A Level H2 Math

Graphs and Transformations Test 3

Q1

The curve C has equation

$$y=\frac{2x^2-3x+5}{x-5}.$$

(i) Express y in the form $px+q+\frac{r}{x-5}$ where p, q and r are constants to be found. [3]

(ii) Sketch C, stating the equations of any asymptotes, the coordinates of any stationary points and any points where the curve crosses the x- and y-axes. [4]

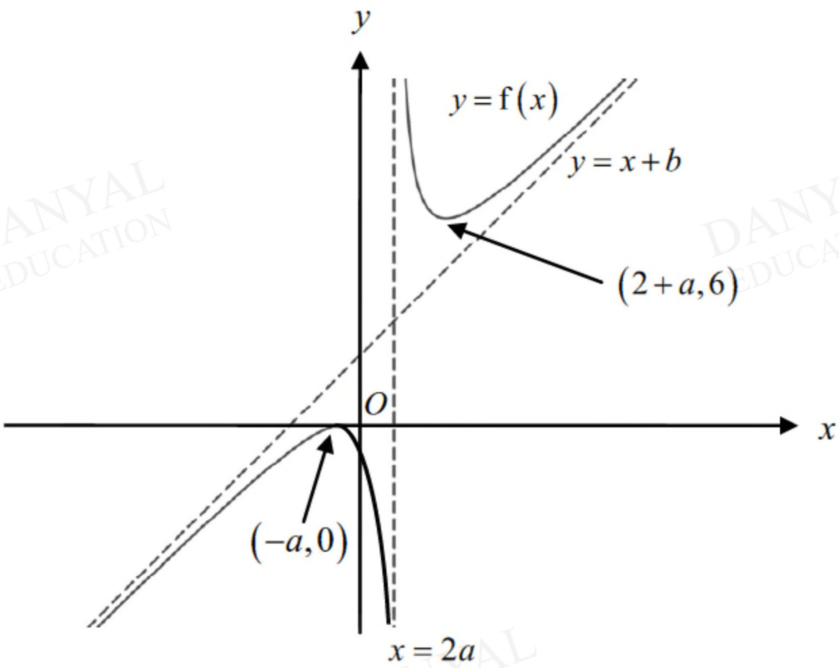
(iii) By sketching another suitable curve on the same diagram in part **(ii)**, state the number of roots of the equation

$$\left(2x^2-3x+5\right)^2=5x\left(x-5\right)^2.$$ [3]

Q2

The diagram shows the graph of the function $y = f(x)$ where, $a,\ b \in \mathbb{R},\ b \geq 2$ and $0 < a < 1$.

The coordinates of the minimum point and maximum point on the curve are $(-a,\ 0)$ and $(2+a,\ 6)$ respectively. The equations of the asymptotes are $y = x + b$ and $x = 2a$.



On separate diagrams, sketch the graphs of the following functions, labelling the coordinates of any points of intersection with the x-axis, the coordinates of any turning points and the equations of any asymptotes.

(i) $y = f(2x-1)+1$, [3]

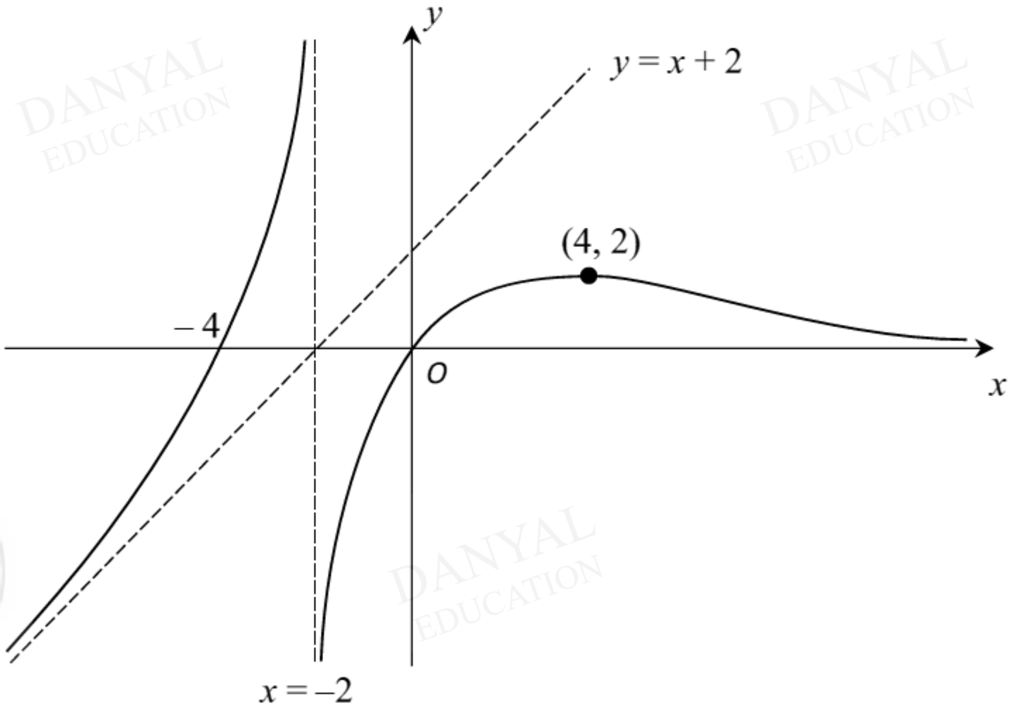
(ii) $y = \dfrac{1}{f(x)}$. [3]

The two asymptotes of $y = f(x)$ intersect at point P. Show that P lies on the line $y = mx + (b + 2a - 2am)$ for all real values of m. Hence, state the range of values of m for which the line $y = mx + (b + 2a - 2am)$ does not cut the curve $y = f(x)$. [3]

Q3

(a) State a sequence of transformations that transform the graph of $x^2+\frac{1}{3}(y-2)^2=1$ to the graph of $(x-2)^2+y^2=1$. [3]

(b) The diagram below shows the curve $y=\mathrm{f}(x)$. It has a maximum point at $(4, 2)$ and intersects the x-axis at $(-4, 0)$ and the origin. The curve has asymptotes $x=-2$, $y=0$ and $y=x+2$.



Sketch on separate diagrams, the graphs of

(i) $y=\mathrm{f}'(x)$, [3]

(ii) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

including the coordinates of the points where the graphs cross the axes, the turning points and the equations of any asymptotes, where appropriate.

Answers

Graphs and Transformations Test 3

Q1

(i)

$$y=\frac{2x^2-3x+5}{x-5}=2x+7+\frac{40}{x-5}$$

$\therefore p=2$

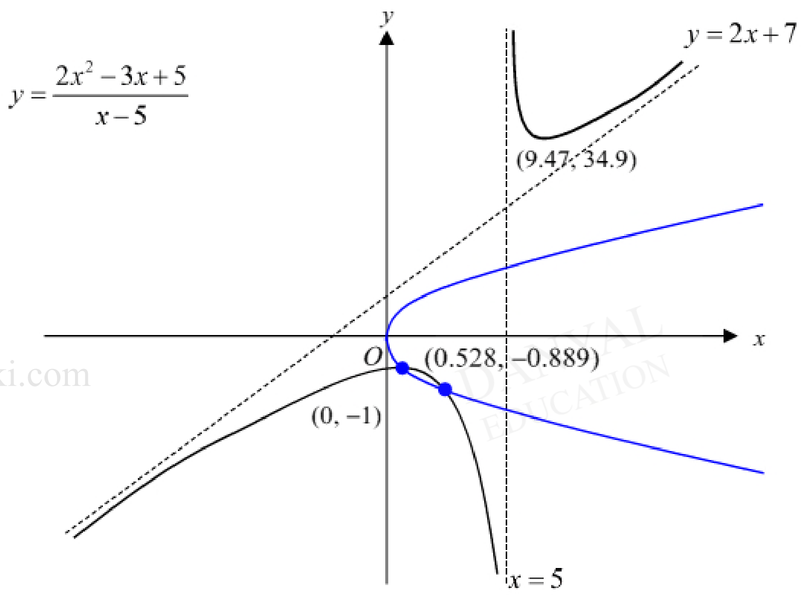
$q=7$

$r=40$

$$\begin{array}{r} 2x+7 \\ x-5\overline{)\,2x^2-3x+5} \\ -\underline{(2x^2-10x)} \\ 7x+5 \\ -\underline{(7x-35)} \\ 40 \end{array}$$

This part was generally well done.

(ii) Asymptotes: $y=2x+7$ and $x=5$



Candidates need to use a ruler for axis and asymptotes to provide an accurate sketch.
Many students did not write down intercept in coordinate form.

(ii)

$$\left(2x^2-3x+5\right)^2=5x\left(x-5\right)^2$$

$$\left(\frac{2x^2-3x+5}{x-5}\right)^2=5x$$

$$y^2=5x$$

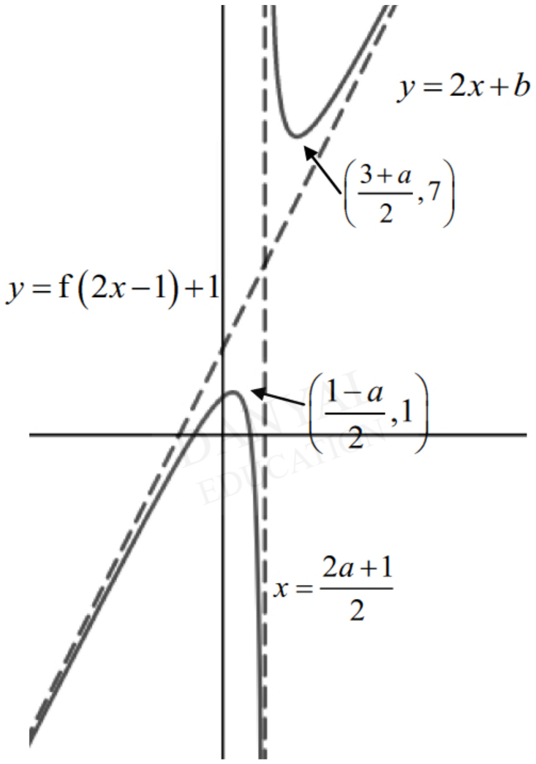
$$y=\pm\sqrt{5x}$$

Sketch $y=\pm\sqrt{5x}$ in part **(ii).**

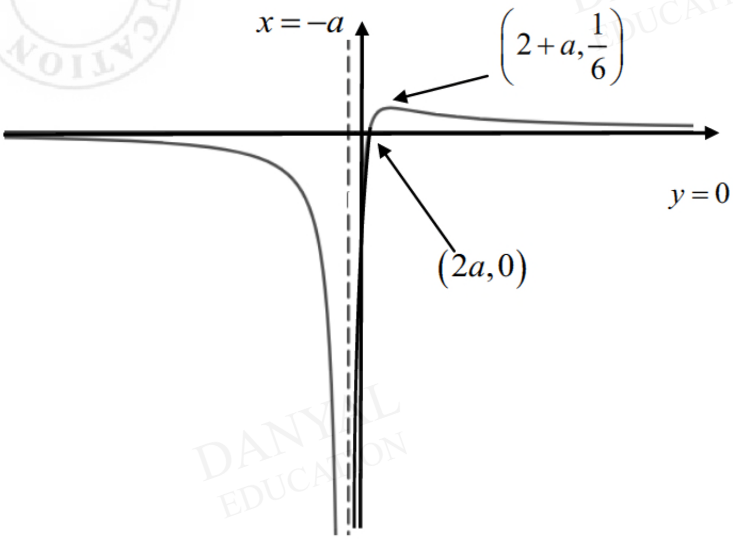
From the diagram, there are 2 points of intersections. Hence, there are 2 roots.

Many students did not include $y=-\sqrt{5x}$.
Some students drew a sketch of $y=\pm\sqrt{5x}$ which did not touch the origin.

Q2



(ii)



(iii)

Point P is $(2a, 2a+b)$

$$\frac{y-(2a+b)}{x-2a}=m \Rightarrow y=mx-2am+2a+b$$

Hence, P lies on the line $y=mx+(b+2a-2am)$ for $m \in \mathbb{R}$.

From the graph, $m \leq 1$ for the line not to cut $y=f(x)$.

Q3

(a) $x^2 + \frac{1}{3}(y-2)^2 = 1$

$\downarrow$ Replace x by $x-2$

$(x-2)^2 + \frac{1}{3}(y-2)^2 = 1$

$\downarrow$ Replace y by $y+2$

$(x-2)^2 + \frac{1}{3}(y)^2 = 1$

$\downarrow$ Replace y by $\sqrt{3}y$

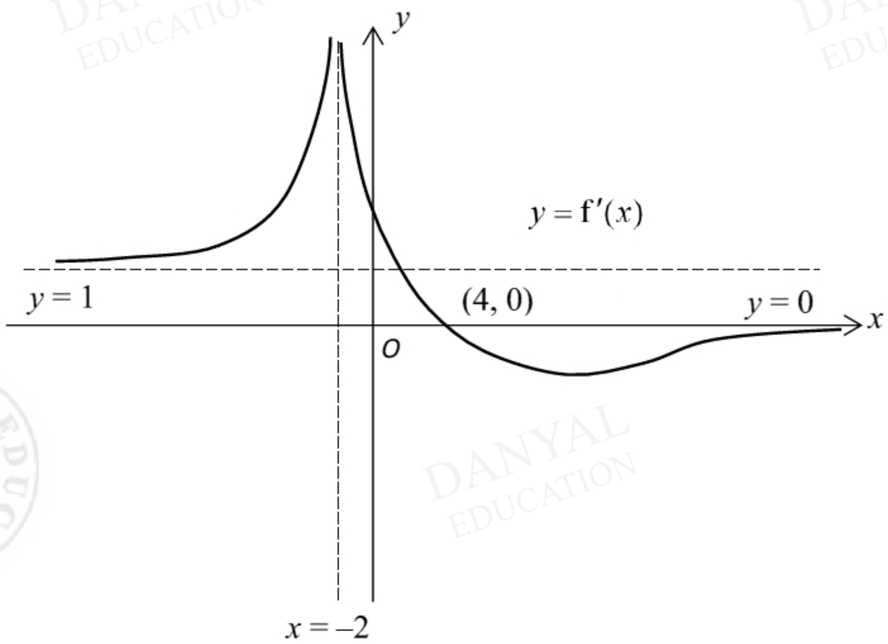
$(x-2)^2 + y^2 = 1$

1. Translate 2 units in the positive x-direction
2. Translate 2 units in the negative y-direction
3. Scale by a factor of $\frac{1}{\sqrt{3}}$ parallel to the y-direction

Alternative:

2. Scale by a factor of $\frac{1}{\sqrt{3}}$ parallel to the y-direction
3. Translate $\frac{2}{\sqrt{3}}$ units in the negative y-direction

(b) (i)



(b) (ii)