semicircle).
3. *Stating that the range of m is* $-3\le m\le\frac{1}{3}$:
   Inaccurate deduction which can be avoided by using the diagram.

Q3

1(i)



(ii)