COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019

ADDITIONAL MATHEMATICS
PAPER 1

Name: ______________________________ (   ) Class: __________

SECONDARY FOUR EXPRESS | **Wednesday 8 May 2019**
SECONDARY FIVE NORMAL ACADEMIC | **08 00 – 10 00**
4047/1 | **2h**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: **Lee Ying Jie**

This paper consists of **5** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

1. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the set of values of x for which $(2+3x)(x-5) > 2+3x$. [3]

2 A curve is such that $\frac{d^2y}{dx^2} = 9x+1$. The gradient of the curve at the point $(2,16)$ is 18. Find the equation of the curve. [6]

3 **(i)** On the same axes sketch the curves of $y = 2x^{-\frac{2}{3}}$ and $y^3 = x$ for $x \geq 0$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

4 The variables x and y are such that when the values of $\frac{x}{y}$ are plotted against x a straight line graph is obtained. It is given that the line passes through the point $(3\sqrt{3}, 4)$ and forms an angle of $60°$ with the x-axis. Express y in terms of x. [4]

5 **(i)** By using long division, divide $2x^3 - 6x^2 + x - 3$ by $x-3$. [1]

(ii) Express $\frac{3x^2+21x+5}{2x^3-6x^2+x-3}$ in partial fractions. [5]

6 The equation of a curve is $y = \frac{1}{2}e^{2x} - 7e^x + 6x$.

(i) Write down expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [2]

(ii) Find the exact x-coordinates of the stationary points on the curve. [4]

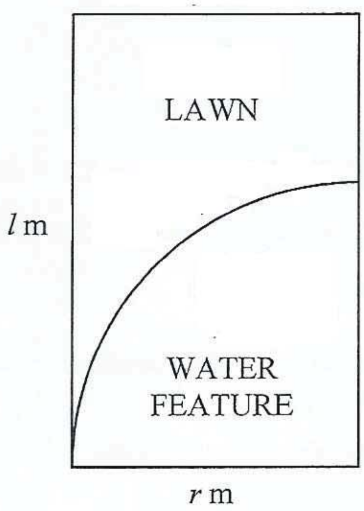
(iii) Determine the nature of each of these stationary points. [3]

7 Find

(i) $\int 3x^4 - 5\cos 6x \, dx$, [2]

(ii) $\int \sin x + \sec^2(5+2\pi x) \, dx$. [2]

8 As part of a garden design, there are plans to put aside a rectangular space which has sides of lengths r m and l m. This rectangular space is to include a quadrant-shaped water feature and a lawn. The area of the lawn is to be 360 m^2.



(i) Show that the perimeter, P m, of the lawn is given by $P = \frac{720}{r} + \pi r$. [4]

A hedge is to be planted along the perimeter of the lawn.

(ii) Given that r can vary, find the dimensions of the rectangular space which can allow the shortest length of hedge to be planted along the perimeter of the lawn. [6]

9 (i) Show that $\frac{\tan^2 x - 1}{\tan^2 x + 1} = 1 - 2\cos^2 x$. [3]

(ii) Hence find, for $0 \le x \le 5$, the values of x in radians for which $\frac{\tan^2 x - 1}{\tan^2 x + 1} = \frac{1}{2}$. [3]

10 The roots of the quadratic equation $4x^2 - 6x + 3 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Find a quadratic equation with roots α^3 and β^3. [6]

11 A rectangular piece of cardboard has the dimensions 18 cm by $5-\sqrt{3}$ cm.

(i) Find the exact value of the square of its diagonal. [2]

A small square piece with sides $1+\sqrt{3}$ cm is cut out from the rectangular piece of cardboard.

(ii) Express in the form $\frac{a+b\sqrt{3}}{c}$, the area of the small square as a fraction of the area of the rectangular piece of cardboard. [3]

12 **(i)** Given that $y=x\sqrt{5x^2-6}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Hence, evaluate $\int_2^4 \frac{5x^2-3}{\sqrt{5x^2-6}}\,\mathrm{d}x$. [3]

13 The equation of the tangent to a circle at the point $A(8,9)$ is given by $4y+3x=60$. The line $y=4x-7$ passes through the centre, P, of the circle.

(i) Find the coordinates of P. [4]

(ii) Find the equation of the circle. [3]

The tangent to the circle at A meets the y-axis at point B.

(iii) Find the equation of another circle with BP as diameter. [4]

END OF PAPER

COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019

ADDITIONAL MATHEMATICS
PAPER 2

Name: ______________________________ (   )   Class:__________

SECONDARY FOUR EXPRESS | **Monday 13 May 2019**
SECONDARY FIVE NORMAL ACADEMIC | **08 00 – 10 30**
4047/02 | **2h 30min**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Name of setter: Ms Kelly Zhang

This paper consists of **5** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 3 - \sin x$.

(i) Explain why the curve $y = \mathrm{f}(x)$ has no turning points. [2]

(ii) Given that the curve passes through the origin, find an expression for $\mathrm{f}(x)$. [3]

2 The equation of a curve is $y = \dfrac{2x+1}{x-4}$.

(i) State, with explanation, whether y is an increasing or decreasing function. [4]

(ii) A particle moves along the curve $y = \dfrac{2x+1}{x-4}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.5 units per second.
Find the rate at which the x-coordinate of the particle is changing at the instant when $x = 2$. [3]

3 (i) Using $\sin 3x = \sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

(ii) Find all the values of x between 0 and 2π for which $\sin 3x = \sin^2 x$. [4]

4 (i) Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants to be found. [3]

(ii) Using your values of R and α, evaluate $\int_0^{\pi} \sqrt{3}\sin x + \cos x \, \mathrm{d}x$, leaving your answer in exact form. [4]

5 (i) By considering the general term in the binomial expansion of $\left(kx - \dfrac{1}{x^3}\right)^7$, where k is a constant, explain why there are no even powers of x in this expansion. [5]

(ii) Given that the coefficient of the third term is thrice the coefficient of the second term, find the value of k. [3]

6 (i) Find the coordinates of all the points at which the graph of $y = |2x-3| - 4$ meets the coordinates axes. [4]

(ii) Sketch the graph of $y = |2x-3| - 4$. [2]

(iii) Solve the equation $x + 4 = |2x-3|$. [3]

7 It is given that $f(x) = 2x^3 + 6x^2 + 6x + 5$.

(i) Find the remainder when $f(x)$ is divided by $(x+1)$. [2]

(ii) Hence, show that $f(x)$ can be expressed in the form $f(x) = a(x+1)^3 + b$. [2]

(iii) Find the coordinates of the stationary point(s) and ~~state~~ determine the nature of the stationary point(s). [5]

(iv) Using your answer in part (iii), explain why the graph of $y = f(x) + k$ will always cut the x-axis only once for all real values of k. [1]

8 **Answer the whole of this question on a piece of graph paper.**

A cuboid box of volume V cm^3 has a height of x cm and a rectangular base of area $(ax^2 + b)$ cm^2. Corresponding values of x and V are shown in the table below.

x	2	4	6	8
V	24	72	168	336

(i) Using suitable variables, draw, on graph paper, a straight line graph and hence estimate the value of each of the constants a and b. [6]

(ii) When a ball is placed in the box, the ball touches every inner side of the box. By drawing a suitable line, find the diameter of the ball. [4]

(iii) Using your values of a and b, calculate the value of x for which the condition in (ii) can be satisfied. [2]

9 (a) Solve the equation $\log_2(2x+1)-\log_4(x+1)=1$. [4]

(b) Given that $(\log_b \mathrm{e})(\log_b a)(\ln a)=16$, express a in terms of b. [4]

(c) On the same axes, sketch the graphs of $y=e^x$ and $e^y=\dfrac{1}{x}$.

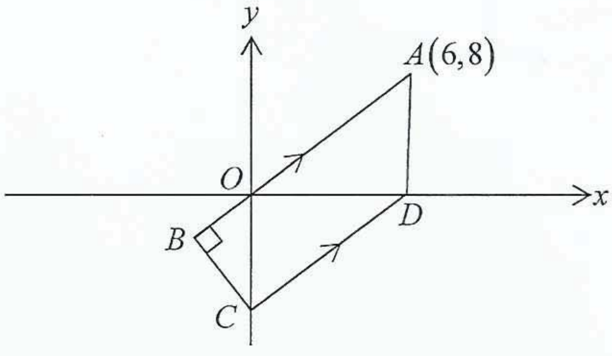
Hence, determine the number of solutions for $\mathrm{e}^x+\ln x=0$. [4]

10 (a) Without using a calculator, find the rational numbers, a and b, for which $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}$ can be expressed as $a\sqrt{2}+b\sqrt{30}$. [4]

(b) (i) Express $5^{2x+3}=5^{x-1}+1$ as a quadratic equation in 5^x and hence find, correct to 2 decimal places, the value of x which satisfies the equation $5^{2x+3}=5^{x-1}+1$. [5]

(ii) Find the range of values of k such that the equation $5^{2x+3}-5^{x-1}+k=0$ has no solution. [3]

11



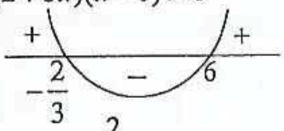
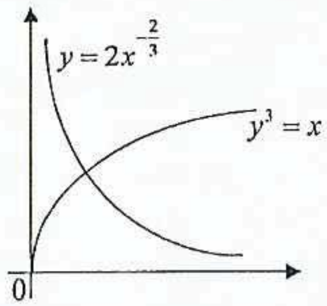
The diagram shows a trapezium $ABCD$ with vertex $A(6,8)$. The sides AB and DC are parallel. AB cuts through the origin O and its length is given to be 15 units. C and D lies on the y-axis and x-axis respectively.

(i) Find the coordinates of vertex B. [4]

(ii) Show that equation of BC is $4y+3x+25=0$. [3]

(iii) Find the coordinates of vertices C and D. [4]

END OF PAPER

Qn	Solution
1	$(2+3x)(x-5)>2+3x$ $(2+3x)(x-5)-(2+3x)>0$ $(2+3x)(x-6)>0$  $\therefore x<-\frac{2}{3}$ or $x>6$
2	$\frac{d^2y}{dx^2}=9x+1$ $\frac{dy}{dx}=\int(9x+1)\ dx$ $=\frac{9}{2}x^2+x+c$ When $x=2$, $\frac{dy}{dx}=18$ $\frac{9}{2}(2)^2+2+c=18$ $c=-2$ $\therefore \frac{dy}{dx}=\frac{9}{2}x^2+x-2$ $y=\int\left(\frac{9}{2}x^2+x-2\right)dx$ $=\frac{3}{2}x^3+\frac{1}{2}x^2-2x+d$ When $x=2$, $y=16$, $16=\frac{3}{2}(2)^3+\frac{1}{2}(2)^2-2(2)+d$ $d=6$ $\therefore y=\frac{3}{2}x^3+\frac{1}{2}x^2-2x+6$
3(i)	

3(ii)	$x^{\frac{1}{3}} = 2x^{-\frac{2}{3}}$ $x = 2$ $y = \sqrt[3]{2}$ $=1.26$ (3s.f.) Coordinates of the point of intersection are $(2, 1.26)$
4	$\frac{x}{y} = mx + c ------(1)$ $m = \tan 60° = \sqrt{3}$ Sub $m = \sqrt{3}$, $(3\sqrt{3}, 4)$ into (1): $4 = \sqrt{3}(3\sqrt{3}) + c$ $c = -5$ $\frac{x}{y} = \sqrt{3}x - 5$ $y = \frac{x}{\sqrt{3}x - 5}$
5(i)	Quotient: $2x^2 \quad +1$ $x-3 \overline{)2x^3 - 6x^2 + x - 3}$ $-(2x^3 - 6x^2)$ $x - 3$ $-(x-3)$ 0 $\frac{2x^3 - 6x^2 + x - 3}{x-3} = 2x^2 + 1$

5(ii)	$\dfrac{3x^2+21x+5}{2x^3-6x^2+x-3}=\dfrac{3x^2+21x+5}{(x-3)(2x^2+1)}$ let $\dfrac{3x^2+21x+5}{(x-3)(2x^2+1)}=\dfrac{A}{x-3}+\dfrac{Bx+C}{2x^2+1}$ $3x^2+21x+5=A(2x^2+1)+(Bx+C)(x-3)$ Sub $x=3$: $95=19A$ $A=5$ Sub $x=0$: $5=5-3C$ $C=0$ Sub $x=1$: $29=15-2B$ $B=-7$ $\dfrac{3x^2+21x+5}{2x^3-6x^2+x-3}=\dfrac{5}{x-3}-\dfrac{7x}{2x^2+1}$
6(i)	$y=\dfrac{1}{2}e^{2x}-7e^x+6x$ $\dfrac{dy}{dx}=e^{2x}-7e^x+6$ $\dfrac{d^2y}{dx^2}=2e^{2x}-7e^x$
6(ii)	For stationary points, $\dfrac{dy}{dx}=0$ $e^{2x}-7e^x+6=0$ let $e^x=u$ $u^2-7u+6=0$ $(u-6)(u-1)=0$ $\Rightarrow u=6$ or $u=1$ $e^x=6$ or $e^x=1$ $x=\ln 6$ or $x=0$
6(iii)	$\left.\dfrac{d^2y}{dx^2}\right\|_{x=\ln 6}=2e^{2\ln 6}-7e^{\ln 6}=30>0$ The curve has a minimum point at $x=\ln 6$. $\left.\dfrac{d^2y}{dx^2}\right\|_{x=0}=2e^0-7e^0=-5<0$ The curve has a maximum point at $x=0$.
7(i)	$\int(3x^4-5\cos 6x)\,dx=\dfrac{3}{5}x^5-\dfrac{5}{6}\sin 6x+c$
7(ii)	$\int(\sin x+\sec^2(5+2\pi x))\,dx=-\cos x+\dfrac{1}{2\pi}\tan(5+2\pi x)+c$

8(i)	Area of lawn: $lr-\frac{\pi r^2}{4}=360$ $l=\frac{360-\frac{\pi r^2}{4}}{r}$ $l=\frac{360}{r}+\frac{\pi r}{4}$ --------(1) Perimeter of lawn: $P=l+r+(l-r)+\frac{1}{4}(2\pi r)$ $P=2l+\frac{\pi r}{2}$ ------------(2) Sub (1) into (2): $P=2\left(\frac{360}{r}+\frac{\pi r}{4}\right)+\frac{\pi r}{2}$ $P=\frac{720}{r}+\pi r$ (shown)
8(ii)	$\frac{dP}{dr}=-\frac{720}{r^2}+\pi$ $\frac{dP}{dr}=0$ for stationary values $-\frac{720}{r^2}+\pi=0$ $r^2=\frac{720}{\pi}$ $r=15.1$ (3s.f.) (rej. -15.1) $\frac{d^2P}{dr^2}=1440r^{-3}$ $\left.\frac{d^2P}{dr^2}\right\|_{r=15.138}=0.415$ (3s.f.) >0 $\Rightarrow$ Shortest perimeter $l=\frac{360}{15.138}+\frac{\pi(15.138)}{4}$ $=35.7$ (3s.f.) The shortest hedge can be planted when the rectangular space is 15.1cm by 35.7cm.

9(i)	$\dfrac{\tan^2 x-1}{\tan^2 x+1}$ $=\dfrac{\dfrac{\sin^2 x}{\cos^2 x}-1}{\dfrac{\sin^2 x}{\cos^2 x}+1}$ $=\dfrac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x}$ $=\sin^2 x-\cos^2 x$ $=1-\cos^2 x-\cos^2 x$ $=1-2\cos^2 x$ (shown)
9(ii)	$1-2\cos^2 x=\dfrac{1}{2}$ $\cos^2 x=\dfrac{1}{4}$ $\cos x=\pm\dfrac{1}{2}$ $\alpha=\cos^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{3}$ $x=\dfrac{\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3}$

10	$4x^2-6x+3=0$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{2}$ ---------(1) $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{3}{4}$ -------(2) From (1): $\frac{\alpha+\beta}{\alpha\beta}=\frac{3}{2}$ ------(3) From (2): $\alpha\beta=\frac{4}{3}$ ----------(4) Sub (3) into (4): $\alpha+\beta=2$ --------(5) $\alpha^3+\beta^3=(\alpha+\beta)\left(\alpha^2-\alpha\beta+\beta^2\right)$ $=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ $=(2)\left((2)^2-3\left(\frac{4}{3}\right)\right)$ $=0$ $(\alpha)^3(\beta)^3=\left(\frac{4}{3}\right)^3$ $=\frac{64}{27}$ The equation is $x^2+\frac{64}{27}=0$.
11(i)	By Pythagora's Theorem, Square of Diagonal $=18^2+\left(5-\sqrt{3}\right)^2$ $=324+25-10\sqrt{3}+3$ $=352-10\sqrt{3}$
11(ii)	$\frac{\left(1+\sqrt{3}\right)^2}{18\left(5-\sqrt{3}\right)}=\frac{4+2\sqrt{3}}{18\left(5-\sqrt{3}\right)}$ $=\frac{4+2\sqrt{3}}{18\left(5-\sqrt{3}\right)}\left(\frac{5+\sqrt{3}}{5+\sqrt{3}}\right)$ $=\frac{20+10\sqrt{3}+4\sqrt{3}+6}{18(25-3)}$ $=\frac{26+14\sqrt{3}}{396}$ $=\frac{13+7\sqrt{3}}{198}$

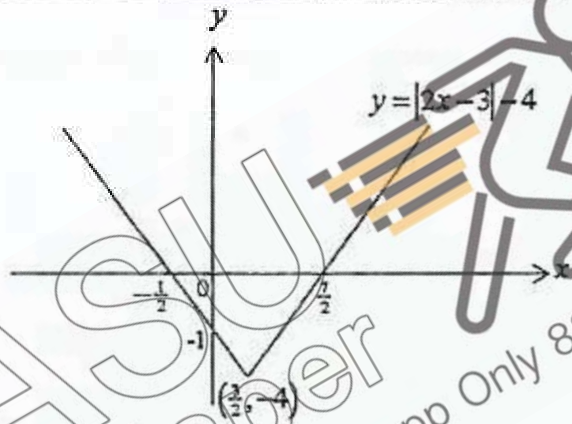
12(i)	$y = x\sqrt{5x^2-6}$ $\frac{dy}{dx} = (5x^2-6)^{\frac{1}{2}} + x\left(\frac{1}{2}\right)(5x^2-6)^{-\frac{1}{2}}(10x)$ $= \frac{5x^2-6+5x^2}{\sqrt{5x^2-6}}$ $= \frac{2(5x^2-3)}{\sqrt{5x^2-6}}$
12(ii)	$\int_2^4 \frac{5x^2-3}{\sqrt{5x^2-6}}\,dx$ $= \frac{1}{2}\int_2^4 \frac{2(5x^2-3)}{\sqrt{5x^2-6}}\,dx$ $= \frac{1}{2}\left[x\sqrt{5x^2-6}\right]_2^4$ $= \frac{1}{2}\left[4\sqrt{74} - 2\sqrt{14}\right]$ $= 13.5$ (3s.f.)
13(i)	$4y + 3x = 60$ $y = -\frac{3}{4}x + 15$ $m_{tangent} = -\frac{3}{4}$ $m_{normal} = \frac{4}{3}$ $y - 9 = \frac{4}{3}(x-8)$ The equation of the normal is $y = \frac{4}{3}x - \frac{5}{3}$. -----(1) $y = 4x - 7$ -------(2) (1) = (2): $\frac{4}{3}x - \frac{5}{3} = 4x - 7$ $x = 2$ $y = 1$ $P(2,1)$
13(ii)	$(x-2)^2 + (y-1)^2 = r^2$ Sub $(8,9)$: $r^2 = 100$ Equation of circle is $(x-2)^2 + (y-1)^2 = 100$.

13(iii)	When $x=0$, $4y+3(0)=60$ $y=15$ $B(0,15)$ Centre of circle $=\left(\frac{2+0}{2},\frac{1+15}{2}\right)$ $=(1,8)$ $BP=\sqrt{(2-0)^2+(1-15)^2}$ $=10\sqrt{2}$ Radius $=5\sqrt{2}$ $(x-1)^2+(y-8)^2=(5\sqrt{2})^2$ Equation of circle is $(x-1)^2+(y-8)^2=50$.

CWSS 4E AM 2019 MYE P2 SOLUTION

1(i)	$-1 \le \sin x \le 1$ $-1 \le -\sin x \le 1$ $-1+3 \le -\sin x+3 \le 1+3$ $2 \le f'(x) \le 4$ Since $f'(x)$ is always more than 0, there are no stationary points on the curve.
1(ii)	$f(x)=\int 3-\sin x\,dx$ $f(x)=3x+\cos x+c$ Since curve passes through origin, $0=3(0)+\cos(0)+c$ $c=-1$ $f(x)=3x+\cos x-1$
2(i)	$y=\frac{2x+1}{x-4}$ $\frac{dy}{dx}=\frac{2(x-4)-1(2x+1)}{(x-4)^2}$ $\frac{dy}{dx}=-\frac{9}{(x-4)^2}$ Since $(x-4)^2 \ge 0$ for all real values of x, $\frac{dy}{dx}<0$ for all real values of x. Hence, y is a decreasing function.
2(ii)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$
	$0.5=-\frac{9}{(2-4)^2}\times\frac{dx}{dt}$
	$\frac{dx}{dt}=-\frac{2}{9}$ units per second
3(i)	$\sin 3x=\sin 2x\cos x+\cos 2x\sin x$ $\sin 3x=2\sin x\cos x\cos x+(1-2\sin^2 x)\sin x$ $\sin 3x=\sin x(2\cos^2 x+1-2\sin^2 x)$ $\sin 3x=\sin x[2(1-\sin^2 x)+1-2\sin^2 x]$ $\sin 3x=3\sin x-4\sin^3 x$

3(ii)	$\sin 3x = \sin^2 x$ $3\sin x - 4\sin^3 x = \sin^2 x$ $\sin x(4\sin x - 3)(\sin x + 1) = 0$ $\sin x = 0,\ \sin x = \frac{3}{4},\ \sin x = -1$ $x = \pi,\ x = 0.848(3sf),\ x = \frac{3\pi}{2}$
4(i)	$R = \sqrt{(\sqrt{3})^2 + 1^2}$ $R = 2$ $\alpha = \tan^{-1}\frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$ $\sqrt{3}\sin\theta + \cos\theta = 2\sin\left(\theta + \frac{\pi}{6}\right)$
4(ii)	$\int_0^{\pi} \sqrt{3}\sin x + \cos x \, dx$ $= \int_0^{\pi} 2\sin\left(x + \frac{\pi}{6}\right) dx$ $= 2\left[-\cos\left(x + \frac{\pi}{6}\right)\right]_0^{\pi}$ $= 2\left\{\left[-\cos\left(\pi + \frac{\pi}{6}\right)\right] - \left[-\cos\left(0 + \frac{\pi}{6}\right)\right]\right\}$ $= 2\left[-\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\right]$ $= 2\sqrt{3}$
5(i)	$\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(kx)^{7-r}\left(-\frac{1}{x^3}\right)^r + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-r}(x^{-3})^r + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-r-3r} + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-4r} + \ldots$ Since the power of x is $7 - 4r = 2(3 - 2r) + 1$ will always be odd, there are no even powers of x for this expansion.

5(ii)	$\binom{7}{2}(k)^{7-2}(-1)^2 = 3\binom{7}{1}(k)^{7-1}(-1)^1$ $21(k)^5 = -3(7)(k)^6$ $k = -1$
6(i)	When $x=0$, $y=\lvert 0-3\rvert-4$ $y=-1$ When $y=0$, $0=\lvert 2x-3\rvert-4$ $\lvert 2x-3\rvert=4$ $2x-3=4$ or $2x-3=-4$ $x=\frac{7}{2}$, $x=-\frac{1}{2}$ Points are $(0,-1)$, $\left(-\frac{1}{2},0\right)$ and $\left(\frac{7}{2},0\right)$.
6(ii)	
6(iii)	$x+4=\lvert 2x-3\rvert$ $x+4=3-2x$ or $x+4=2x-3$ $3x=-1$ or $x=2x-3-4$ $x=-\frac{1}{3}$, $x=7$
7(i)	$f(-1)=2(-1)^3+6(-1)^2+6(-1)+5$ $f(-1)=3$
7(ii)	$f(x)=(x+1)(2x^2+4x+2)+3$ $f(x)=2(x+1)^3+3$

7(iii)

$f'(x) = 6x^2 + 12x + 6$ or $f'(x) = 6(x+1)^2$

$6x^2 + 12x + 6 = 0$ or $(x+1)^2 = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1$

$y = 2(-1+1)^3 + 3$

$y = 3$

By first derivative test,

x	-1^-	-1	-1^+
$\frac{dy}{dx}$	positive	0	positive
shape			

$(-1, 3)$ is a point of inflexion.

7(iv)

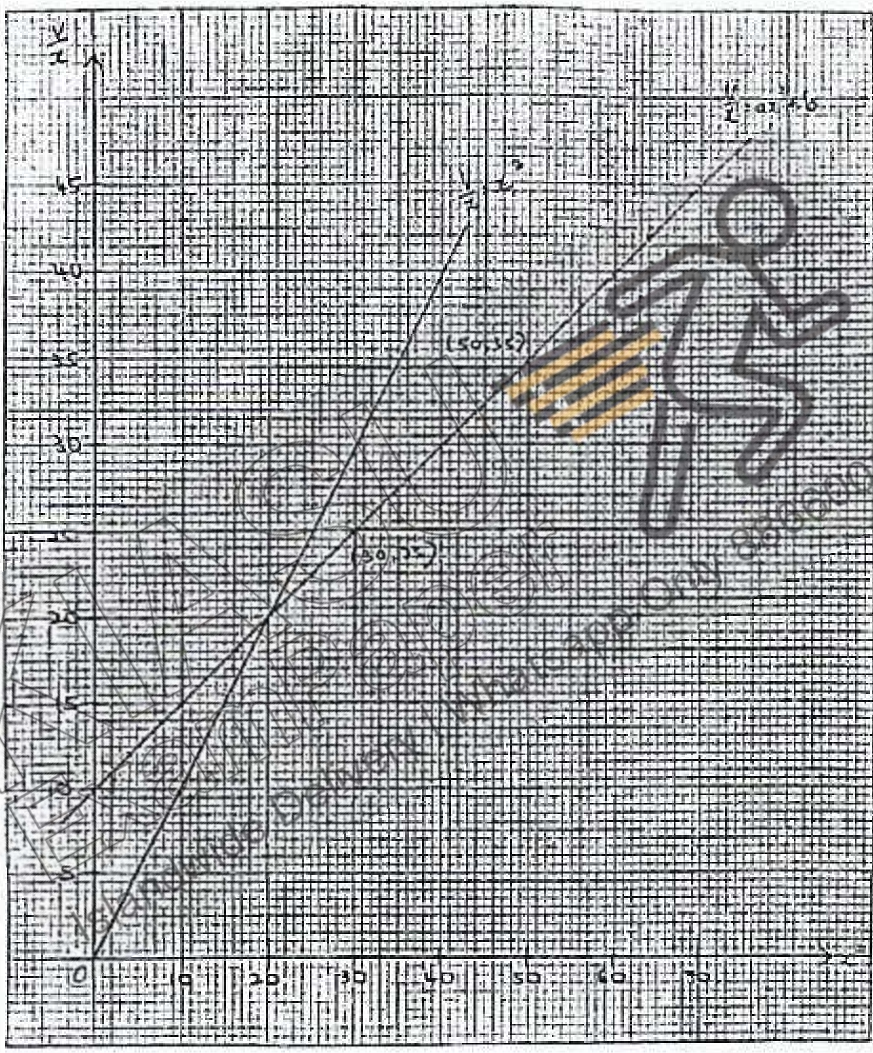
As the graph is an increasing function with no turning points, it will always only cut the x-axis only once.

8(i)

$V = x\left(ax^2 + b\right)$

$\frac{V}{x} = ax^2 + b$

x^2	4	16	36	64
$\frac{V}{x}$	12	18	28	42

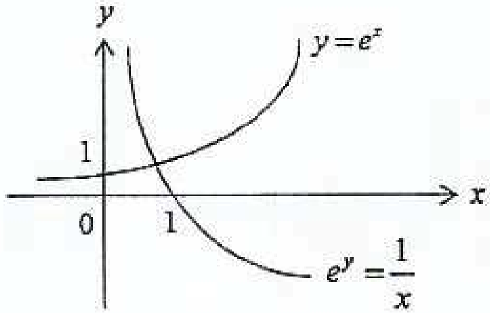


From the graph

$a = \frac{35-25}{50-30}$

$a = \frac{1}{2}$ and $b = 10$

8(ii)	$V = x^3$ Add the line $\frac{V}{x} = x^2$ onto graph From the graph, $x^2 = 20$ Diameter of the ball $= \sqrt{20} = 2\sqrt{5}$ cm
8(iii)	$x^3 = x\left(\frac{1}{2}x^2 + 10\right)$ $\frac{1}{2}x^3 - 10x = 0$ $x = 0$ or $x^2 = 20$ Since $x \neq 0$, the diameter of the ball $= \sqrt{20} = 2\sqrt{5}$ cm
9(a)	$\log_2(2x+1) - \log_4(x+1) = 1$ $\log_2(2x+1) - \frac{\log_2(x+1)}{\log_2 4} = \log_2 2$ $\log_2(2x+1) - \frac{1}{2}\log_2(x+1) = \log_2 2$ $\log_2 \frac{(2x+1)}{\sqrt{x+1}} = \log_2 2$ $2x+1 = 2\sqrt{x+1}$ $4x^2 + 4x + 1 = 4x + 4$ $4x^2 = 3$ $x = \frac{\sqrt{3}}{2}$ or $x = -\frac{\sqrt{3}}{2}$ (reject)
9(b)	$(\log_b e)(\log_b a)(\ln a) = 16$ $\left(\frac{\ln e}{\ln b}\right)\left(\frac{\ln a}{\ln b}\right)(\ln a) = 16$ $(\ln a)^2 = 16(\ln b)^2$ $\ln a = 4\ln b$ or $\ln a = -4\ln b$ $a = b^4$ or $a = \frac{1}{b^4}$

9(c)	$y=e^x$ $e^y=\frac{1}{x}$ $y\ln e=\ln x^{-1}$ $y=-\ln x$  $e^x+\ln x=0$ $e^x=-\ln x$ From the graph, there is only one point of intersection, hence there is only one solution.
10(a)	$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\frac{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{10}+\sqrt{6}\right)}{\left(\sqrt{10}\right)^2-\left(\sqrt{6}\right)^2}$ $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\frac{\sqrt{50}+\sqrt{30}+\sqrt{30}+\sqrt{18}}{10-6}$ $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\frac{5\sqrt{2}+2\sqrt{30}+3\sqrt{2}}{4}$ $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=2\sqrt{2}+\frac{1}{2}\sqrt{30}$
10(b)(i)	$5^{2x+3}=5^{x-1}+1$ $5^3\left(5^x\right)^2=\frac{5^x}{5}+1$ Let $y=5^x$ $625y^2-y-5=0$ $y=0.090246$ and $y=-0.088646$ $5^x=0.090246$ and $5^x=-0.088646$ (reject) $x=\frac{\ln 0.090246}{\ln 5}$ $x=-1.49$ (2 dp)

10(b)(ii)	Let $y=5^x$ $125y^2-\frac{y}{5}+k=0$ $625y^2-y+5k=0$ $(-1)^2-4(625)(5k)<0$ $1-12500k<0$ $k>\frac{1}{12500}$
11(i)	Length of $AO=\sqrt{6^2+8^2}=10$ Length of OB $=15-10=5$ $\sqrt{x^2+y^2}=5$ and $\frac{y}{x}=\frac{4}{3}$ By substitution or by Pythagoras triplets, $x=-3$ and $y=-4$ $B(-3,-4)$
11(ii)	Gradient of $BC=\frac{-1}{\frac{4}{3}}$ $-4=-\frac{3}{4}(-3)+c$ Equation of BC is $y=-\frac{3}{4}x-\frac{25}{4}$ $4y+3x+25=0$
11(iii)	From equation of BC, C is y-int. Hence, $C\left(0,-\frac{25}{4}\right)$ Equation of CD is $y=\frac{4}{3}x-\frac{25}{4}$ since AB is parallel to DC. $0=\frac{4}{3}x-\frac{25}{4}$ $x=\frac{75}{16}$ $D\left(\frac{75}{16},0\right)$