A Level H2 Math

Graphs and Transformations Test 5

Q1

The curve C has equation $4y^2-8y-x^2-4x-4=0$.

(i) Using an algebraic method, find the set of values that y cannot take. [3]

(ii) Showing any necessary working, sketch C and indicate the equations of the asymptotes. [4]

Q2

It is given that $f(x)=\begin{cases} 2x-1 & 0\le x\le 2, \\ 2-(x-3)^3 & 2<x\le 4, \\ 1 & \text{otherwise.} \end{cases}$

Sketch, on separate diagrams, for $0\le x\le 8$, the graphs of

(i) $y=f(x)$ and state the range of f, [5]

(ii) $y=\dfrac{1}{f(x)}$. [4]

In each graph, indicate clearly the coordinates of the end points, points of intersection with the axes and stationary point, if any. State clearly the equation of any asymptote.

(iii) Deduce the value of $\int_{-6}^{-4} f(-x)\,dx$. [1]

Q3

A curve C has equation $y=f(x)$, where

$$f(x)=\frac{a}{(x+b)^2}+cx,$$

and a, b and c are constants. It is given that C has a vertical asymptote $x=-1$ and a minimum point at $(0,\ 1)$.

(i) Find the values of a, b and c. [4]

(ii) Sketch the graph of $y=f(|x|)$, stating the coordinates of any point(s) of intersection with the axes and the equation(s) of any asymptote(s). [3]

(iii) Hence, solve the inequality $f(|x|)-4>0$. [2]

Answers
Graphs and Transformations Test 5

Q1

(i) $-x^2-4x+\left(4y^2-8y-4\right)=0$

For values that y cannot take, there are no real solutions for x and discriminant<0.

Therefore, $(-4)^2-4(-1)\left(4y^2-8y-4\right)<0$

$$16+16y^2-32y-16<0$$
$$y^2-2y<0$$
$$y(y-2)<0$$
$$\therefore 0<y<2$$

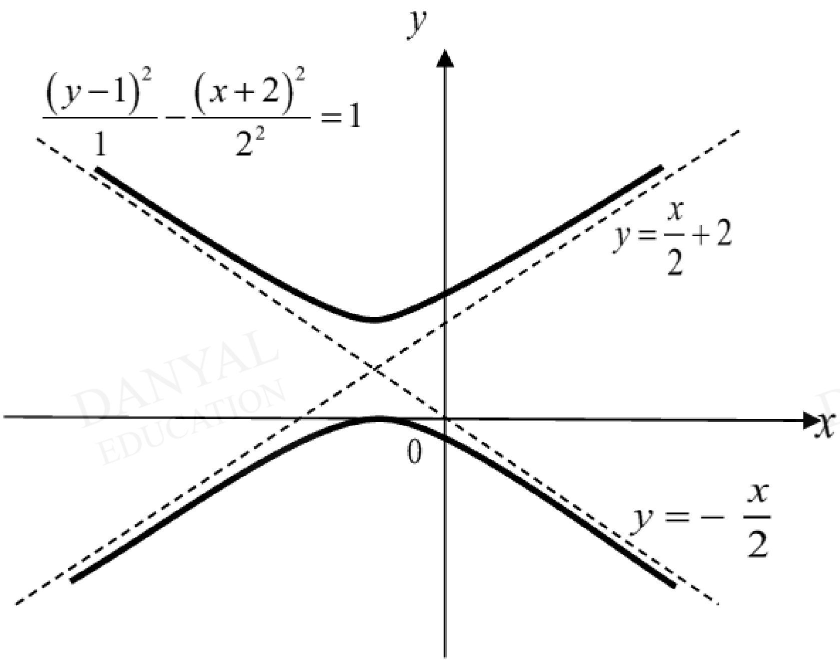
Set of values that y cannot take is $\left\{y\in\mathbb{R}: 0<y<2\right\}$.

(ii) $4y^2-8y-x^2-4x-4=0$

$4\left[\left(y-1\right)^2-1\right]-\left[\left(x+2\right)^2-4\right]-4=0$

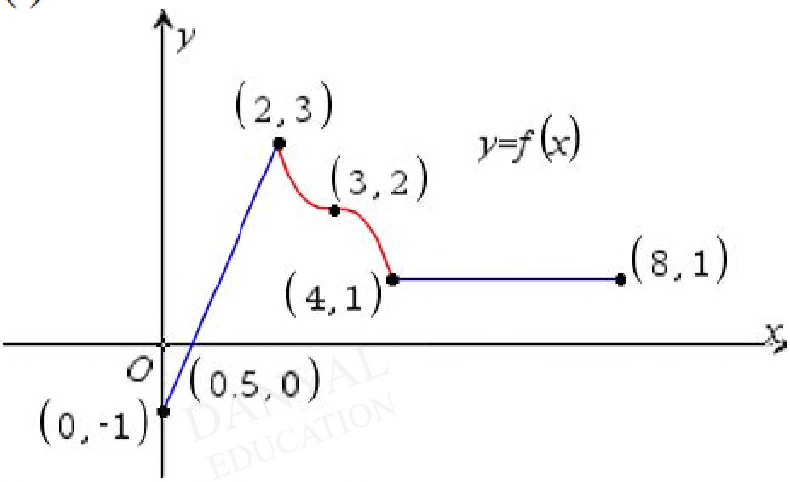
$4\left(y-1\right)^2-4-\left(x+2\right)^2=0$

$\dfrac{\left(y-1\right)^2}{1}-\dfrac{\left(x+2\right)^2}{2^2}=1$



Q2

(i)

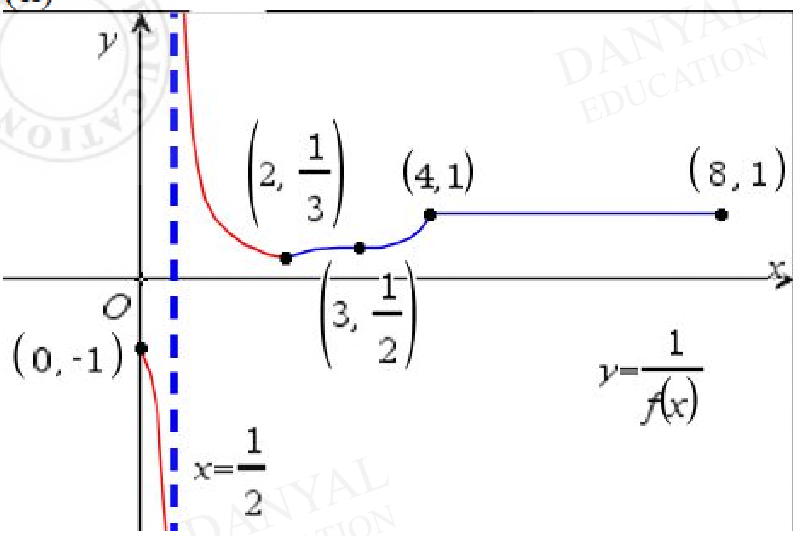


Range of f is $[-1, 3]$

or $R_f = [-1, 3]$

or $R_f = \{y : -1 \le y \le 3\}$

(ii)



(iii)

$$\int_{-6}^{-4} f(-x)\,dx = \int_{4}^{6} f(x)\,dx$$

$$= \text{area of rectangle}$$

$$= 2$$

Q3

(i) $y=\frac{a}{(x+b)^2}+cx$

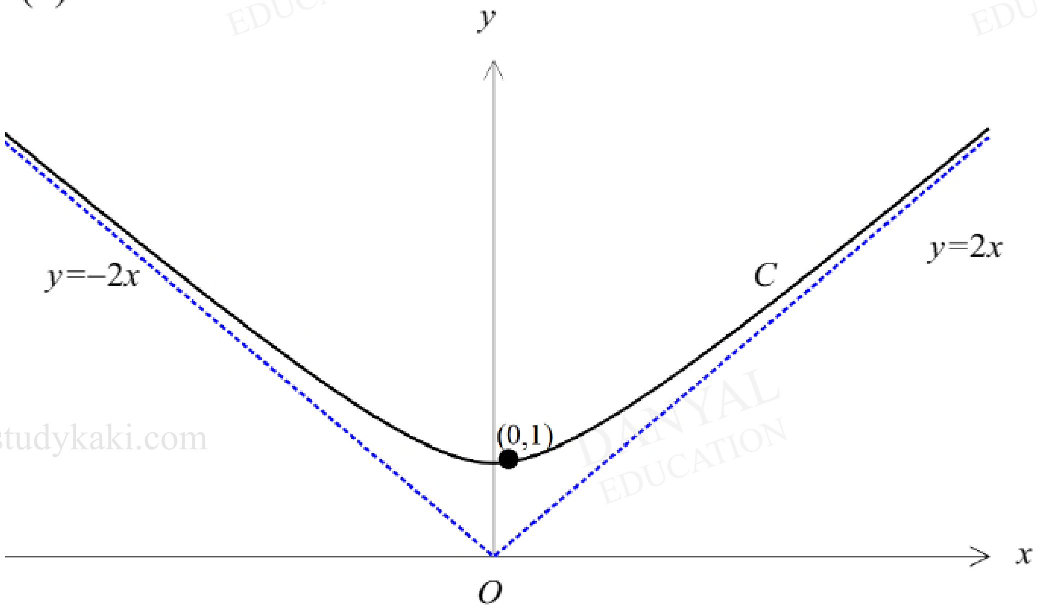
C has a vertical asymptote $x=-1 \Rightarrow b=1$

C passes through $(0,1) \Rightarrow a=1$

$\frac{dy}{dx}=-\frac{2}{(x+1)^3}+c$

At (0,1), $\frac{dy}{dx}=0 \Rightarrow c=\frac{2}{1^3}=2$

(ii)



(iii)

$f(|x|)-4>0 \Leftrightarrow f(|x|)>4$

The line $y=4$ cuts the graph of $y=f(|x|)$ at $x=\pm 1.94$ (3sf).

$\therefore f(|x|)-4>0 \Leftrightarrow x<-1.94$ or $x>1.94$