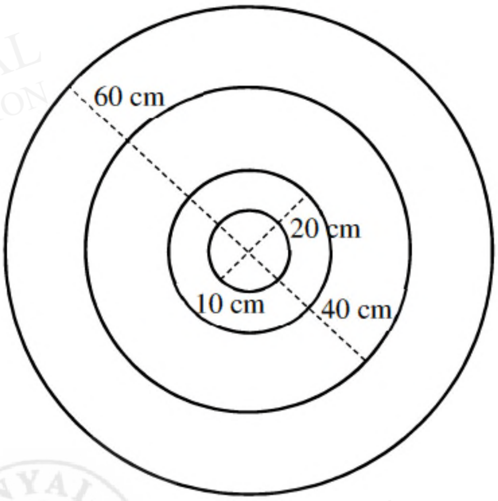
A Level H2 Math

Discrete Random Variable Test 4

Q1

An archer shoots an arrow into a circular target board that has a radius of 60 cm. The target board further consists of three inner concentric circular sections, with radii 40 cm, 20 cm and 10 cm respectively as shown in the diagram.



The archer scores

- 50 points if the arrow lands in the centre circle of radius 10 cm,
- 20 points if the arrow lands in the ring with outer radius 20 cm,
- 10 points if the arrow lands in the ring with outer radius 40 cm,
- 0 point otherwise.

Assume that the arrow will definitely hit the target board and is equally likely to hit any portion of the target board.

(i) Let X be the number of points scored for one arrow shot. Find the expectation of X, leaving your answer in 4 significant figures. [3]

(ii) Interpret, in this context, the value obtained in part **(i)**. [1]

(iii) The archer shot at the target board forty times. Find the probability that the average score obtained by the archer is between 10 and 20 points (inclusive). [4]

Q2

An unbiased cubical die has the number 1 on one face, the number 2 on two faces and the number 3 on three faces. Adrian invites Benny to play a game. In each round, Benny rolls the die twice. Adrian pays Benny \$$a$ if the total score is 2 and \$3 if the total score is 3. However, if the total score is 4, Benny pays Adrian \$2. No payment is made otherwise.

(i) Find the probability that Adrian pays Benny at least 5 times in 20 rounds. [4]

The random variable X represents Benny's winnings in each round.

(ii) Given that $a = 6$, find the probability distribution of X. Hence, help Benny decide if he should accept Adrian's invitation to play the game. Justify your answer. [5]

(iii) Determine the value of a for the game to be fair. [1]

Q3

A fairground game involves trying to hit a moving target with a gunshot. A round consists of a **maximum** of 3 shots. Ten points are scored if a player hits the target. The **round** ends **immediately** if the player misses a shot. The probability that Linda hits the target in a single shot is 0.6. All shots taken are independent of one another.

(i) Find the probability that Linda scores 30 points in a round. [2]

The random variable X is the number of points Linda scores in a round.

(ii) Find the probability distribution of X. [3]

(iii) Find the mean and variance of X. [4]

(iv) A game consists of 2 rounds. Find the probability that Linda scores more points in round 2 than in round 1. [2]

Answers
Discrete Random Variable Test 4

Q1

(i) Given that X is the number of points scored for one arrow shot.

$$P(X=50)=\frac{\pi(10)^2}{\pi(60)^2}=\frac{1}{36}$$

$$P(X=20)=\frac{\pi(20)^2-\pi(10)^2}{\pi(60)^2}=\frac{1}{12}$$

$$P(X=10)=\frac{\pi(40)^2-\pi(20)^2}{\pi(60)^2}=\frac{1}{3}$$

$$E(X)=(10)\left(\frac{1}{3}\right)+(20)\left(\frac{1}{12}\right)+(50)\left(\frac{1}{36}\right)$$
$$=6.389 \quad \text{(4 sig fig)}$$

(ii)
If the archer is to shoot at the target board repeatedly, then in the long run his average score will be 6.389 points.

(iii)

$$\text{Var}(X)=(10)^2\left(\frac{1}{3}\right)+(20)^2\left(\frac{1}{12}\right)+(50)^2\left(\frac{1}{36}\right)-(6.38888)^2$$
$$=95.2932$$

Let $\bar{X}=\dfrac{X_1+X_2+...+X_{40}}{40}$.

Since $n = 40$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(6.38888, \dfrac{95.2932}{40}\right)$ approximately.

Required probability
$=P(10<\bar{X}<20)$
$=0.00965$ (3 sig fig)

Q2
(i)

Die shows	1	2	3
Probability	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

P(Adrian pays Benny in a round)
= P(total score is 2) + P(total score is 3)

$$= \frac{1}{6}\times\frac{1}{6} + \left(\frac{1}{6}\times\frac{1}{3}\right)2$$

$$= \frac{5}{36}$$

Let Y be the number of rounds, out of 20, that Adrian pays Benny.

$$Y \sim \mathrm{B}\left(20, \frac{5}{36}\right)$$

$\mathrm{P}(Y \geq 5)$
$= 1 - \mathrm{P}(Y \leq 4)$
$= 0.134$ (3 s.f.)

(ii)

P(total score is 4) $= \left(\frac{1}{6}\times\frac{1}{2}\right)2 + \frac{1}{3}\times\frac{1}{3} = \frac{5}{18}$

Method 1 :

P(total score is 5 or 6) $= 1 - \left(\frac{1}{36} + \frac{1}{9} + \frac{5}{18}\right) = \frac{7}{12}$

Method 2 :

P(total score is 5 or 6) $= \left(\frac{1}{3}\times\frac{1}{2}\right)2 + \frac{1}{2}\times\frac{1}{2} = \frac{7}{12}$

Given : X represents Benny's winnings in each round

x	6	3	-2	0
$\mathrm{P}(X = x)$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{7}{12}$

$\mathrm{E}(X) = -\frac{1}{18}$

Since $\mathrm{E}(X) < 0$, Benny is expected to lose in the long run. Thus, Benny should not accept Adrian's invitation to play the game.

(iii)
For the game to be fair, $\mathrm{E}(X) = 0$.

$$\frac{a}{36} + \frac{1}{3} - \frac{10}{18} = 0$$

$$a = 8$$

Q3

(i)

P(Linda scores 30 points) $= P(\{\text{hit, hit, hit}\})$

$= 0.6^3$

$= \frac{27}{125}$ (0.216)

(ii)

Let X be the number of points scored by Linda in a round.

X	0	10	20	30
$P(X=x)$	0.4	0.6×0.4 $=0.24$	$0.6^2\times0.4$ $=0.144$	0.216

(iii)

$E(X) = 0\times0.4 + 10\times0.24 + 20\times0.144 + 30\times0.216$
$=11.76$

$E(X^2) = 0^2\times0.4 + 10^2\times0.24 + 20^2\times0.144 + 30^2\times0.216$
$= 276$

$\text{Var}(X) = E(X^2) - [E(X)]^2$
$= 276 - 11.76^2 = 137.7024$

(iv)

Let X_1 be the number of points scored by Linda in Round 1 and let X_2 be the number of points scored by Linda in Round 2.

P(Linda scores more in round 2 than in round 1)

$= P(X_1 = 0\ \&\ X_2 \geq 10)$

$+P(X_1 = 10\ \&\ X_2 \geq 20)$

$+P(X_1 = 20\ \&\ X_2 = 30)$

$= P(X_1 = 0)P(X_2 \geq 10)$

$+P(X_1 = 10)P(X_2 \geq 20)$

$+P(X_1 = 20)P(X_2 = 30)$

$= 0.4\times(1-0.4)$

$+0.24\times(0.144+0.216)+0.144\times0.216$

$= 0.357504 = 0.358$ (3 s.f.)