A Level H2 Math

Equations and Inequalities Test 2

Q1

There are 3 bike-sharing companies in the current market. For each ride, α- bike charges a certain amount per 5 min block or part thereof, β- bike charges a certain amount per 10 min block or part thereof and μ- bike charges a certain amount per 15 min block or part thereof. Rebecca rode each of the bike-sharing companies' bikes once in each month. The table below shows the amount of time Rebecca clocked for each ride and her total spending for each month. In celebration of the company's first anniversary, the pricings in February and March 2017 of μ- bikes are a 5% discount off the immediate previous month's pricing.

	January 2017	**February 2017**	**March 2017**
α- bike	25 min	17 min	36 min
β- bike	30 min	10 min	39 min
μ- bike	15 min	44 min	33 min
Total spending	$5.70	$5.72	$9.71

Determine which bike-sharing company offers the cheapest rate (without any discount) for a 40-min ride. Justify your answer clearly. [4]

Q2

Using an algebraic method, find the set of values of x that satisfies the inequality

$$2-x \le \frac{x}{2-x}.$$

[3]

Hence solve $2-x^2 \le \dfrac{x^2}{2-x^2}$. [2]

Q3

(i) By first expressing $3x-x^2-4$ in completed square form, show that $3x-x^2-4$ is always negative for all real values of x. [2]

(ii) Hence, or otherwise, without the use of a calculator, solve the inequality

$$\frac{\left(3x-x^2-4\right)(x-1)^2}{x^2-2x-5} \le 0,$$

leaving your answer in exact form. [4]

Answers
Equations and Inequalities Test 2

Q1

Let α, β and μ be the original amount charged per 5 min, 10 min and 15 min block for each ride by α- bike, β- bike and μ- bike respectively.

$5\alpha + 3\beta + \mu = 5.7$ ---------- (1)

$4\alpha + \beta + 3(0.95\mu) = 5.72$ ---------- (2)

$8\alpha + 4\beta + 3(0.95^2\mu) = 9.71$ ---------- (3)

Solving the above 3 equations simultaneously by GC,

$\alpha = \$0.4079329609, \beta = \$0.8402234637, \mu = \$1.139664804$

$\alpha = \$0.41, \beta = \$0.84, \mu = \$1.14$.

Original pricing per 40-min block:

Using calculator values

α- bike: $\$0.4079329609 \times 8 = \$3.26$

β- bike: $\$0.84 \times 4 = \$3.36$

μ- bike: $\$1.14 \times 3 = \$3.42$

Thus, α- bike offers the cheapest rate for a 40-min ride.

Q2

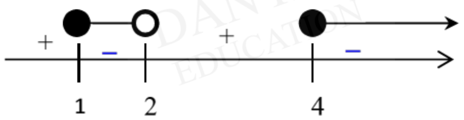
(i) $2 - x \le \dfrac{x}{2-x}$

$2 - x - \dfrac{x}{2-x} \le 0$

$\dfrac{(2-x)^2 - x}{2-x} \le 0$

$\dfrac{x^2 - 5x + 4}{2-x} \le 0$

$\dfrac{(x-4)(x-1)}{2-x} \le 0$



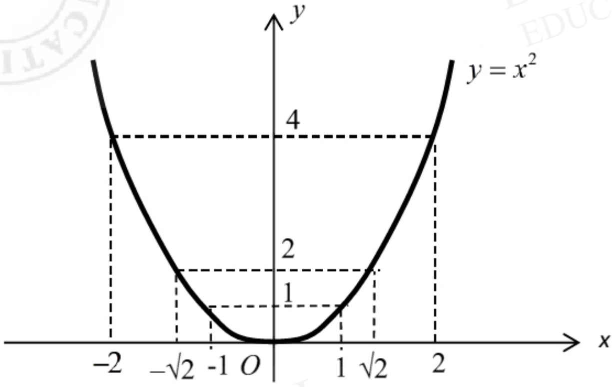
Set of values of x: $\{1 \le x < 2$ **or** $x \ge 4\}$

(ii) Let $y = x^2$.

$2 - x^2 \le \dfrac{x^2}{2-x^2} \Rightarrow 2 - y \le \dfrac{y}{2-y}$

$1 \le y < 2$ or $y \ge 4$

Method 1: Using $y = x^2$ graph



The range of values of x is $x \le -2$ or $-\sqrt{2} < x \le -1$ or $1 \le x < \sqrt{2}$ or $x \ge 2$

Method 2: Using definition of $|x|$

Since $x^2 = |x|^2$

For $1 \le |x|^2 < 2$

$\Rightarrow 1 \le |x| < \sqrt{2} \Rightarrow -\sqrt{2} \le x < -1$ **or** $1 \le x < \sqrt{2}$

For $|x|^2 \ge 4$

$|x| \ge 2 \Rightarrow x \le -2$ **or** $x \ge 2$

Hence, the range of values of x is $x \le -2$ or $-\sqrt{2} < x \le -1$ or $1 \le x < \sqrt{2}$ or $x \ge 2$

Q3

(i)

$$3x-x^2-4=-\left(x^2-3x+4\right)$$
$$=-\left(\left(x-\frac{3}{2}\right)^2+\frac{7}{4}\right)$$
$$=-\left(x-\frac{3}{2}\right)^2-\frac{7}{4}$$

Since $\left(x-\frac{3}{2}\right)^2\geq 0$ for all $x\in\mathbb{R}$, $-\left(x-\frac{3}{2}\right)^2\leq 0$

Hence $3x-x^2-4=-\left(x-\frac{3}{2}\right)^2-\frac{7}{4}\leq-\frac{7}{4}<0$

$\therefore$ $3x-x^2-4$ is always negative for all values of x.

(ii)

$$\frac{\left(3x-x^2-4\right)\left(x-1\right)^2}{x^2-2x-5}\leq 0$$

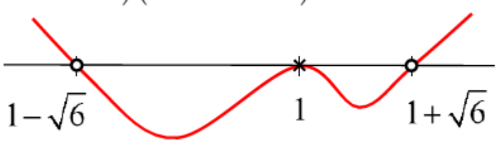
Since $3x-x^2-4$ is always negative, $\dfrac{\left(x-1\right)^2}{x^2-2x-5}\geq 0$

Method 1 (Quadratic formula)

Let $x^2-2x-5=0$

$$\therefore\ x=\frac{2\pm\sqrt{(-2)^2-4(1)(-5)}}{2(1)}=\frac{2\pm\sqrt{24}}{2}=1\pm\sqrt{6}$$

Hence $\dfrac{\left(x-1\right)^2}{\left(x-(1-\sqrt{6})\right)\left(x-(1+\sqrt{6})\right)}\geq 0$

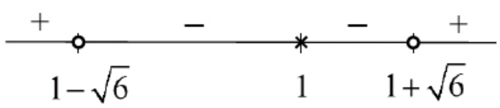


$\therefore$ $x<1-\sqrt{6}$ or $x>1+\sqrt{6}$ or $x=1$

Method 2 (Complete the square)

$$\frac{\left(x-1\right)^2}{\left(x-1\right)^2-6}\geq 0$$

$$\frac{\left(x-1\right)^2}{\left(x-(1-\sqrt{6})\right)\left(x-(1+\sqrt{6})\right)}\geq 0$$



$\therefore$ $x<1-\sqrt{6}$ or $x>1+\sqrt{6}$ or $x=1$