Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2016
Secondary Three

MATHEMATICS PAPER 1   **1 hr 30 min**

Thursday   **06 October 2016**   **08 45 – 10 15**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Write your name, register number and class in the spaces at the top of this page.
2. Answer **all** the questions.
3. Write your answers in the spaces provided on the question paper.
4. **All working must be written in dark blue or black ink.**
5. **Omission of essential working will result in loss of marks.**
6. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
7. The use of calculators is allowed for this paper.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 60.
3. You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

MENSURATION

Area of triangle $ABC = \dfrac{1}{2} ab \sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \dfrac{1}{2} r^2 \theta$, where θ is in radians

TRIGONOMETRY

$$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Answer **all** the questions.

1   Simplify $25n^2 + 1 - (5n - 1)^2$.

Answer _____ [2]

2   Solve the equation $\dfrac{49}{(3x-2)} - (3x-2) = 0$

Answer $x =$ _____ or _____ [3]

3   Given that $16 \div 32^k = \dfrac{1}{4}$, find the value of k.

Answer $k =$ _____ [3]

4   (a)   Solve $-15 < 5 - 3x \leq 2x$.

Answer (a) _____ [2]

(b)   Write down the largest prime number which satisfies $-15 < 5 - 3x \leq 2x$.
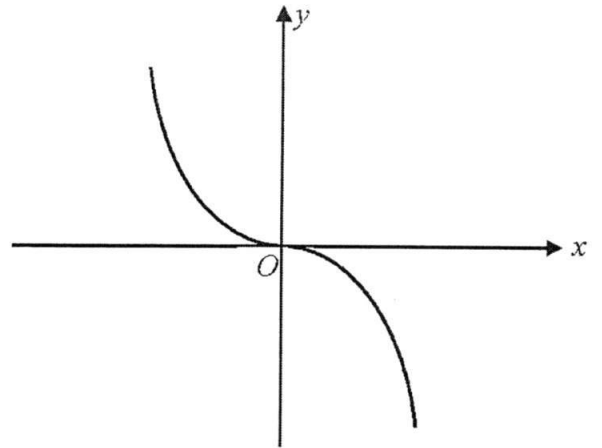
Answer (b) _____ [1]

5   (a)   Express $x^2 - 6x - 2$ in the form $(x - p)^2 - q$, where p and q are constants.

Answer (a) _____ [2]

(b)   Hence write down the value of k for which $x^2 - 6x - 2 \geq k$ for all values of x.
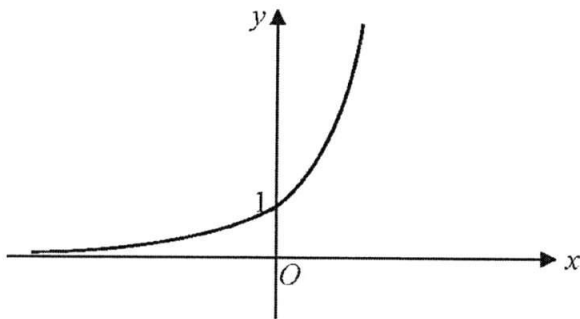
Answer (b) $k =$ _____ [1]

6   **(a)**   The sketch represents the graph of $y = kx^n$.

Write down a possible value of k and of n.

Answer (a) $k = $ _____ , $n = $ _____ [2]

(b)   Write down a possible equation for this graph.

Answer (b) _____ [1]

7   Two geometrically similar fruit juice bottles have base areas of 54 cm^2 and 96 cm^2 respectively.

    (a)   Find, in its simplest integer form, the ratio of the height of the smaller bottle to the height of the larger bottle.

<div align="right">

Answer (a) _____ [2]

</div>

    (b)   The capacity of the larger bottle is 1.6 litres.
Find the capacity of the smaller bottle in cubic centimetres.

<div align="right">

Answer (b) _____ cm^3 [2]

</div>

---

8   The points A and B are $(1, -3)$ and $(7, 6)$ respectively.

    (a)   Given that the straight line $m(x + 1) = 4y - 5$ has the same gradient as the line AB, find the value of m.

<div align="right">

Answer (a) $m =$ _____ [3]

</div>

    (b)   Triangle ABC has a line of symmetry $y = -3$.
Find the coordinates of C.

<div align="right">

Answer (b) C (_____ , _____) [1]

</div>

<div align="right">

[Turn over

</div>

9   Weekday admission charges for a marine park attraction are $16 per adult, $10 per child and $6 per senior citizen. Weekend admission charges are $20 per adult, $12 per child and $10 per senior citizen.

This information can be represented by the matrix $\mathbf{P} = \begin{pmatrix} 16 & 20 \\ 10 & 12 \\ 6 & 10 \end{pmatrix}$ $\begin{matrix} \text{Adult} \\ \text{Child} \\ \text{Senior Citizen} \end{matrix}$.

Wkday Wkend

(a)   Mr Tan intends to purchase tickets for 3 adults, 2 children and 2 senior citizens. Mr Lim intends to purchase tickets for 2 adults, 4 children and 1 senior citizen. Represent their intended purchases in a 2×3 matrix $\mathbf{R}$.

Answer (a) R = _____ [1]

(b)   Evaluate the matrix $\mathbf{Q} = \mathbf{RP}$.

Answer (b) Q = _____ [1]

(c)   State what the elements of $\mathbf{Q}$ represent.

Answer (c) _____

_____

_____ [1]

(d)   For a certain month, a 10% discount was given per adult, a 20% discount per child and a 50% discount per senior citizen.
Write down a matrix $\mathbf{D}$, which, when multiplied with matrix $\mathbf{P}$, will show the total savings for a family consisting of an adult, a child and a senior citizen who are considering a weekday or a weekend visit during that month.

Answer (d) D = _____ [1]

10   The diagram shows the locations of four signposts A, B, C and D on a map. A, B and D form an equilateral triangle. B, C and D form an isosceles triangle and $\angle CBD = 90°$. The bearing of D from A is $102°$ and $AB = 16$ km.
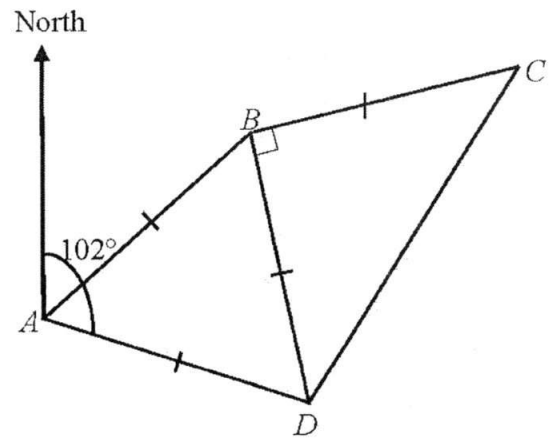
North

(a)   Find the bearing of

    (i) B from D,

Answer (a) (i) _____ ° [2]

    (ii) C from B.

Answer (a) (ii) _____ ° [2]

(b)   Find how far B is north of A.
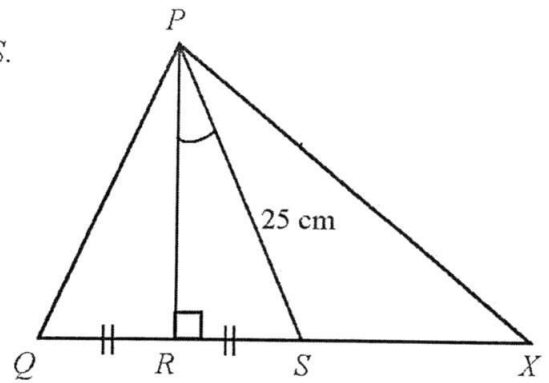
Answer (b) _____ km [2]

[Turn over

11 In the diagram, *QRSX* is a straight line.

$\angle PRS = 90°$, $PS = 25$ cm, $\sin \angle RPS = \dfrac{3}{5}$ and $QR = RS$.

(a) Find

    (i) the length of *QS*,

P

25 cm

Q   *R*   *S*   *X*

Answer (a) (i)_____ cm **[2]**

    (ii) the exact value of tan $\angle PSX$.

Answer (a) (ii) tan $\angle PSX =$ _____ **[2]**

(b) If $2 \sin \angle PXS = 3 \sin \angle XPS$, find the length of *SX*.
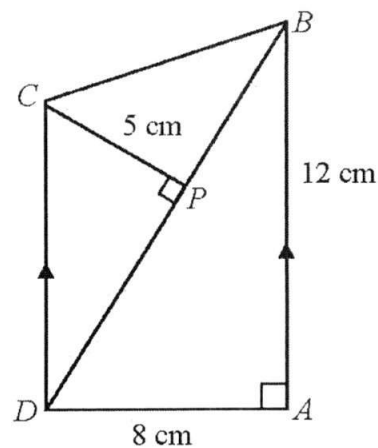
Answer (b) $SX =$ _____ cm **[2]**

12 In the diagram, *ABCD* is a trapezium and *BPD* is a straight line.

AB is parallel to *DC*, *PC* = 5 cm, *AD* = 8 cm and *AB* = 12 cm.

$\angle CPD = \angle DAB = 90^\circ$.

(a) Show that $\triangle ABD$ is similar to $\triangle PDC$.

Answer (a)
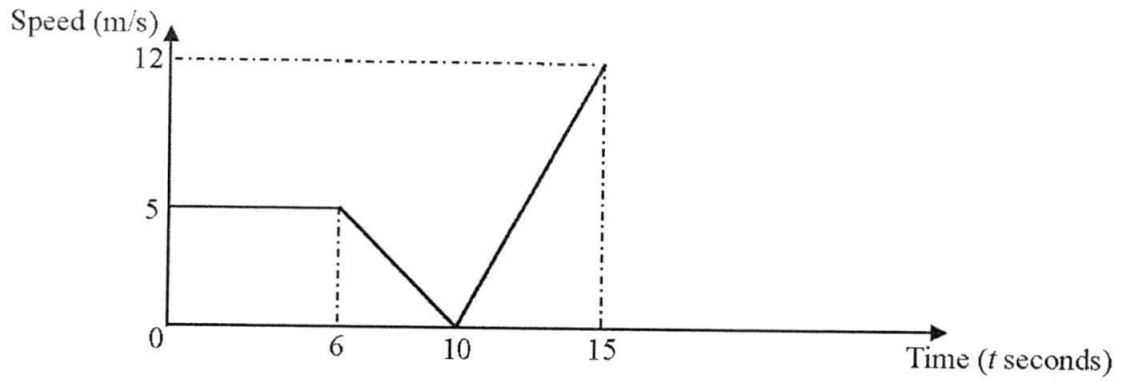
_____

_____

_____

_____

_____

_____ [2]

(b) Find the length of *PD*.

Answer (b) *PD* = _____ cm [2]

(c) Find the radius of the circle which passes through the points *A*, *B* and *D*.

Answer (c) _____ cm [2]

[Turn over

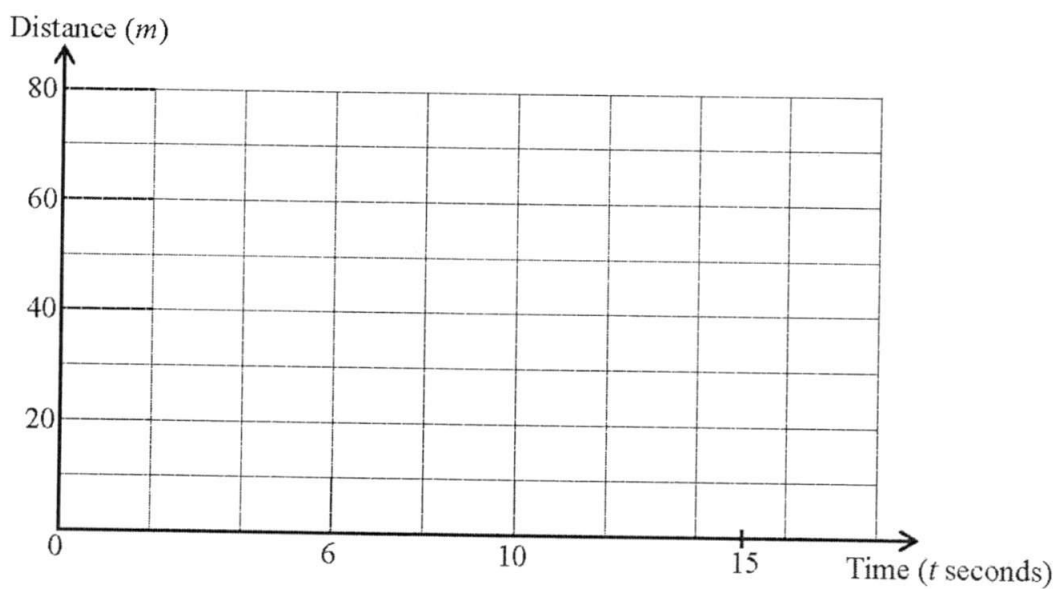13   The diagram shows the speed-time graph for the first 15 seconds of a journey.

(a)   Find the deceleration when $t = 7$.

Answer (a) _____ m/s^2 [1]
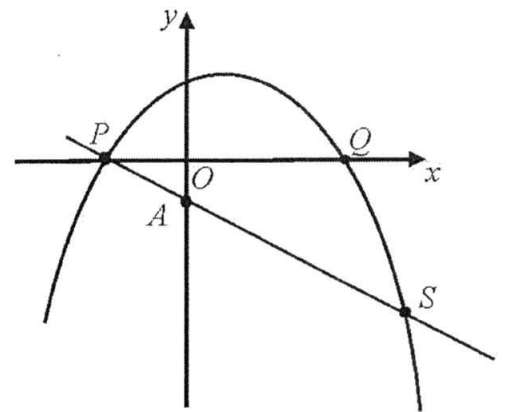
(b)   Find the average speed for the first 15 seconds of the journey.

Answer (b) _____ m/s [2]

(c)   On the grid below, draw the distance-time graph for the same journey.   [3]

14 The diagram shows a sketch of part of the graph of
$y = (2 + x)(3 - x)$. The curve cuts the x-axis at P and Q.

(a) Write down the coordinates of P and Q.

Answer (a) P (_____ , _____) [1]

Q (_____ , _____) [1]

(b) Find the coordinates of the highest point on the graph.

Answer (b) (_____ , _____) [2]

(c) Write down the range of values of x for which the gradient of the curve is negative.

Answer (c) _____ [1]

(d) A straight line PS cuts the y-axis at A $(0, -2)$.
Write down, but do not simplify, an equation in x which has the x-coordinates of the
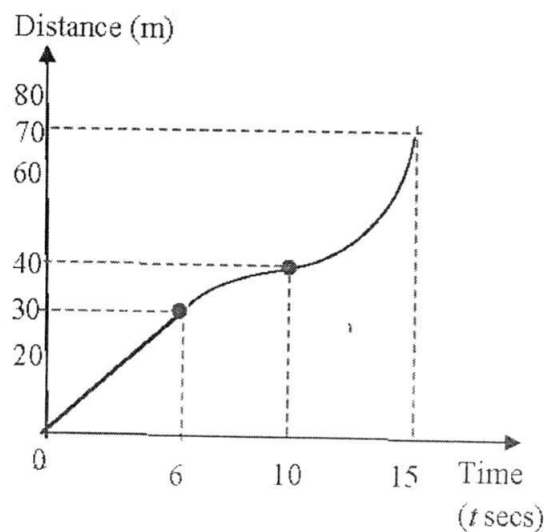points P and S as its solutions.

Answer (d) _____[2]

END OF PAPER

1	$10n$
2	$x = -1\dfrac{2}{3}$ or $x = 3$
3	$k = \dfrac{6}{5} = 1\dfrac{1}{5}$
4a	$1 \le x < 6\dfrac{2}{3}$
4b	5
5a	$(x-3)^2 - 11$
5b	$k = -11$
6a	$k = $ (any -ve number) $n = 3$
6b	$y = $ (any no. > 1)$^{(\text{any +ve no.}) x}$ eg $y = e^x$, $y = 3^{2x}$ etc
7a	$3 : 4$
7b	675
8a	$m = 6$
8b	$C(7, -12)$
9a	$\mathbf{R} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 4 & 1 \end{pmatrix}$
9b	$\mathbf{Q} = \begin{pmatrix} 80 & 104 \\ 78 & 98 \end{pmatrix}$
9c	The total amounts payable by Mr Tan for weekday tickets and for weekend tickets are $80 and $104 respectively. The total amounts payable by Mr Lim for weekday tickets and for weekend tickets are $78 and $98 respectively.
9d	$\begin{pmatrix} 0.1 & 0.2 & 0.5 \end{pmatrix}$
10ai	$342°$
aii	$072°$

10b	$11.9 \, \text{km}$
11ai	$30 \, \text{cm}$
11aii	$-\dfrac{4}{3}$
11b	$16\dfrac{2}{3}$ cm
12a	$\angle BAD = \angle DPC$ (given $90°$) $\angle ABD = \angle PDC$ (alternate angles, $AB \, // \, CD$) $\therefore$ triangles $\mathbf{ABD}$ and $\mathbf{PDC}$ are similar
12b	$7\dfrac{1}{2}$ cm
12c	Radius $= 7.21$
13a	Deceleration $= 1\dfrac{1}{4} \text{m/s}^2$
13b	$4\dfrac{2}{3}$ m/s
14a	$P(-2, 0)$ $Q(3, 0)$
14b	$\left(\dfrac{1}{2}, 6\dfrac{1}{4} \right)$
14c	$x > \dfrac{1}{2}$
14d	$(2 + x)(3 - x) = -x - 2$

13c

Distance (m) vs Time (t secs)

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南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-Of-Year Examination 2016
Secondary Three

MATHEMATICS
Paper 2
Monday

1 hour 30 minutes

10 October 2016

0845 - 1015

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer **all** the questions.

2. Write your answers in and working on the separate answer paper provided.

3. Write your name, register number and class on each separate sheet of paper that you use and fasten the separate sheets together with the string provided. Do not staple your answer sheets together.

4. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

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Arc length $= r\theta$, where θ is in radians

Sector area $= \dfrac{1}{2} r^2\theta$, where θ is in radians

TRIGONOMETRY

Formulae for $\triangle ABC$

$$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

1 (a) Given that $q = 2\sqrt{\dfrac{3r}{p-r}}$, express r in terms of p and q . [2]

   (b) Factorise $2x^3 - x^2 - 8x + 4$ completely. [2]

   (c) Express $\dfrac{6}{x^2-9} - \dfrac{1}{(x-3)}$ as a single fraction in the simplest form. [3]

2 O and C are the centres of two circles with radii 6 cm and 3 cm respectively. P is a point on the circumference of the smaller circle while Q is the point where the two circles meet, as shown in the diagram below.

   Angle $POQ = \dfrac{\pi}{6}$ radian.

   (a) Find the length of minor arc PQ, in cm. [2]

   (b) Calculate the shaded area, in cm^2. [3]
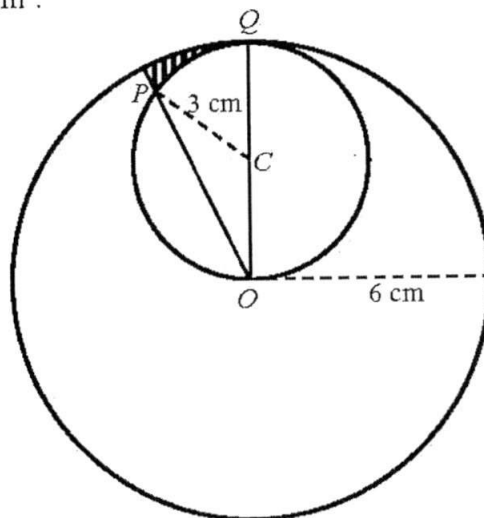
3 An express bus travels a 240 km journey daily at an average speed of x km/h.

(a) Write down, in terms of x, the time needed (in hours) for the bus to complete the daily journey. [1]

Last Monday, the bus left the bus terminal 10 minutes after the scheduled time of departure. The driver decided to increase the average speed by 5 km/h and managed to reach the destination at the usual time of arrival.

(b) Write down, in terms of x, the time taken to complete the delayed journey. [1]

(c) Using the information given above, show that $x^2 + 5x - 7200 = 0$. [2]

(d) Showing your method clearly, solve the equation $x^2 + 5x - 7200 = 0$, giving both solutions correct to two decimal places. [3]

(e) Find the time taken to complete the delayed journey, giving your answer in hours and minutes, correct to the nearest minute. [2]

4
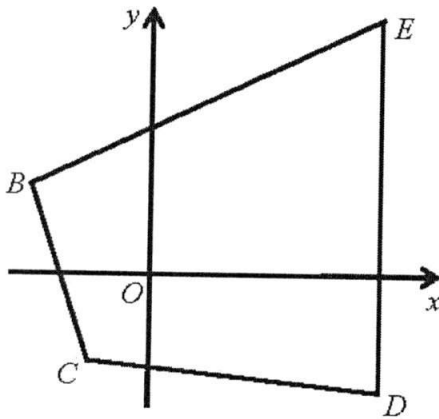
The diagram above shows a quadrilateral $BCDE$, where B is $(-4, 3)$, C is $(-2, -3)$, D is $(6, -4)$ and E is $(6, 8)$.

(a) Find the equation of the line BE. [2]

(b) BE is extended to a point P where the coordinates are equal. Find the coordinates of P. [2]

(c) Calculate the area of triangle CDE. [2]

(d) A line through C and parallel to DE intersects BE at the point Q.

Determine the value of $\dfrac{\text{area of } \Delta CQB}{\text{area of } \Delta CQE}$. [2]

5   (a)   In the diagram, O is the centre of the circle which passes through A, B, C and

D. SAT is a tangent at A, BOD is a straight line and AC intersects BD at P.

$\angle ACB = 56°$ and $\angle CAD = 26°$.

Explain briefly why $\angle OAS = 90°$.   [1]

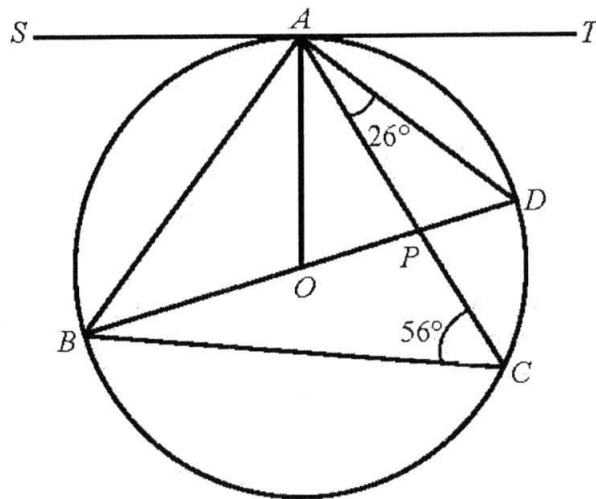Find

(i) $\angle AOB$,

(ii) $\angle BAS$,
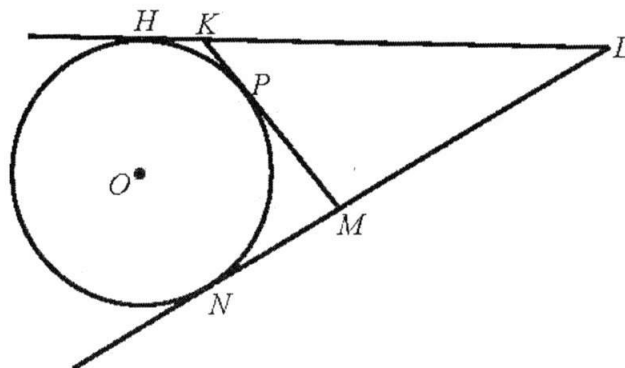
(iii) $\angle OAC$.   [3]
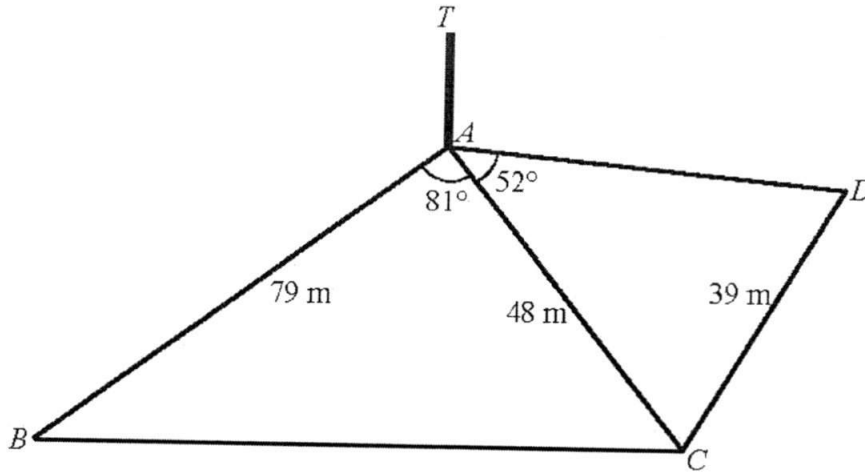

(b)

The diagram shows tangents HKL, KPM and LMN touching a circle at H,

P and N respectively. The centre of the circle is O. $KL = 8$ cm, $KM = 5$ cm

and $LM = 7$ cm.

Find the length of MN.   [3]

[Turn over

6

In the diagram, *ABCD* represents a horizontal field and *T* represents the top of a vertical tree at the corner *A*. A footpath runs along the edge *BC* of the field. $AB = 79$ m, $AC = 48$ m and $CD = 39$ m. $\angle BAC = 81°$ while $\angle CAD = 52°$.

(a) Given that the angle of depression of *B* from *T* is 6°, calculate the height of the tree. [2]

(b) Find $\angle ADC$. [2]

(c) Calculate the length of the footpath *BC*. [2]

(d) Calculate the area of triangle *ABC*. [2]

(e) Find the shortest distance from *A* to the footpath *BC*. [2]

(f) Find the greatest possible angle of elevation of the top of the tree when viewed from a point on the footpath *BC*. [2]

7   **Answer the whole of this question on a sheet of graph paper.**
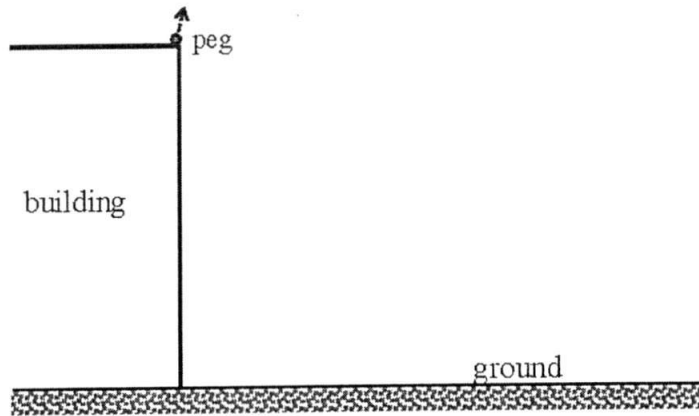
A peg is projected upwards from the edge of the top of a building, as shown in the diagram above. The vertical height of the peg above the building, h metres, at t seconds after it is projected, is given by the equation $h = 6t(4 - t)$.

The table below shows some values of t and the corresponding values of h.

t (seconds)	0	0.5	1	1.5	2	3	3.5	5
h (metres)	0	10.5	18	22.5	24	18	10.5	−30

(a)   Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for
       $0 \le t \le 5$. Using a scale of 2 cm to represent 10 metres, draw a vertical
       h-axis for $-30 \le h \le 30$.

       On your axes, plot the points given in the table above and join them with
       a smooth curve.   [3]

(b)   Use your graph to find

       (i)   the greatest distance of the peg above the top of the building, and
              the value of t when this happens,

       (ii)   the duration of time when the peg is at least 21 metres above the
              top of the building.   [3]

(c)   (i)   By drawing a tangent, find the gradient of the curve when $t = 3$.

       (ii)   Explain what your answer to **c(i)** tells you about the motion of the
              peg when $t = 3$.   [3]

(d)   The peg hits the ground 5 seconds after it is projected. Deduce the height
       of the building.   [1]

(e)   By inserting a straight line on the same axes, solve the equation
       $3t^2 - 17t + 5 = 0$ for $0 \le t \le 5$.   [2]

END OF PAPER

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	Answers
1(a)	$r = \dfrac{pq^2}{12+q^2}$.
1(b)	$(2x-1)(x-2)(x+2)$
1(c)	$-\dfrac{1}{x+3}$

Qn	Answers
2(a)	π cm.
2(b)	The shaded area $= 0.815$ cm^2

| Qn	Answers
3(a)	$\dfrac{240}{x}$ h
3(b)	$\dfrac{240}{x+5}$ h
3(c)	$\dfrac{240}{x} - \dfrac{240}{x+5} = \dfrac{1}{6}$ To show $\therefore x^2 + 5x - 7200 = 0$.
3(d)	$x = 82.39$ or 87.39
3(e)	Time taken $= 2$ hours 45 minutes.

Qn	Answers
4(a)	$y = \dfrac{1}{2}x + 5$.
4(b)	$\therefore$ The coordinates of P are $(10, 10)$.
4(c)	The area of $\triangle CDE = 48$ square units
4(d)	$\dfrac{\text{area of } \triangle CQB}{\text{area of } \triangle CQE} = \dfrac{BQ}{QE} = \dfrac{1}{4}$.

Qn	Answers
5(a)	The <u>radius</u> of a circle is <u>perpendicular to the tangent</u> at the <u>point of contact</u>.
(i)	$\angle AOB = 112°$
(ii)	$\angle BAS = 56°$
(iii)	$\angle OAC = 30°$
5(b)	$\therefore MN = 3$ cm.

6(a)	Height ≈ 8.30 m
6(b)	$\angle ADC \approx 75.9°$
6(c)	$BC \approx 85.8$ m.
6(d)	The area of $\triangle ABC$ ≈ 1873 m².
6(e)	The shortest distance ≈ 43.7 m.
6(f)	The greatest angle of elevation ≈ 10.8°.

t (seconds)	0	0.5	1	1.5	2	3	3.5	5
h (metres)	0	10.5	18	22.5	24	18	10.5	−30
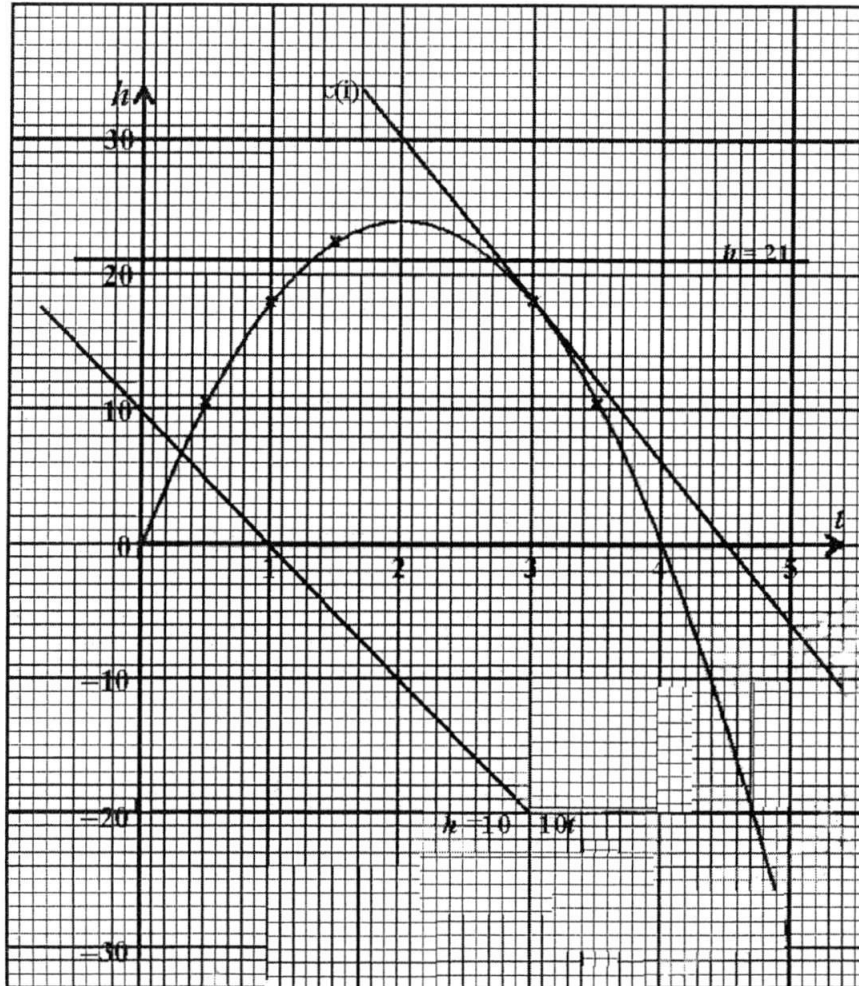
(b) (i)   The greatest distance = 24 m, when $t = 2$

    (ii) $1.3 \leq t \leq 2.7$ OR 1.4 sec

(c) (i)   The gradient = −12

    (ii) When $t = 3$, the peg is <u>falling</u> at a speed of <u>12 m/s</u>

(d) The height of the building is 30 m.

(e) From $3t^2 - 17t + 5 = 0$, get $10 - 10t = 6t(4 - t)$.

    Correct line of $h = 10 - 10t$

    $t = 0.3$