A Level H2 Math

Functions Test 6

Q1

It is given that

$$f(x)=\begin{cases}(x-2)^2-1, & \text{for } 0< x\leqslant 3,\\ x-3, & \text{for } 3<x\leqslant 6,\end{cases}$$

and that $f(x)=f(x+6)$ for all real values of x.

(i) Sketch the graph of $y=f(x)$ for $0<x\leqslant 10$. [3]

(ii) On a separate diagram, sketch the graph of $y=1+f\left(\frac{1}{2}x\right)$ for $0<x\leqslant 10$. [2]

Q2

The function h is defined by

$$h: x \mapsto e^{x-2}-1, \qquad \text{for } x\in\mathbb{R}.$$

(i) Find $h^{-1}(x)$ and state the domain of h^{-1}. [3]

(ii) Sketch, on the same diagram, the graphs of $y=h(x)$ and $y=h^{-1}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the x- and y-axes. [3]

(iii) Find the set of values of x such that $h^{-1}(x)>h(x)$. [2]

Q3

The functions f and g are defined by

$$\mathrm{f}: x \mapsto \mathrm{e}^{-x^2}, \quad x \in \mathbb{R},\ x < 0,$$

$$\mathrm{g}: x \mapsto \frac{1}{x+3}, \quad x \in \mathbb{R},\ x \neq -3.$$

(i) Show that g^{-1} exists, and define g^{-1} in a similar form. [2]

(ii) State the solution set for $\mathrm{g}\mathrm{g}^{-1}(x) = x$. [1]

(iii) Explain why $\mathrm{f}\mathrm{g}^{-1}$ does not exist. [1]

Let the function h be defined by

$$\mathrm{h}: x \mapsto \mathrm{g}(x), \quad x \in \mathbb{R},\ x < k,$$

where k is a real constant.

(iv) Given that $\mathrm{f}\mathrm{h}^{-1}$ exists, state the maximum value of k. [1]

(v) For the value of k found in **(iv)**,

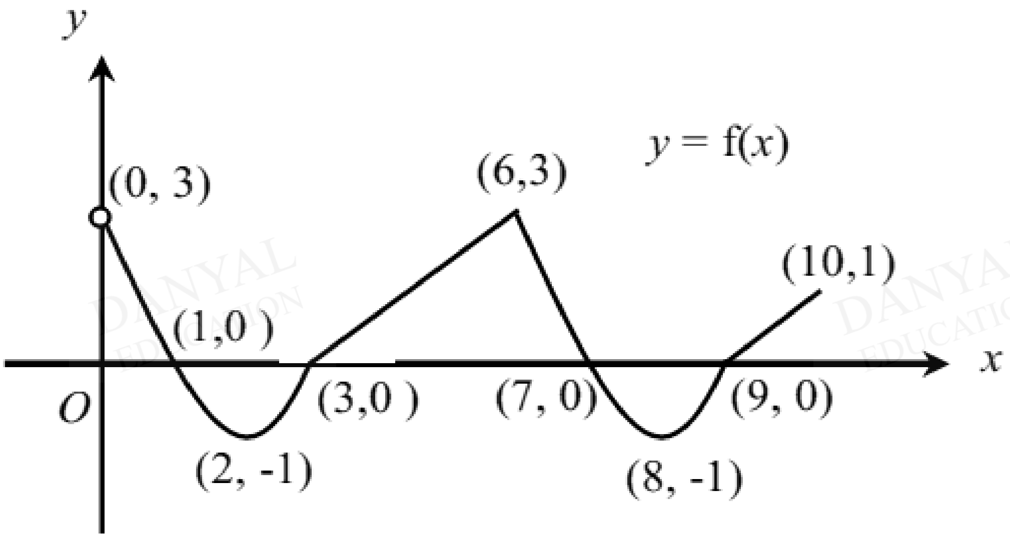
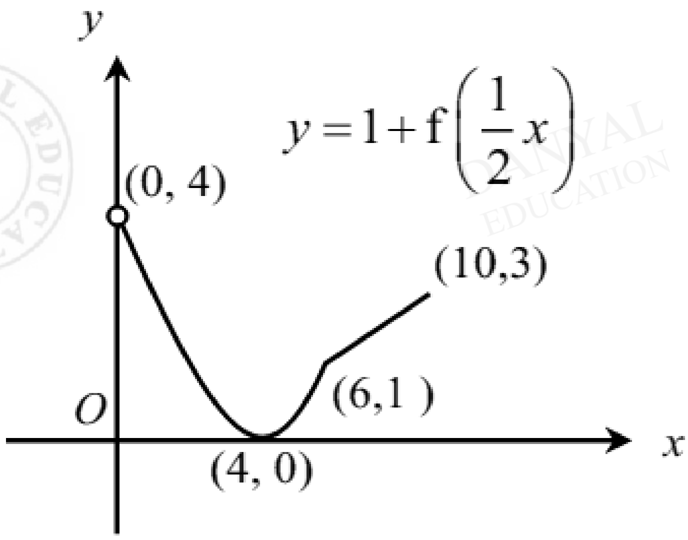
(a) find the exact range of $\mathrm{f}\mathrm{h}^{-1}$, [2]

(b) solve $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$. [2]

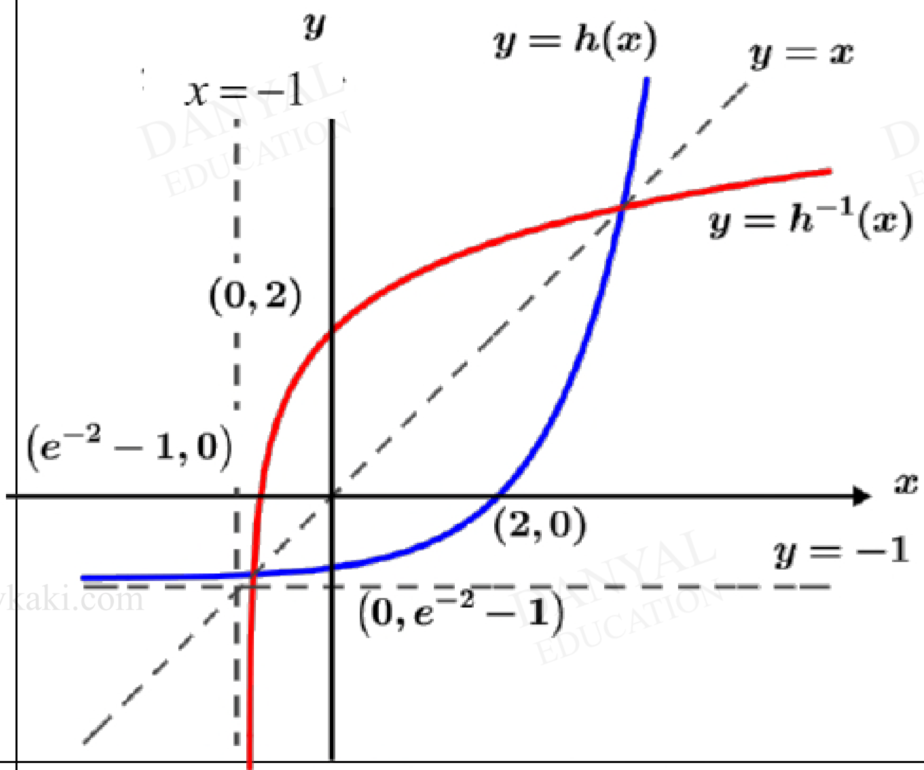
Answers

Functions Test 6

Q1

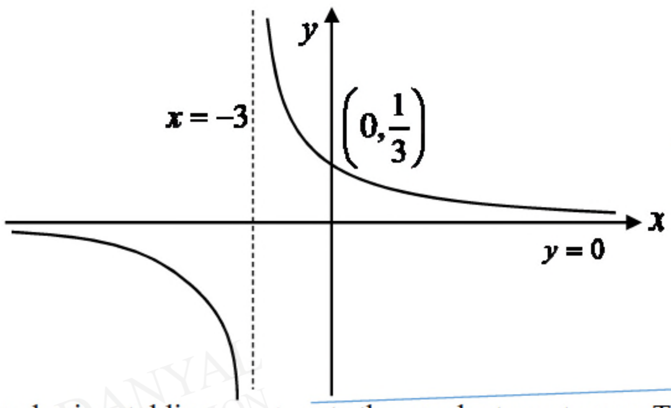
i	
ii	

Q2

)i	$y = e^{x-2} - 1$ $x = \ln(y+1) + 2$ $h^{-1}(x) = \ln(x+1) + 2$ Domain of h^{-1} = range of $h = (-1, \infty)$
ii	
iii	Using G.C, $y = h^{-1}(x)$ and $y = h(x)$ intersects at $x = -0.94753$ and $x = 3.50524$ Set of values of $x = \{x \in \mathbb{R} : -0.948 < x < 3.51\}$.

Q3

(i)



Every horizontal line $y = k$ cuts the graph at most once. This implies g is one-one. Therefore g^{-1} exists

$g^{-1} : x \mapsto \frac{1}{x} - 3, x \in \mathbb{R}, \ x \neq 0$

> There is a need to draw the graph to show that any horizontal line will cut the graph at most once.

> To show that function is 1-1, there is a need to have a general equation, $y = k$.

> This is different from "at only one point", as the line $y = 0$ does not cut the graph.

(ii) $\{x \in \mathbb{R} \mid x \neq 0\}$

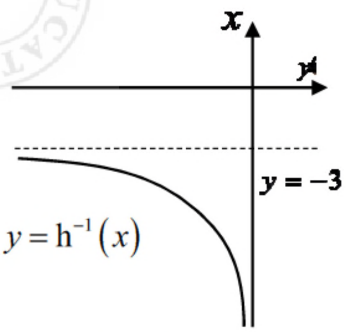
> Question asked for "solution set" therefore answer must be written in sets.

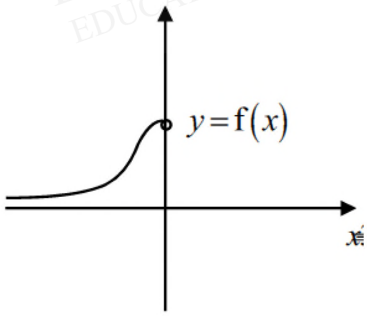
(iii) $R_{g^{-1}} = D_g = \mathbb{R} \backslash \{-3\}$, $D_f = \mathbb{R}^-$.

Since $R_{g^{-1}} \not\subset D_f$, fg^{-1} does not exist.

(iv) $k = -3$

(v) a)





$R_{fh^{-1}} = \left(0, e^{-9}\right)$

(iv) b)

$$h(x) = h^{-1}(x)$$
$$h(x) = x$$
$$\frac{1}{x+3} = x$$
$$x^2 + 3x - 1 = 0$$

Since $x < -3$, $x = -3.30$ (3sf)

> Need to reject the other root which does not lie in to range $x < -3$.