A Level H2 Math

Equations and Inequalities Test 1

Q1

Without using a calculator, solve the inequality

$$\frac{3x^2+7x+1}{x+3}<2x-1.$$ [4]

Q2

A curve C has equation $y=\frac{2x^2+3}{x-1}, \quad x\in\mathbb{R}, \quad x\neq 1.$

(i) Sketch C, stating the equations of the asymptotes, axial intercepts and the coordinates of the turning points, if any. [3]

(ii) Using part **(i)**, solve the inequality $2x+2\leq e^x-\frac{5}{x-1}$. [2]

(iii) Hence, solve the inequality $2x+4\leq e^{x+1}-\frac{5}{x}$. [2]

Q3

The curve with equation $y=f(x)$, where $f(x)$ is a cubic polynomial, has a maximum point with coordinates $\left(-2,\frac{34}{3}\right)$ and a minimum point with coordinates $\left(3,-\frac{19}{2}\right)$. Find the equation of the curve. [4]

Answers
Equations and Inequalities Test 1

Q1

$$\frac{3x^2+7x+1}{x+3}<2x-1$$

$$\frac{3x^2+7x+1}{x+3}-(2x-1)<0$$

$$\frac{3x^2+7x+1-(2x-1)(x+3)}{x+3}<0$$

$$\frac{x^2+2x+4}{x+3}<0$$

$$\frac{(x+1)^2+3}{x+3}<0$$

Since $(x+1)^2+3>0$ for all real x, the inequality reduces to:

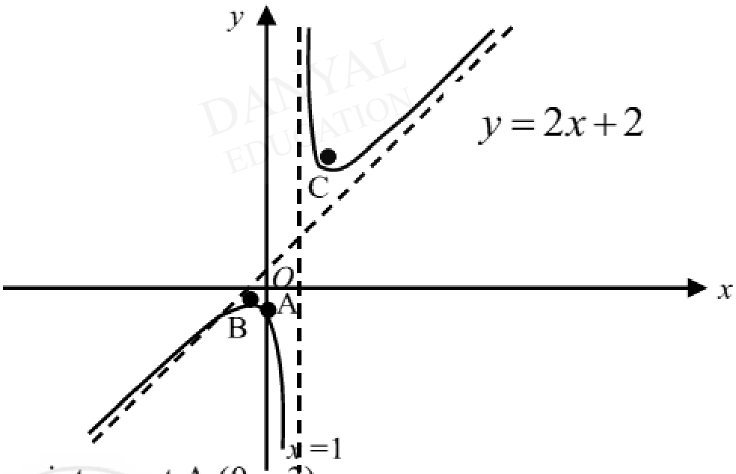
$$x+3<0$$

$$\Rightarrow\ x<-3$$

Q2

(i)
By long division,

$$y = \frac{2x^2+3}{x-1}$$
$$= 2x + 2 + \frac{5}{x-1}$$

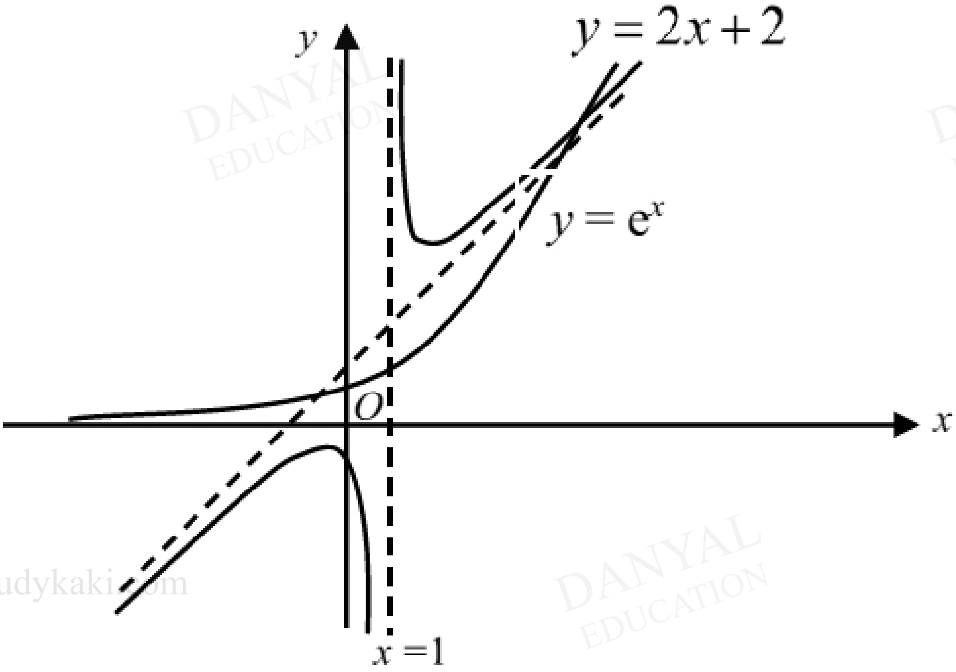


y-intercept A (0, −3)
Max point B (−0.581, −2.32)
Min point C (2.58, 10.3)

(ii)

$$2x+2 \le e^x - \frac{5}{x-1}$$

$$2x+2+\frac{5}{x-1} \le e^x$$



Intersection of both curves: (2.34, 10.4)

$x < 1$ or $x \ge 2.34$

(iii)
Replacing x by $x + 1$

$x+1<1$ or $x+1 \ge 2.34$
$x<0$ or $x \ge 1.34$

Q3

(i) $y = ax^3 + bx^2 + cx + d$

Curve passes through $\left(-2, \frac{34}{3}\right)$:

$a(-2)^3 + b(-2)^2 + c(-2) + d = \frac{34}{3}$

$-8a + 4b - 2c + d = \frac{34}{3}$ — ①

Curve passes through $\left(3, -\frac{19}{2}\right)$:

$a(3)^3 + b(3)^2 + c(3) + d = -\frac{19}{2}$

$27a + 9b + 3c + d = -\frac{19}{2}$ — ②

$\frac{dy}{dx} = 3ax^2 + 2bx + c$

Curve has maximum point $\left(-2, \frac{34}{3}\right)$:

$3a(-2)^2 + 2b(-2) + c = 0$

$12a - 4b + c = 0$ — ③

Curve has minimum point $\left(3, -\frac{19}{2}\right)$:

$3a(3)^2 + 2b(3) + c = 0$

$27a + 6b + c = 0$ — ④

Solving, $a = \frac{1}{3}, b = -\frac{1}{2}, c = -6, d = 4$

$\therefore\ y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$

Most common mistake:
- Some students assumed the coeff of x^3 is 1, eg, $y = x^3 + bx^2 + cx + d$

Some attempt to form ONLY 2 or 3 equations to solve for 4 unknowns; note that at least 4 eqns are needed to solve for 4 unknowns.

A few students left their eqn as $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$ instead of $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$