A Level H2 Math

AP and GP Test 1

Q1

Abbie and Benny each take a \$50 000 study loan for their 3-year undergraduate program, disbursed on the first day of the program. The terms of the loan are such that during the 3-year period of their studies, interest is charged at 0.1% of the outstanding amount at the end of each month. Upon graduation, interest is charged at 0.375% of the outstanding amount at the end of each month.

(a) Since the interest rate is lower during her studies, Abbie decides that she will make a constant payment at the beginning of each month from the start of the program for its entire duration.

(i) Find the amount, correct to the nearest cent, Abbie needs to pay at the beginning of each month so that the outstanding amount after interest is charged remains at \$50 000 at the end of every month. [2]

(ii) After graduating, Abbie intends to increase her payment to a constant \$$k$ at the beginning of every month. Show that the outstanding amount Abbie owes the bank at the end of n months after graduation, and after interest is charged, is

$$\$\left[1.00375^n(50000)-\frac{803}{3}k\left(1.00375^n-1\right)\right].$$ [2]

(iii) Abbie plans to repay her loan within 10 years after graduation. Determine if she can do this with a monthly instalment of \$500, justifying your answer. [1]

Find the amount she needs to pay so that she fully repays her loan at the end of exactly 10 years after graduation, leaving your answer to the nearest cent. [2]

(b) Benny wishes to begin his loan repayment only after graduation. Like Abbie, he aims to repay the loan at the end of exactly 10 years after graduation.

Leaving your answer to the nearest cent, find

(i) the constant amount Benny needs to pay each month in order to do this, [3]

(ii) the amount of interest Benny pays altogether. [2]

Q2

Timber cladding is the application of timber planks over timber planks to provide the layer intended to control the infiltration of weather elements.

(a) Using method *A*, 20 rectangular planks are used and the lengths of the planks form an arithmetic progression with common difference *d* cm. The shortest plank has length 65 cm and the longest plank has length 350 cm.

(i) Find the value of *d*. [2]

(ii) Find the total length of all the planks. [2]

(b) Using method *B*, a long plank of 2000 cm is sawn off by a machine into *n* smaller rectangular planks. The length of the first plank is a cm and each successive plank is $\frac{8}{9}$ as long as the preceding plank.

(i) Show that the total length of the planks sawn off can never be greater than *k* times the length of the first plank, where *k* is an integer to be determined. [2]

(ii) Given that $a = 423$, find the greatest possible integral value of *n* and the corresponding length of the shortest plank. [4]

Q3

Henry and Isaac take part in a marathon race. In their first training session, they run a distance of 2.4 km each.

(a) Henry increases the distance he runs in each subsequent training session by 400 m.

(i) Find the distance he runs in the 20th session. [2]

(ii) Find the minimum number of sessions he needs to attend in order to run a total distance of 99 km. [3]

(b) **(i)** Isaac increases the distance he runs in each subsequent session by x %. Find x if Isaac runs a total distance of 200 km at the end of 20 sessions. [3]

(ii) Isaac feels that the training is too tough after the first session. He decides to decrease the distance he runs in each subsequent session by 5% and increase the numbers of sessions. Will he be able to run a total distance of 200 km? Justify your answer. [2]

Answers

AP and GP Test 1

Q1

(a)(i)	After one month, if she pays $\$x$ at the beginning of the month, she will owe the bank $\$(50000-x)\times(1.001)$ Hence $(50000-x)\times(1.001)=50000 \quad \Rightarrow \quad x=49.95$ Abbie needs to pay \$49.95 (to the nearest cent) a month.	Many students were confused about the interest rate, and hence multiplied by 1.1 or 1.01. Some merely took 0.1% of \$50,000.
(a)(ii)	One month after graduating, she owes $(50000-k)\times(1.00375)$. n months after graduating, she will owe $1.00375^n(50000-k)-1.00375^{n-1}k-\ldots-1.00375k$ $=1.00375^n(50000)-k\left(1.00375^n+1.00375^{n-1}+\ldots+1.00375\right)$ $=1.00375^n(50000)-k\left[\frac{1.00375\left(1.00375^n-1\right)}{1.00375-1}\right]$ $=1.00375^n(50000)-\frac{803}{3}k\left(1.00375^n-1\right)$ (shown).	While many students were able to deduce that this was the sum of a GP, a common mistake was thinking that the last/first term of the GP was 1 instead of 1.00375.
(a)(iii)	Sub n = 120, and k = 500: $1.00375^{120}(50000)-\frac{803}{3}(500)\left(1.00375^{120}-1\right)=2467.11>0$. No, she cannot. A monthly payment of \$500 is not enough. When n = 120, $1.00375^{120}(50000)-\frac{803}{3}k\left(1.00375^{120}-1\right)=0$ $\Rightarrow \quad k=516.26$ (nearest cent) She needs to pay \$516.26 per month.	Many students did not realise n was in months, and used n = 10.
(b)(i)	Oustanding amount upon graduation $=1.001^{36}(50000)$ $=51831.86$ Using Abbie's formula, but with a starting outstanding amount of \$51831.86, $1.00375^{120}(51831.86)-\frac{803}{3}k\left(1.00375^{120}-1\right)=0$ $\Rightarrow \quad k=535.17$ (nearest cent) He needs to pay \$535.17 per month.	Some students used 1.00375^{36}. Some took the 35^{th} power. Many students did not realise they could use the same formula as (a)(iii) but with a different starting amount. As with the previous parts, some interpreted the interest rate wrongly and used 1.1 or 1.01, and some thought n was in years.
(b)(ii)	$120\times535.17-50000=14220.43$ (to 2 d.p.) He paid \$14220.43 in interest altogether.	Some students had very involved ways of calculating the interest, including summing the GP all over again. Many students did not subtract 50,000.

Q2

(a)(i)

$$u_{20} = a + (n-1)d$$
$$350 = 65 + 19d$$
$$d = 15$$

(a)(ii)

$$S_{20} = \frac{20}{2}(65+350)$$
$$= 4150 \text{ cm} \qquad \text{(Accept: 41.5 m)}$$

(b)(i)

$$S_\infty = \frac{a}{1-8/9}$$
$$= 9a$$

$\therefore$ integer $k = 9$.

(i)

Method 1:

Number of ways $= \binom{14}{3} \times 3! = 2184$

Method 2:

Number of ways $= 14 \times 13 \times 12 = 2184$

(b)(ii)

$$S_n \le 2000$$
$$\frac{423\left[1-\left(8/9\right)^n\right]}{1-8/9} \le 2000$$
$$1-\left(8/9\right)^n \le \frac{2000}{3807}$$
$$\left(8/9\right)^n \ge \frac{1807}{3807}$$
$$n \le \frac{\ln\left(1807/3807\right)}{\ln\left(8/9\right)}$$
$$n \le 6.3267$$

$\therefore$ Largest integer $n = 6$.

Length of shortest plank is $u_6 = 423\left(\frac{8}{9}\right)^{6-1}$

$$= 235 \text{ cm (3 s.f.)}$$

Q3

(ai)
AP: $a = 2.4$, $d = 0.4$
Distance he runs in the 20th session
$= 2.4 + (20-1)(0.4)$
$= 10$ km

(aii)
$S_n \geq 99$

$\Rightarrow \quad \frac{n}{2}\left[2(2.4)+(n-1)(0.4)\right] \geq 99$

$\Rightarrow \quad n\left[4.8+0.4n-0.4\right] \geq 198$

$\Rightarrow \quad 0.4n^2 + 4.4n - 198 \geq 0$

$\Rightarrow \quad n \leq -28.4 \quad$ or $\quad n \geq 17.4$

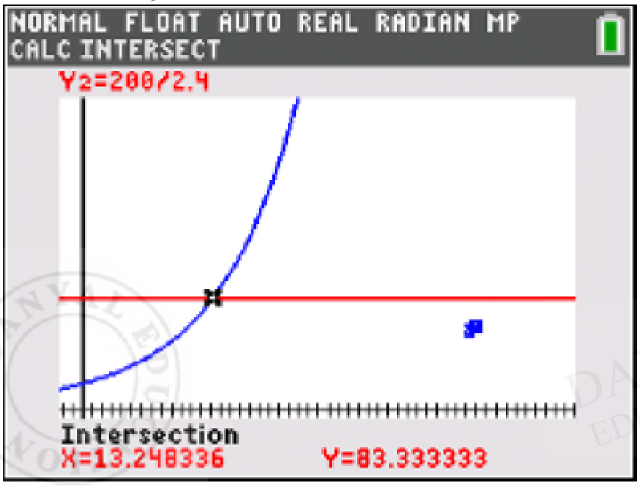
(rejected as $n > 0$)
Least value of $n = 18$
He needs a minimum of 18 sessions.

(bi)

$$S_{20}=\frac{2.4\left(\left(1+\frac{x}{100}\right)^{20}-1\right)}{\left(1+\frac{x}{100}\right)-1}=200$$

$$\frac{\left(1+\frac{x}{100}\right)^{20}-1}{\frac{x}{100}}=\frac{200}{2.4}$$

From GC,



$x = 13.2\%$

(bii)

$$\text{Sum to infinity} = \frac{2.4}{1-0.95} = 48$$

Hence, total distance can never be greater than 200 km.