A Level H2 Math

Binomial Distribution Test 4

Q1

In a large consignment of mangoes, 4.5% of the mangoes are damaged.

(i) A total of 21 mangoes are selected at random. Calculate the probability that not more than 3 mangoes are damaged. [2]

(ii) The mangoes are randomly selected and packed into boxes of 21. For shipping purposes, the boxes are packed into cartons, with each carton containing 12 boxes. A box containing more than 3 damaged mangoes is considered low standard. Calculate the probability that, in a randomly selected carton, there are at least 2 boxes which are of low standard. [3]

(iii) Find the probability that a randomly chosen box that is of low standard contains no more than five damaged mangoes. [3]

Q2

At a hospital, records show that 84.5% of patients turn up for their appointments. It is known that on any day, the doctor has time to see 20 patients.

On one particular day, there are 20 patients who make appointments to see the doctor.

(i) State, in this context, one condition that must be met for the number of patients who turn up for their appointments to be well modelled by a binomial distribution. [1]

For the remainder of this question, assume that the condition stated in part **(i)** is met.

(ii) Find the probability that more than 15 patients turn up for their appointments. [2]

(iii) Given that at least 12 patients turn up for their appointments, find the probability that more than 2 patients fail to turn up for their appointments. [3]

(iv) To improve efficiency, the hospital decides to increase the number of appointments that can be made on each day. Given that there will still be enough time for the doctor to see 20 patients, find the greatest number of appointments that can be made so that there is a probability of at least 0.85 of the doctor having time to see all patients who turn up. [2]

Q3

It is a common practice for airlines to sell more plane tickets than the number of seats available. This is to maximise their profits as it is expected that some passengers will not turn up for the flight.

The plane used by Victoria Airline for her daily 10 am flight from Singapore to Hong Kong has a maximum capacity of 150 seats. For this particular flight, 154 tickets are sold every day. On average, p out of 100 customers who have purchased a plane ticket for this flight turn up. Customers who turn up after the flight is full will be turned away. The number of customers who turn up for the 10 am flight, on a randomly chosen day, is denoted by X.

(i) State, in the context of this question, two assumptions needed to model X by a binomial distribution. [2]

(ii) Explain why one of the assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that these assumptions do in fact hold.

(iii) It is known that there is a 0.05 probability that at least 153 customers will turn up for the 10 am flight. Write down an equation for the value of p, and find this value numerically. [3]

It is given instead that $p = 94$.

(iv) Find the probability that, on a randomly chosen day,

(a) there are at least 141 but not more than 148 customers who turn up for the 10 am flight, [2]

(b) every customer who turns up gets a seat on the 10 am flight. [1]

(v) Find the probability that every customer who turns up gets a seat on the 10 am flight on more than 5 days in a week. [3]

Answers

Binomial Distribution Test 4

Q1

(i) Let X be the random variable "number of damaged mangoes out of 21 mangoes".

$$X \sim B(21, 0.045)$$

$$P(X \le 3) = 0.98673 = 0.987 \text{ (3 s.f.)}$$

(ii) Let Y be the random variable "number of boxes of mangoes out of 12 boxes which are of low standard".

$$Y \sim B(12, 1-0.98673) \Rightarrow Y \sim B(12, 0.013268)$$

$$P(Y \ge 2) = 1 - P(Y \le 1)$$
$$= 1 - 0.98936 = 0.01064 = 0.0106 \text{ (3 s.f.)}$$

(iii) $\text{P(required)} = P(X \le 5 \mid \text{box is of low standard})$

$$= P(X \le 5 \mid X > 3)$$
$$= \frac{P(X \le 5 \cap X > 3)}{P(X > 3)}$$
$$= \frac{P(X = 4) + P(X = 5)}{1 - P(Y \le 3)}$$
$$= \frac{0.011219 + 0.0017975}{1 - 0.98673}$$
$$= 0.981$$

Q2

(i)

Whether a randomly chosen patient turns up for an appointment is independent of any other patient.

(ii)

Let X be the number of patients who turn up for their appointments, out of 20 appointments.

$X \sim B(20, 0.845)$

$P(X > 15)$

$= 1 - P(X \le 15)$

$= 0.812$ (3 sig fig)

(iii)

Required probability

$= P(X \le 17 \mid X \ge 12)$

$= \dfrac{P(12 \le X \le 17)}{P(X \ge 12)}$

$= \dfrac{P(X \le 17) - P(X \le 11)}{1 - P(X \le 11)}$

$= 0.618$ (3 sig fig)

(iv)

Let Y be the number of patients who turn up for their appointments, out of n appointments.

$Y \sim B(n, 0.845)$

$P(Y \le 20) \ge 0.85$ --- (*)

Using GC,

When $n = 21$, $P(Y \le 20) = 0.9709$ (> 0.85)

When $n = 22$, $P(Y \le 20) = 0.8762$ (> 0.85)

When $n = 23$, $P(Y \le 20) = 0.7146$ (< 0.85)

$\therefore$ Largest n is 22.

Q3

)i The assumptions are

(1) The probability that a customer turn up for the flight is $\frac{p}{100}$ <u>for all the 154 customers</u>.

(2) Customers turn up independently of each other.

)ii Customers may be travelling in a group or as a family. Therefore, customers may not turn up independently of the others in their group.

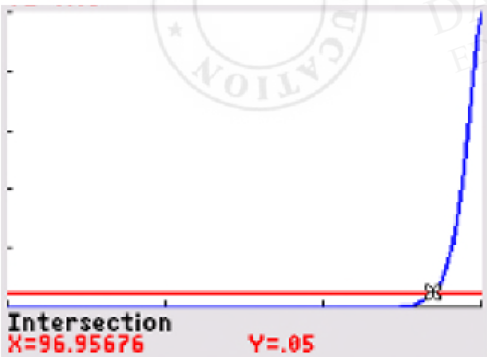
)iii $X \sim \mathrm{B}\left(154, \frac{p}{100}\right)$

Given $\mathrm{P}(X \geqslant 153) = 0.05$

$$\mathrm{P}(X = 153) + \mathrm{P}(X = 154) = 0.05$$

$$\binom{154}{153}\left(\frac{p}{100}\right)^{153}\left(1 - \frac{p}{100}\right) + \binom{154}{154}\left(\frac{p}{100}\right)^{154} = 0.05$$

$$154\left(\frac{p}{100}\right)^{153}\left(1 - \frac{p}{100}\right) + \left(\frac{p}{100}\right)^{154} = 0.05$$



From the GC, $p = 96.9568 = 97.0$ (to 3 s.f.)

)iv a $X \sim \mathrm{B}(154, 0.94)$

$$\mathrm{P}(141 \leqslant X \leqslant 148) = \mathrm{P}(X \leqslant 148) - \mathrm{P}(X \leqslant 140)$$
$$= 0.825$$

)iv b $\mathrm{P}(X \leqslant 150) = 0.98443 = 0.984$ (to 3 s.f.)

)v Let Y be the number of days (out of 7) in which every customer who turns up gets a seat on the flight

$$Y \sim \mathrm{B}(7, 0.98443)$$

$$\mathrm{P}(Y > 5) = 1 - \mathrm{P}(Y \leqslant 5)$$
$$= 0.995$$