

1 A map is drawn to a scale of 1 : 20 000.

(a) (i) This scale can be expressed as 1 cm represents  $n$  km.  
Find  $n$ . [1]

(ii) The distance between two towns on the map is 20 cm.  
Find the actual distance, in kilometres, between the two towns. [1]

(iii) A garden has a map area of  $0.25 \text{ cm}^2$ .  
Find the actual area, in square kilometres, of the garden. [2]

(b) Lee is making a pond in the garden.  
He has a maximum of \$100 to spend on a water pump for the pond.  
The pump must have a flow rate of at least 250 litres per minute.

His local garden centre has four water pumps for sale.

Pump	Cost	Flow rate
A	\$92.50	4.2 litres per second
B	\$104.99	4.4 litres per second
C	\$80.75	13 000 litres per hour
D	\$89.99	15 120 litres per hour

Explain which pump he should buy. [2]

2 The table below shows information about the population and area of four different countries in 2016.

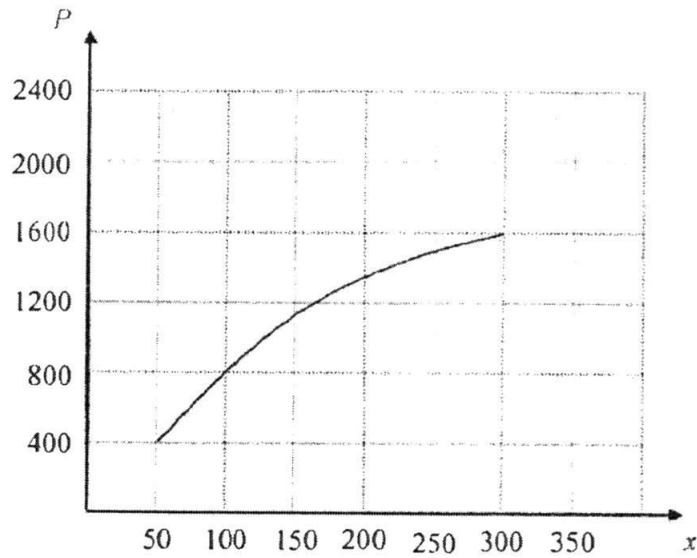
	Singapore	Malaysia	UK	Australia
Population	$5.6 \times 10^6$	$2.9 \times 10^7$	$6.4 \times 10^7$	$2.8 \times 10^7$
Area ( $\text{km}^2$ )	700	$3.4 \times 10^5$	$2.4 \times 10^5$	$7.7 \times 10^6$

(a) Find the ratio of the population of Singapore to the population of Australia.  
Give your answer in the form of 1 :  $n$ . [1]

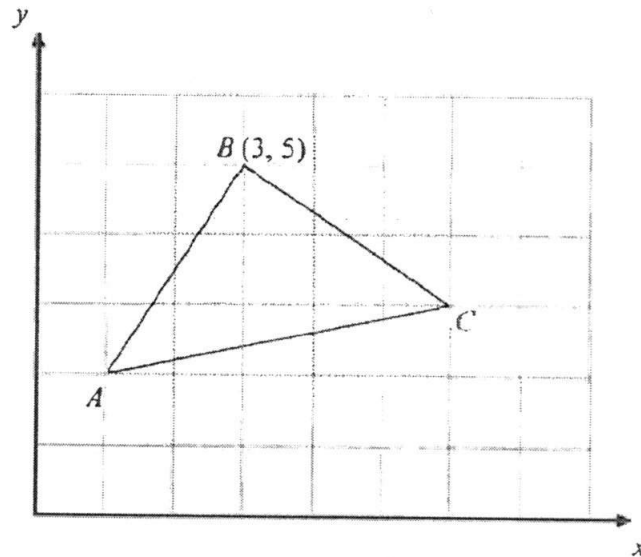
(b) How many more people live in UK than in Singapore?  
Give your answer in standard form. [2]

(c) Calculate the average number of people per square kilometres in Malaysia. [2]

- 3 (a) A company earns a weekly profit of  $P$  dollars by selling  $x$  items. The graph below shows how  $P$  varies with  $x$ .



- (i) Find the weekly profit per item for 100 items. [1]
- (ii) Estimate the weekly profit for 350 items. [1]
- (b) Find the area of triangle  $ABC$  below. [2]



- 4 Part of a number grid is shown below.

A square can be placed anywhere on the grid outlining four numbers.

The numbers in opposite corners of the square are multiplied together and the difference between the products is found.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20				

$$9 \times 14 - 8 \times 15 = 126 - 120$$

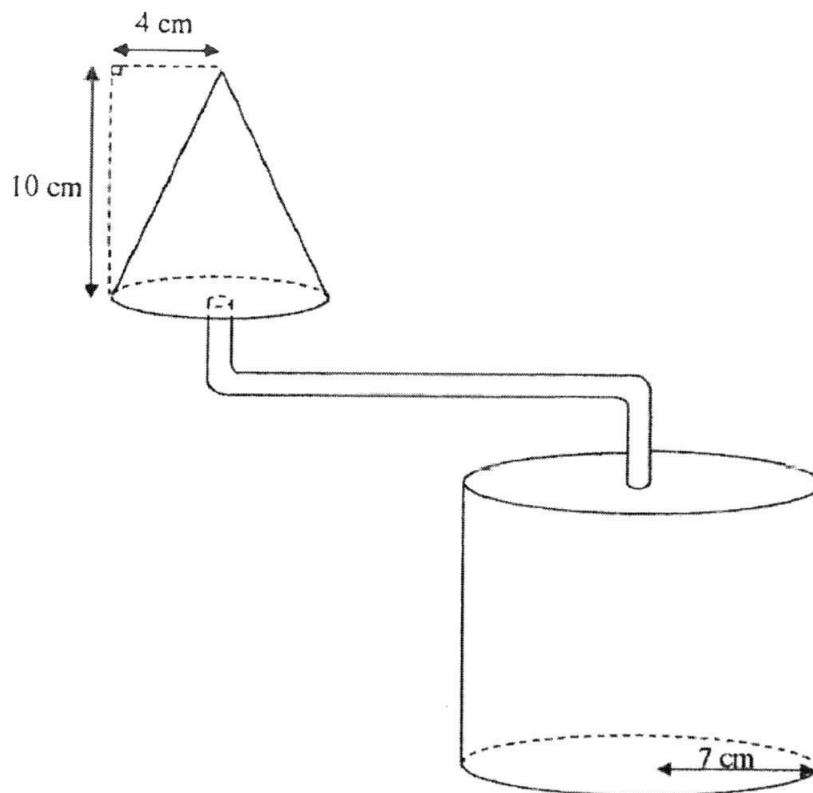
$$= 6$$

The grid is continued downwards.

- (a) If  $n$  represents the number in the top left corner of the square, write down an expression in terms of  $n$ , for the number in the bottom right corner of the square. [1]
- (b) Show that the difference between the products of the numbers in the opposite corners of the square is always 6. [2]
- (c) Show that the sum of the four numbers in the square cannot be 260. [3]

- 5 The diagram shows a cone with radius 4 cm and height 10 cm which is full of water. A pipe connects the cone to a cylinder.

The cylinder has a radius of 7 cm.



- (a) Calculate the volume of water in the cone. [1]
- (b) Water flows from the cone through the pipe into the empty cylinder at a constant rate of  $2 \text{ cm}^3$  per second.
- (i) Calculate the time taken to empty the cone in seconds. [1]
- (ii) Find the height of the water in the cylinder when the cone is empty. [2]

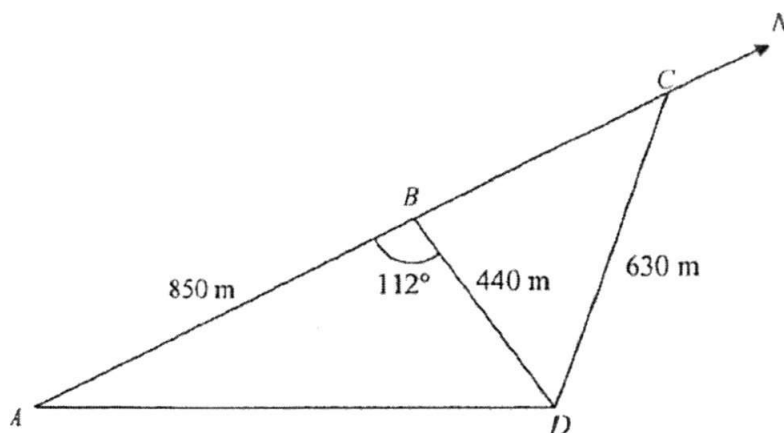
- 6 (a) (i) Solve the inequality  $17 - 3x < 3x + 1 \leq 46 - 5x$ . [2]
- (ii) Represent your answer on a number line. [1]
- (b) Given that  $-6 \leq a \leq 10$  and  $2 \leq b \leq 7$ , find the smallest value of  $\frac{a}{b}$ . [1]
- (c) A rectangle has length 127 cm and width 87 cm, both correct to the nearest whole number. Calculate the least and greatest possible perimeter of the rectangle. [3]
- (d) Given that  $7^m = 3$  and  $7^n = 8$ , find the value of  $7^{2m - \frac{n}{3}}$ . [3]

7 The distance between London and York is 320 km.

A train takes  $x$  hours to travel from London to York.

- (a) Write down an expression, in terms of  $x$ , for the average speed of the train in km/h. [1]
- (b) A car takes  $2\frac{1}{2}$  hours longer than the train to travel from London to York.  
Write down an expression, in terms of  $x$ , for the average speed of the car in km/h. [1]
- (c) The average speed of the train is 80 km/h greater than the average speed of the car. Form an equation in  $x$  and show that it simplifies to  $2x^2 + 5x - 20 = 0$ . [3]
- (d) Solve the equation  $2x^2 + 5x - 20 = 0$ . [3]
- (e) Find the average speed of the car correct to the nearest km/h. [2]

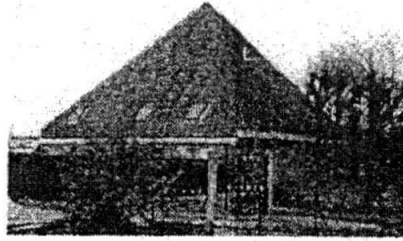
- 8 Four points  $A$ ,  $B$ ,  $C$  and  $D$  on level ground are as shown in the diagram below.



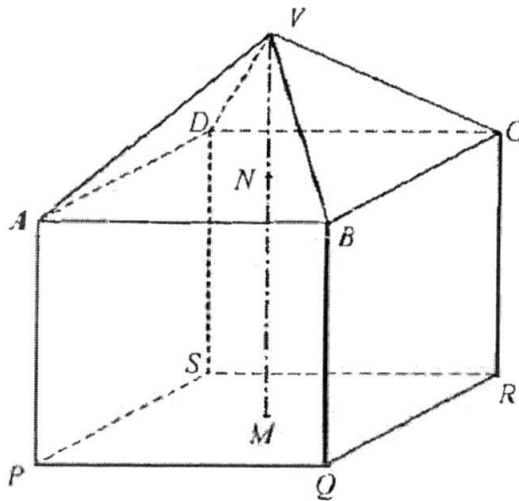
$ABC$  is a straight line.  $C$  is due North of  $A$  and  $\angle ABD = 112^\circ$ . It is given that  $AB = 850\text{ m}$ ,  $BD = 440\text{ m}$  and  $CD = 630\text{ m}$ .

- (a) Find
- (i)  $AD$ , [3]
  - (ii)  $\angle BCD$ , [2]
  - (iii) the bearing of  $D$  from  $C$ , [1]
  - (iv) the bearing of  $C$  from  $D$ . [1]
- (b) A vertical building at point  $B$  has a height of  $380\text{ m}$ . Find the greatest angle of elevation of the top of the building from a point along  $AD$ . [4]

- 9 A man wants to build a farmhouse as shown in the diagram below.



The farmhouse can be modelled by a square pyramid sitting on a cuboid.



Its roof is represented by a square-based pyramid,  $VABCD$ . The vertical line,  $VNM$ , passes through the centres,  $N$  and  $M$ , of the horizontal squares  $ABCD$  and  $PQRS$ .  
 $AB = 60$  m and  $VN = 40$  m.

- (a) Show that the total surface area of the slant roof in the model is  $6000$  m<sup>2</sup>. [3]
- (b) (i) The slant roof in the model is to be covered with tiles with dimensions of  $30$  cm by  $20$  cm by  $1$  cm. Information about the tiles is shown below.

**Useful information**

- Density of tiles :  $2.6$  g/cm<sup>3</sup>
- $1$  kg is equivalent to  $9.81$  N

Find the weight of each tile, in N. [3]

- (ii) The roof in the model will collapse if the total weight of the tiles exceeds  $1600$  kN. Justify whether this type of tile is suitable. [5]

10 Answer the whole of this question on a piece of graph paper.

The table below gives some values of  $x$  and  $y$  where  $y = 12 + 8x + \frac{27}{x}$ .

$x$	1	1.5	2	2.25	3	3.5
$y$	$p$	42	41.5	42	45	47.7

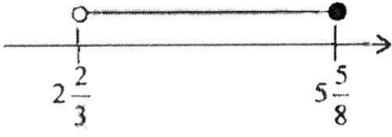
- (a) Find the value of  $p$ . [1]
- (b) Using a scale of 2 cm to 0.5 units, draw a horizontal  $x$ -axis for  $0 \leq x \leq 4$ .  
Using a scale of 2 cm to 1 unit, draw a vertical  $y$ -axis for  $38 \leq y \leq 48$ .  
On your axes, plot the points given in the table and join them with a smooth curve. [3]
- (c) Find the gradient of the curve at the point where  $x = 1.5$ . [2]
- (d) Use your graph to find the values of  $x$  for which  $12 + 8x + \frac{27}{x} = 46$ . [2]
- (e) Explain why the line  $x = 1.875$  is not a line of symmetry for the curve  
 $y = 12 + 8x + \frac{27}{x}$ . [1]
- (f) The line  $AB$  has a gradient of  $-1$  and passes through the point  $(0, 52)$ . The  $x$  coordinates of the points where the line  $AB$  intersects the curve  
 $y = 12 + 8x + \frac{27}{x}$  are the solutions of the equation  $9x^2 + ax + b = 0$ .  
Find the values of  $a$  and  $b$ . [4]

End of Paper



## Answers

No	Solution	No	Solution
1(a)(i)	$n = 0.2$	7(a)	$\frac{320}{x}$
1(a)(ii)	4 km	7(b)	$\frac{320}{x + 2.5}$
1(a)(iii)	0.01 km <sup>2</sup>	7(c)	$\frac{320}{x} - 80 = \frac{320}{x + 2.5}$
1(b)	Flow rate for A = 252 l/min Flow rate for C = $216\frac{2}{3}$ l/min Flow rate for D = 252 l/min He should buy pump D.	7(d)	$x = 2.15, -4.65$
2(a)	1 : 5	7(e)	69 km/h
2(b)	$5.84 \times 10^7$ people	8(a)(i)	$AD = 1090$ or $1093.75762$ or $1093$ m
2(c)	85.3 people/km <sup>2</sup>	8(a)(ii)	$\angle BCD = 40.4^\circ, 40.36^\circ, 40.35766^\circ$
3(a)(i)	\$8 per item	8(a)(iii)	$139.6^\circ, 139.64^\circ, 139.64234^\circ$
3(a)(ii)	\$1620 - \$1750	8(a)(iv)	$319.6^\circ, 319.64^\circ, 319.64234^\circ$
3(b)	6.5 units <sup>2</sup>	8(b)	$319.6^\circ, 319.64^\circ, 319.64234^\circ$
4(a)	$n + 7$	9(b)(i)	15.30360 N
4(b)	$(n + 1)(n + 6) - n(n + 7)$ $= 6$	9(b)(ii)	1530.36 kN < 1600 kN Tile is suitable
4(c)	$n = 61.5$ $n$ is a whole number/natural number/positive integer/integer. Or $n$ cannot be a fraction/decimal. Therefore the sum cannot be 260.		

No	Solution	No	Solution
5(a)	$168 \text{ cm}^3$	10(a)	$p = 47$
5(b)(i)	$83.8 \text{ s}$	10(c)	$-4$
5(b)(ii)	$h = 1.09 \text{ cm}$	10(d)	$1.0 \text{ to } 1.1, 3.15 \text{ to } 3.25$
6(a)(i)	$2\frac{2}{3} < x \leq 5\frac{5}{8}$	10(e)	$y$ is different at $x = 1.8$ and $x = 1.95$ .  $x = 1.875$ is not a line of symmetry.
6(a)(ii)		10(f)	$a = -40, b = 27$
6(b)	$-3$		
6(c)	$426 \text{ cm}, 430 \text{ cm}$		
6(d)	$4.5$		