A Level H2 Math

AP and GP Test 3

Q1

For this question, you may leave your answers to the nearest dollar.

(a) Mr Foo invested \$25,000 in three different stocks A, B and C. After a year, the value of the stocks A and B grew by 2% and 6% respectively, while the value of stock C fell by 2%. Mr Foo did not gain or lose any money. Let a, b and c denote the amount of money he invested in stocks A, B and C respectively.

(i) Find expressions for a and b, in terms of c. [2]

(ii) Find the values between which c must lie. [2]

(b) Mr Lee is interested in growing his savings amount of \$55,000 and is considering the Singapore Savings Bonds. He is able to enjoy a higher average return per year when he invests over a longer period of time as shown in the following table.

Number of years invested	1	2	3	4	5	6	7	8
Average return per year, %	1.04	1.21	1.35	1.48	1.60	1.71	1.82	1.92

For example, if Mr Lee invests for two years, he is able to enjoy compound interest at a rate of 1.21% per year.

(i) Calculate the compound interest earned by Mr Lee if he were to invest \$55,000 in this bond for a period of five years. [2]

A bank offers a dual-savings account with the following scheme:

" For every \$1,000 deposited into the normal savings account, an individual can deposit \$10,000 into the special savings account to enjoy a higher interest rate. The annual compound interest rates for the normal savings account and the special savings account are 0.19% and 1.8% respectively."

Mr Lee is interested in setting up this dual-savings account and considers an n-year investment plan as such:

At the start of each year, he will place \$1,000 in the normal savings account and \$10,000 in the special savings account.

(ii) Find the respective amount of money in the normal savings account and special savings account at the end of n years. Leave your answers in terms of n. [4]

(iii) Find the least value of n such that the compound interest earned in dual-savings account is more than the compound interest earned in part **(i)**. [2]

Q2

A manual hoist is a mechanical device used primarily for raising and lowering heavy loads, with the motive power supplied manually by hand. Three hoists, A, B and C are used to lift a load vertically.

(i) For hoist A, the first pull will raise the load by a vertical distance of 45 cm. On each subsequent pull, the load will raise 1.6 cm lesser than the vertical distance covered by the previous pull. Determine the number of pulls needed for the load to achieve maximum total height. Hence find this maximum total height. [4]

(ii) For hoist B, the first pull will raise the load by a vertical distance of 45 cm. On each subsequent pull, the vertical distance raised will be 95% of the distance covered by the previous pull. Find the theoretical maximum total height that the load can reach. [2]

(iii) For hoist C, every pull will raise the load by a constant vertical distance of 45 cm. However, after each pull, the load will slip and drop by 2% of the total vertical height the load has reached. Show that just before the 4^{th} pull, the load would have reached a total vertical height of 130 cm, correct to 3 significant figures.

Hence show that before the $(n+1)^{\text{th}}$ pull, the load would have reached a total vertical height of $X + Y(0.98)^{n+1}$, where X and Y are integers to be determined. [5]

(iv) Explain clearly if hoist C can lift the load up a building of height 25 metres. [2]

Q3

The sum, S_n, of the first n terms of a sequence u_1, u_2, u_3, … is given by

$$S_n = b - \frac{3a}{(n+1)!},$$

where a and b are constants.

(i) It is given that $u_1 = k$ and $u_2 = \frac{2}{3}k$, where k is a constant. Find a and b in terms of k. [3]

(ii) Find a formula for u_n in terms of k, giving your answer in its simplest form. [2]

(iii) Determine, with a reason, if the series $\sum_{r=1}^{\infty} u_r$ converges. [1]

Answers

AP and GP Test 3

Q1

(a)(i) $a+b+c=25000$ ----------(1)

$0.02a+0.06b-0.02c=0$ ----------(2)

[or $1.02a+1.06b+0.98c=25000$]

Solving SLE,

$a=37500-2c$

$b=c-12500$

(ii) Since a and b must both be positive, it implies that c must lie between 12500 and 18750.

(b)(i) Since Mr Lee invested in a period of five years, the average return per year will be 1.6%.

Total amount of interest earned

$=(1.016)^5(55000)-55000$

$=4543$ (to the nearest dollar)

(ii) Amount of money in the normal savings account at the end of n years

$=1000(1.0019+1.0019^2+1.0019^3+\ldots+1.0019^n)$

$=1000(1.0019)\left(\dfrac{1.0019^n-1}{1.0019-1}\right)$

$=527315.79(1.0019^n-1)$

Amount of money in the special savings account at the end of n years

$=10000(1.018)\left(\dfrac{1.018^n-1}{1.018-1}\right)$

$=565555.56\left(1.018^n-1\right)$

(iii) Total interest earned from dual-savings account

$=527315.79(1.0019^n-1)+565555.56\left(1.018^n-1\right)-11000n$

$527315.79(1.0019^n-1)+565555.56\left(1.018^n-1\right)-11000n>4543$

From GC, $n\geq 7$

Least value of n is 7.

Q2

(i)

Method 1

Distance covered at the n^{th} pull $= 45 + (n-1)(-1.6)$
$= 46.6 - 1.6n$

$46.6 - 1.6n \geq 0$

$n \leq 29.125$

Hence number of pulls needed to achieve maximum total height is 29.

Maximum total height

$= \frac{29}{2}\left[2(45) + (29-1)(-1.6)\right]$

$= 655.4 \text{ cm}$

Method 2

Distance covered at the n^{th} pull, $u_n = 45 + (n-1)(-1.6)$
$= 46.6 - 1.6n$

Using GC,

n	u_n
29	0.2
30	−1.4

Hence number of pulls needed to achieve maximum total height is 29.

Maximum total height $= \frac{29}{2}(45 + 0.2) = 655.4 \text{ cm}$

Method 3

Distance covered at the n^{th} pull $= 45 + (n-1)(-1.6) = 0$
$\Rightarrow n = 29.125$

n	u_n
29	0.2
30	−1.4

Hence number of pulls needed to achieve maximum total height is 29.

Maximum total height $= \frac{29}{2}(45 + 0.2) = 655.4 \text{ cm}$

Method 4

Total height after n pulls,

$S_n = \frac{n}{2}\left[2(45) + (n-1)(-1.6)\right] = 45.8n - 0.8n^2$

Using GC,

n	S_n
28	655.2
29	655.4
30	654

Hence the number of pulls needed to achieve maximum total height is 29, and the maximum total height covered is 655.4 cm.

(ii)

Since $r = 0.95 < 1$, sum to infinity of G.P. exists.

$\therefore$ maximum total height $= \dfrac{45}{1-0.95} = 900$ cm

(iii)

	Total height reached
Before 2^{nd} pull	$0.98(45)$
Before 3^{rd} pull	$0.98\left(0.98(45)+45\right)$ $=0.98^2(45)+0.98(45)$
Before 4^{th} pull	$0.98\left(0.98^2(45)+0.98(45)+45\right)$ $=0.98^3(45)+0.98^2(45)+0.98(45)$
$\vdots$	$\vdots$
Before $(n+1)^{\text{th}}$ pull	$0.98^n(45)+0.98^{n-1}(45)+\ldots+0.98(45)$ $=\dfrac{0.98(45)(1-0.98^n)}{1-0.98}$ [sum of G.P. with $a=45$, $r=0.98$]

$\therefore$ before 4^{th} pull, total height reached

$=\dfrac{0.98(45)(1-0.98^3)}{1-0.98}$

$=129.67164$

$=130$ cm (3 s.f.)

Before $(n+1)^{\text{th}}$ pull, total height reached

$=\dfrac{0.98(45)(1-0.98^n)}{1-0.98}$

$=2205-2250(0.98)^{n+1}$, where $X=2205$, $Y=-2250$

(iv)

From **(iii)**,

Total height reached by load using hoist C $= 2205-2250(0.98)^{n+1}$

As $n\to\infty$, $(0.98)^{n+1}\to 0$.

Hence maximum total height $\to 2205$.

Therefore maximum total height reached by load using hoist C will approach 2205 cm. Therefore the hoist C cannot be used to lift the load up the building of 2500 cm

Q3

(i) $S_1 = b - \frac{3a}{2!} = b - \frac{3a}{2} = k \quad \ldots(1)$

$S_2 = b - \frac{3a}{3!} = b - \frac{a}{2} = k + \frac{2}{3}k = \frac{5}{3}k \quad \ldots(2)$

$(2)-(1)$,

$$-\frac{a}{2} - \left(-\frac{3a}{2}\right) = \frac{5}{3}k - k$$

$$\therefore \quad a = \frac{2}{3}k$$

$$\therefore \ b = k + \frac{3a}{2} = k + \frac{3}{2}\left(\frac{2}{3}k\right) = 2k$$

(ii) $S_n = 2k - \frac{2k}{(n+1)!}$

$$u_n = S_n - S_{n-1}$$
$$= \left(2k - \frac{2k}{(n+1)!}\right) - \left(2k - \frac{2k}{n!}\right)$$
$$= \frac{2k}{n!} - \frac{2k}{(n+1)!}$$
$$= \frac{2k}{n!}\left(1 - \frac{1}{n+1}\right)$$
$$= \frac{2k}{n!}\left(\frac{n}{n+1}\right)$$
$$= \frac{2kn}{(n+1)!}$$

(iii) $\sum_{r=1}^{n} u_r = S_n = 2k - \frac{2k}{(n+1)!}$

As $n \to \infty$, $\frac{1}{(n+1)!} \to 0$.

$\therefore \ S_n = 2k - \frac{2k}{(n+1)!} \to 2k$

Hence the series $\sum_{r=1}^{\infty} u_r$ converges.