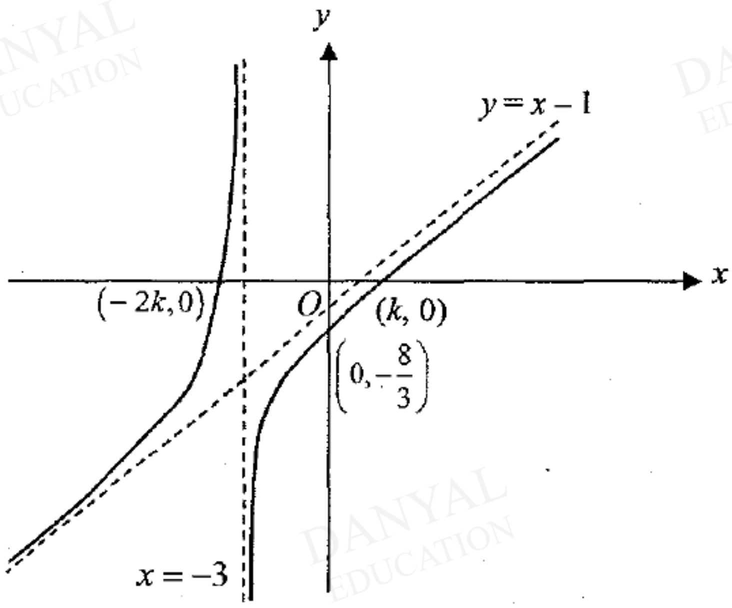
A Level H2 Math

Graphs and Transformations Test 7

Q1

The diagram below shows the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x- and y-axes at the points $(-2k, 0)$, $(k, 0)$ and $\left(0, -\frac{8}{3}\right)$ where $k > 0$. The curve has an oblique asymptote $y = x - 1$ and vertical asymptote $x = -3$.



(i) On separate diagram, sketch the graph of $y = \frac{1}{\mathrm{f}(x)}$, including the coordinates of the points where the graph crosses the axes and the equations of any asymptotes. [3]

(ii) It is further known that $\mathrm{f}(x) = \frac{x^2 + ax + b}{x + c}$ where a, b and c are constants. Find the values of a, b and c. [4]

Q2

The curve C has equation

$$y = 1 + \frac{2x+p}{(x-2)(x+3)},$$

where p is a constant.

(i) Find the range of values of p for which C has more than one stationary point. [4]

(ii) Sketch C for $p = 7$, stating the coordinates of the turning point(s) and the points of intersection with the axes and the equations of any asymptotes. [3]

(iii) By sketching a suitable graph on the same diagram, solve the inequality

$$1 + \sqrt{12 - x^2} \geq \frac{2x+7}{(2-x)(x+3)}.$$ [3]

Q3

A curve C has equation $y = \dfrac{ax+b}{cx+1}$, where a, b and c are positive real constants and $b > \dfrac{a}{c}$.

(i) Sketch C, stating the equations of any asymptotes and the coordinates of the points where the curve crosses the axes. [3]

The curve C is transformed by a scaling parallel to y-axis by factor $\dfrac{1}{2}$ and followed by a translation of 2 units in the positive x-direction.

(ii) Find the equation of the new curve in the form of $y = \mathrm{f}(x)$. [2]

It is given that the new curve $y = \mathrm{f}(x)$ passes through the points with coordinates $\left(3, \dfrac{3}{2}\right)$ and $(6, 1)$, and that $y = \dfrac{3}{4}$ is one of the asymptotes of the new curve $y = \mathrm{f}(x)$.

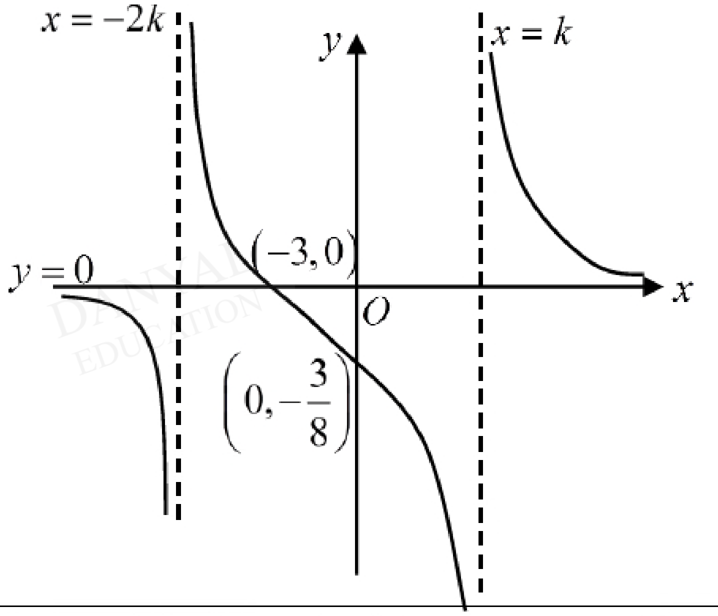
(iii) Find the values of a, b and c. [5]

Answers

Graphs and Transformations Test 7

Q1

7i



ii

Since $x=-3$ is the vertical asymptote, $c=3$

Given that $y=x-1$ is an oblique asymptote,

$$f(x)=x-1+\frac{A}{x+3}$$

$$=\frac{(x-1)(x+3)+A}{x+3}=\frac{x^2+2x-3+A}{x+3}$$

By comparing coefficient of x with $\frac{x^2+ax+b}{x+3}$: $a=2$

Since $\left(0,-\frac{8}{3}\right)$ is on the curve, $\frac{(0)^2+2(0)+b}{(0)+3}=-\frac{8}{3}$

$$b=-8$$

Q2

(i)

$$y = 1 + \frac{2x+p}{x^2+x-6}$$

$$\frac{dy}{dx} = \frac{2\left(x^2+x-6\right)-(2x+p)(2x+1)}{\left(x^2+x-6\right)^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow 2\left(x^2+x-6\right)-(2x+p)(2x+1) = 0$$

$$2x^2+2px+12+p=0$$

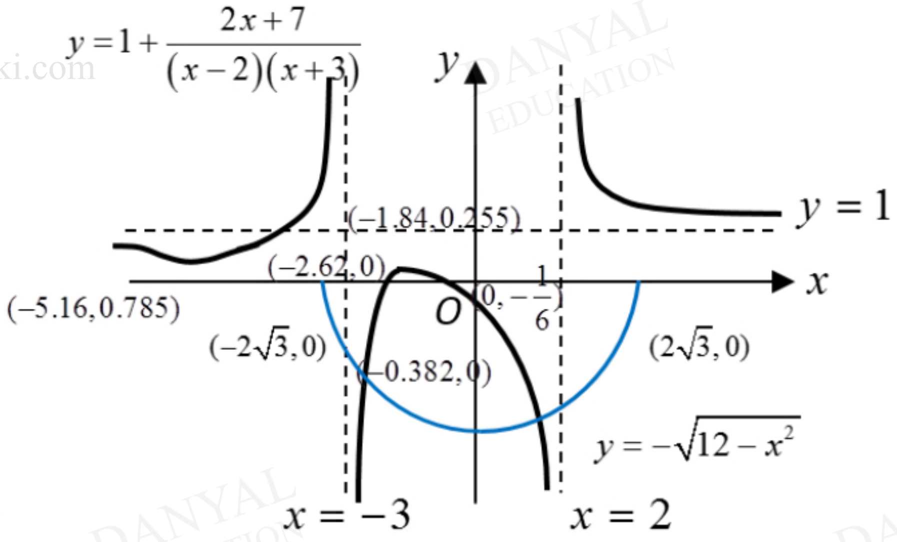
$$4p^2-4(2)(12+p)>0$$

$$p^2-2p-24>0$$

$$(p+4)(p-6)>0$$

$p<-4$ or $p>6$

(ii)



(iii)

$$1+\sqrt{12-x^2} \ge \frac{2x+7}{(2-x)(x+3)}$$

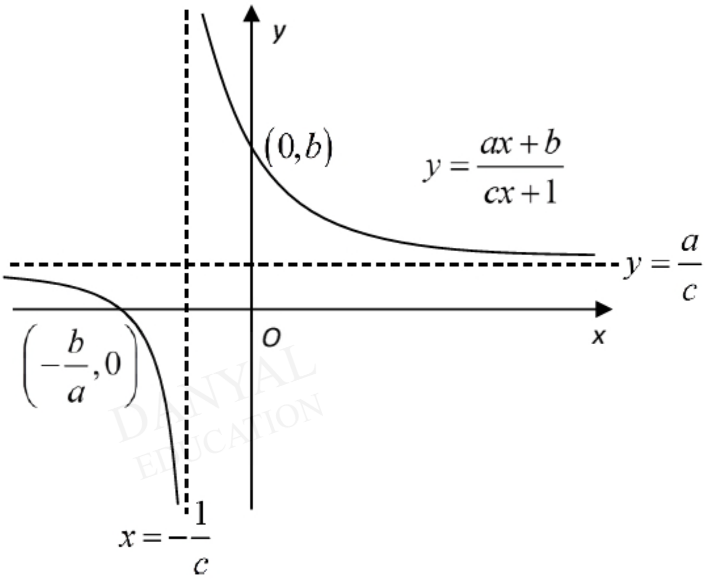
$$1+\frac{2x+7}{(x-2)(x+3)} \ge -\sqrt{12-x^2}$$

Sketch $y=-\sqrt{12-x^2}$ (as above)

$-2\sqrt{3} \le x < -3$ OR $-2.92 \le x \le 1.46$ OR $2 < x \le 2\sqrt{3}$

Q3

(i)



(ii) Equation of new curve: $y=\dfrac{1}{2}\left[\dfrac{a(x-2)+b}{c(x-2)+1}\right]$

(iii) Since the new curve $y=\mathrm{f}(x)$ passes through the points with coordinates $\left(3,\dfrac{3}{2}\right)$ and $(6,1)$:

$\dfrac{3}{2}=\dfrac{1}{2}\left[\dfrac{a(3-2)+b}{c(3-2)+1}\right]$

$3=\dfrac{a+b}{c+1}$

$a+b=3c+3$

$a+b-3c=3$ --------(1)

$1=\dfrac{1}{2}\left[\dfrac{a(6-2)+b}{c(6-2)+1}\right]$

$2=\dfrac{4a+b}{4c+1}$

$4a+b=8c+2$

$4a+b-8c=2$ ---------(2)

Since $y=\dfrac{3}{4}$ is one of the asymptotes of $y=\mathrm{f}(x)$,

$\dfrac{3}{4}=\dfrac{1}{2}\left(\dfrac{a}{c}\right)$

$\dfrac{a}{c}=\dfrac{3}{2}$

$2a-3c=0$ --------(3)

Solving equations (1), (2) and (3) using GC,
$a=3,\ b=6$ and $c=2$.