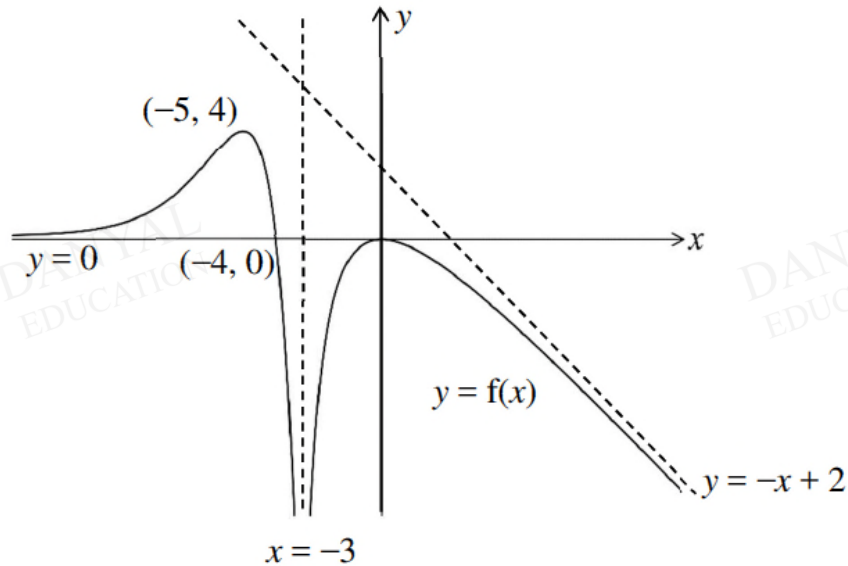


A Level H2 Math

Graphs and Transformations Test 6

Q1



The diagram shows the curve $y = f(x)$. The curve has maximum points at $(-5, 4)$ and the origin, and crosses the x -axis at $(-4, 0)$. The lines $y = 0$, $x = -3$ and $y = -x + 2$ are the horizontal, vertical and oblique asymptotes to the curve respectively.

On separate diagrams, draw sketches of the graphs of

(a) $y = \frac{1}{f(x)}$, [3]

(b) $y = f'(x)$, [3]

(c) $y = f\left(\frac{x+1}{2}\right)$, [3]

labelling clearly the equation(s) of any asymptote(s), coordinates of any axial intercept(s) and turning point(s) where applicable.

Q2

(i) State a sequence of transformations that would transform the curve with equation $y = e^{x^2}$ onto the curve with equation $y = f(x)$, where $f(x) = e^{ax^2} - b$, $a > 0$ and $b > 1$. [2]

(ii) Sketch the curve $y = f(x)$ and the curve $y = \frac{1}{f(x)}$.

You should state clearly the equations of any asymptotes, coordinates of turning points and axial intercepts. [5]

Q3

The curve C has equation $(y+4)^2 - (x+3)^2 = 4$. Sketch C , giving the coordinates of any turning points and the equations of any asymptotes. [3]

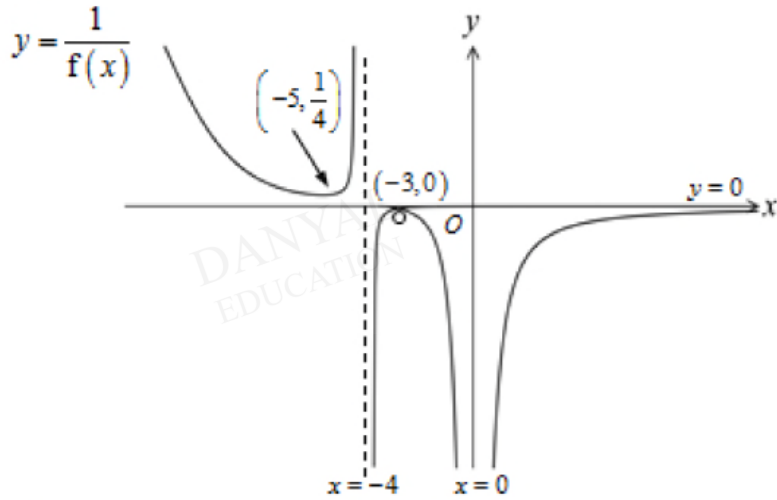
Hence find the set of values of m such that the straight line with gradient m that passes through the point $(-3, -4)$ intersects C at least once. [2]

Answers

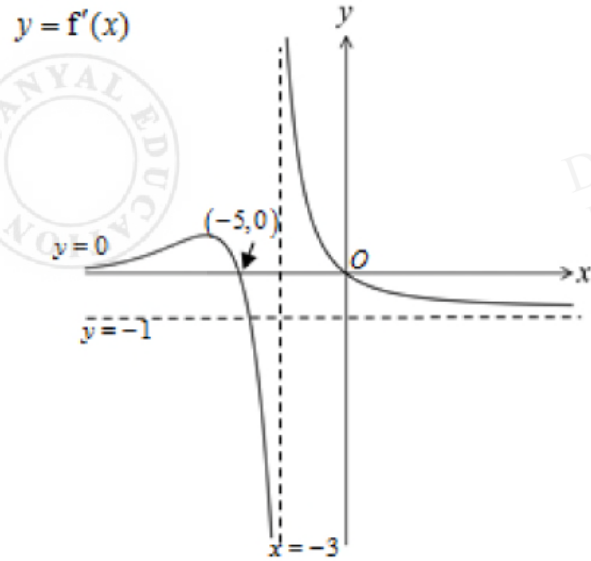
Graphs and Transformations Test 6

Q1

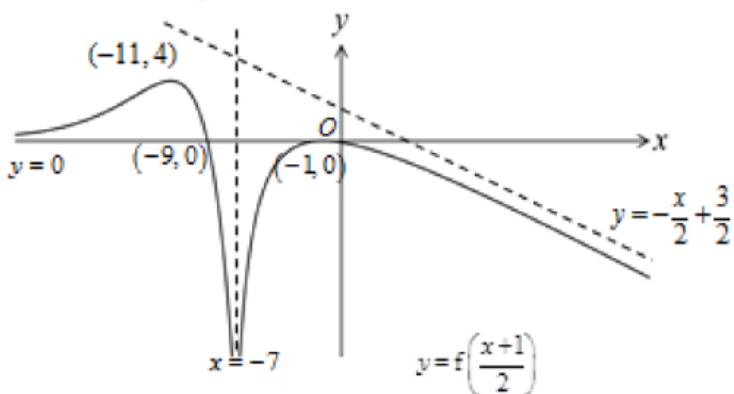
(a)



(b)



(c) $y = f\left(\frac{x+1}{2}\right)$



Q2

(i)

$$y = e^{ax^2} - b = e^{(\sqrt{ax})^2} - b$$

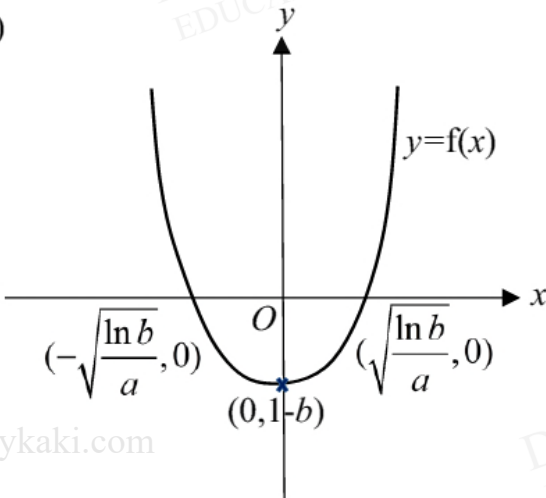
If $f(x) = e^{x^2}$, then $f(\sqrt{ax}) = e^{(\sqrt{ax})^2}$ and so

$$y = f(x) \rightarrow y = f(\sqrt{ax}) \rightarrow y = f(\sqrt{ax}) + b$$

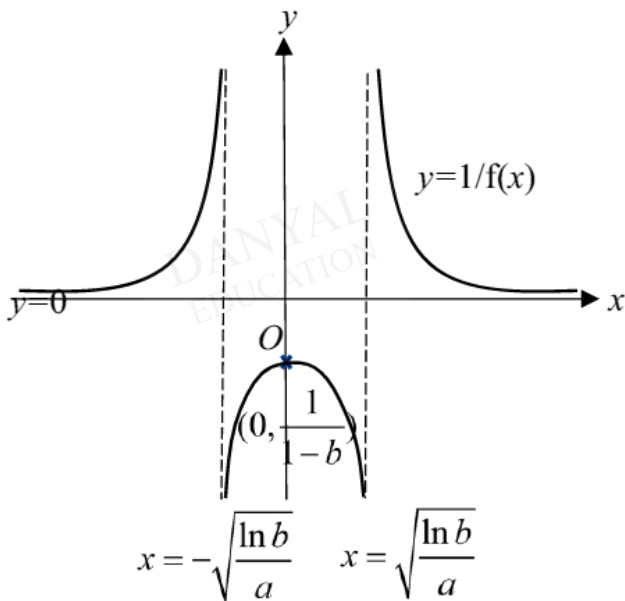
Hence the sequence of transformations are:

1. Scale by a factor of $\frac{1}{\sqrt{a}}$ parallel to the x -axis,
2. Translate the resulting curve by b units in the negative y -direction.

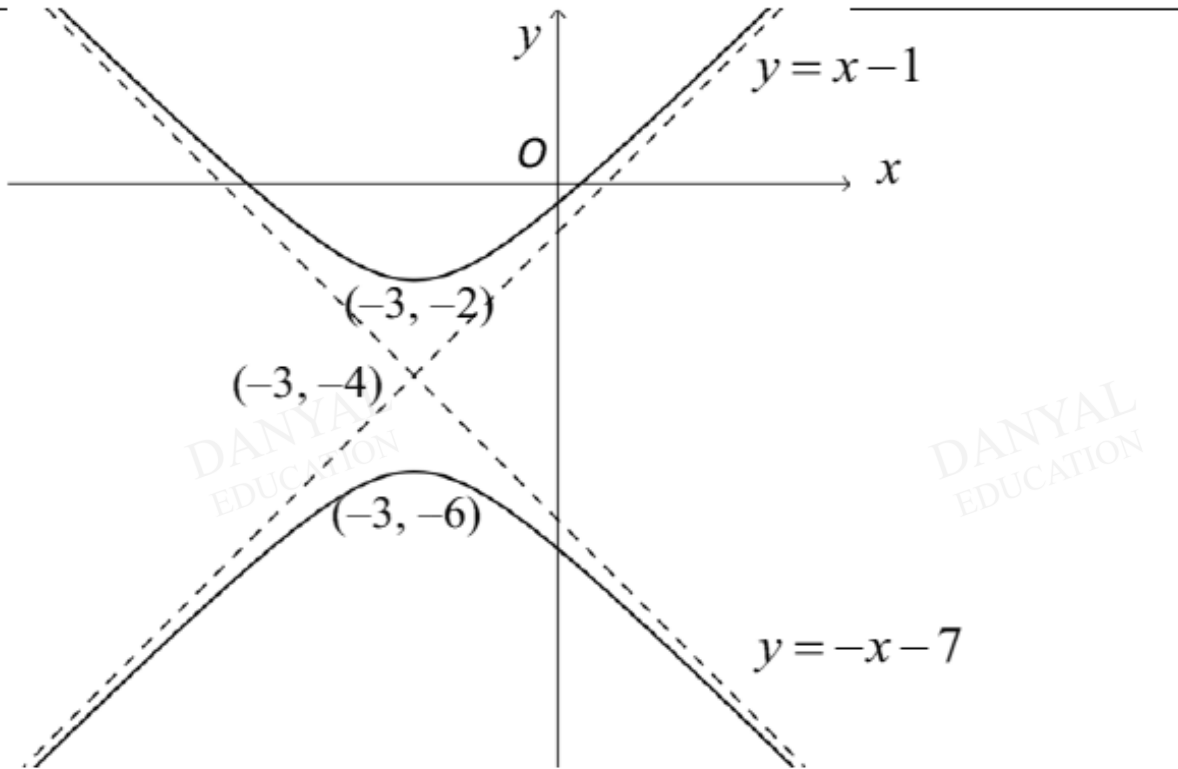
(ii)



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Q3



Intersect at least once: $\{m \in \mathbb{R} : m < -1 \text{ or } m > 1\}$