Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL
SEMESTRAL ASSESSMENT TWO
SECONDARY THREE

MATHEMATICS **4016/01**
Paper 1

07 October 2016

Additional Materials: Nil

2 hours

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Subtotal	
P	
R	
U	

For Examiner's Use

80

This document consists of **16** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. Factorise the expression $16 - 9m^2 - 6mn - n^2$.

Answer ____________________ [2]

2. Given that $7^x = 3$ and $7^y = 5$, find the value of 7^{3x+y}.

Answer ____________________ [2]

3. Given that $m^2 + \frac{1}{m^2} = 11$, find the values of $\frac{1}{4}\left(m - \frac{1}{m}\right)$.

Answer ____________________ [3]

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4. y is inversely proportional to the square root of x, where $x > 0$. It is given that $y = 12$ for a particular value of x. Find the decrease in the value of y when this value of x is increased by 800%.

Answer ________________ [3]

5. Given that $p = 1 - \sqrt{\dfrac{m^2 + n}{m^2}}$, make m the subject of the formula.

Answer $m =$ ________________ [3]

6. If 2 men can make 50 tables in 7 days, how long will 14 men take to make 225 tables?

Answer ________________ days [3]

7. Simplify $\left(\dfrac{2x^2y^2}{54x^5y^{-4}}\right)^{-\frac{1}{3}}$, expressing your answer in the positive index form.

Answer ____________________ [3]

8. **(a)** A polygon has n sides. Three of its interior angles are $148°$, $157°$ and $175°$. The remaining interior angles are $155°$ each. Find the value of n.

Answer $n =$ ____________________ [2]

(b) Explain why the interior angle of a regular polygon cannot be $130°$.

____

____ [1]

9. Express $\frac{5x+2}{3x^2-12}+\frac{1}{2-x}$ as a single fraction in its simplest form.

Answer ____________________ [3]

10. It is given that Cylinder A has a volume of 300 cm^3. Calculate the volume of

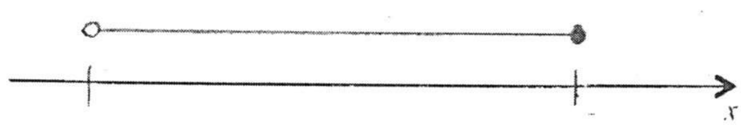
(a) Cylinder B with base radius $\frac{2}{5}$ that of Cylinder A and a height thrice that of Cylinder A.

(b) Cylinder C which is geometrically similar to Cylinder A but has a curved surface area 16 times that of Cylinder A.

Answer (a) ____________________ cm^3 [2]

(b) ____________________ cm^3 [2]

11. **(a)** Solve the inequality $-\frac{1}{3}+x \le \frac{x+3}{2} < x+4$. Represent your answer on the number line below.



Answer (a) ______________________ [4]

(b) Write down all the integers that satisfy $-\frac{1}{3}+x \le \frac{x+3}{2} < x+4$.

Answer (b) ______________________ [1]

12. The radius of a spherical particle is approximately 5 picometres.
Find, leaving your answer in standard form,

(a) the diameter of one such particle in centimetres,

(b) the number of particles that must be placed side by side in order to make a length of 30 millimetres,

(c) the total volume, in cubic centimetres, of 1 million of such particles. Give your answer correct to 3 significant figures.

Answer (a) ______________________ cm [2]

(b) ______________________ [1]

(c) ______________________ cm^3 [2]

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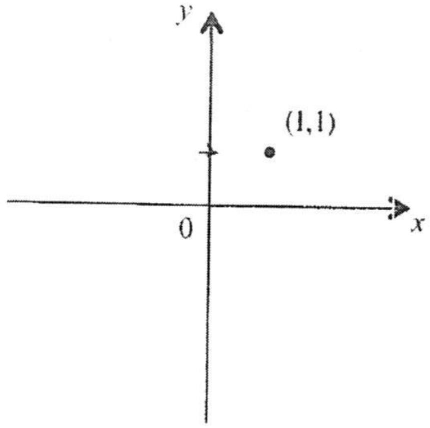
For Examiner's Use

13. The point $(1,1)$ is marked on the diagram below. Sketch the graph of $y = 3^x$.

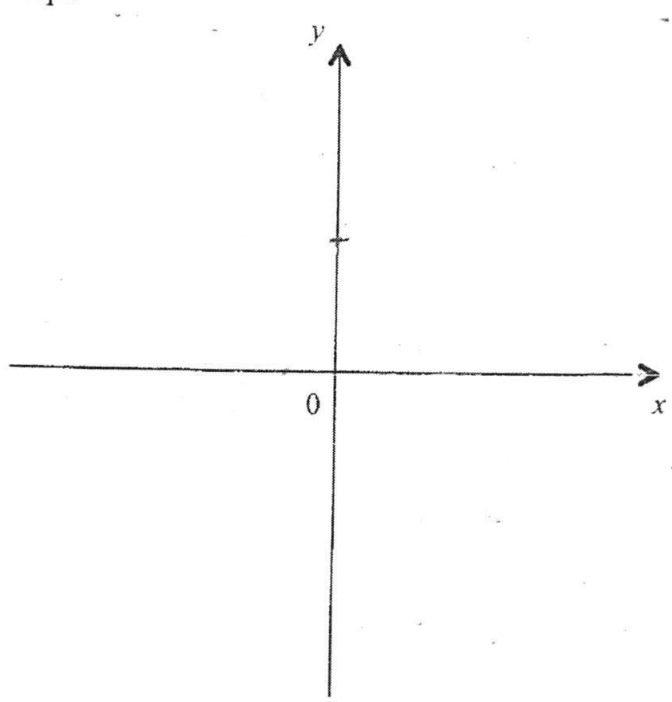


[1]

14. (a) Express $-x^2 + 4x + 7$ in the form of $-(x+h)^2 + k$.

Answer (a) ______________________ [3]

(b) Hence, sketch the graph of $y = -x^2 + 4x + 7$ on the axes below, indicating the turning point and the y – intercept. [2]

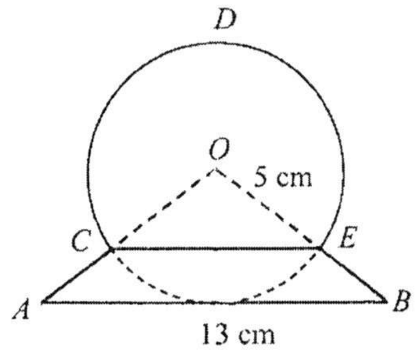


15. The diagram below shows the cross-section of a snowglobe with centre O, of radius 5 cm. The base makes an isosceles triangle OAB. AB is a tangent to the circle and is 13 cm long.

(a) Show that angle $AOB = 1.83$ radians, correct to 3 significant figures. [2]

(b) Calculate

(i) the area of major segment CDE,
(ii) the perimeter of the snowglobe $ABEDC$.



Answer (b) (i) ______________ cm^2 [3]

(ii) ______________ cm [3]

For Examiner's Use

For Examiner's Use

16. **(a)** Express the numbers 66 and 2520 as products of their prime factors.

(b) Find the smallest positive integer, k, such that $2520k$ is a perfect cube.

(c) Find the smallest positive integer, n, such that $66n$ is a multiple of 2520.

Answer (*a*) 66 = ____________________ [1]

2520 = ____________________ [1]

(*b*) k = ____________________ [2]

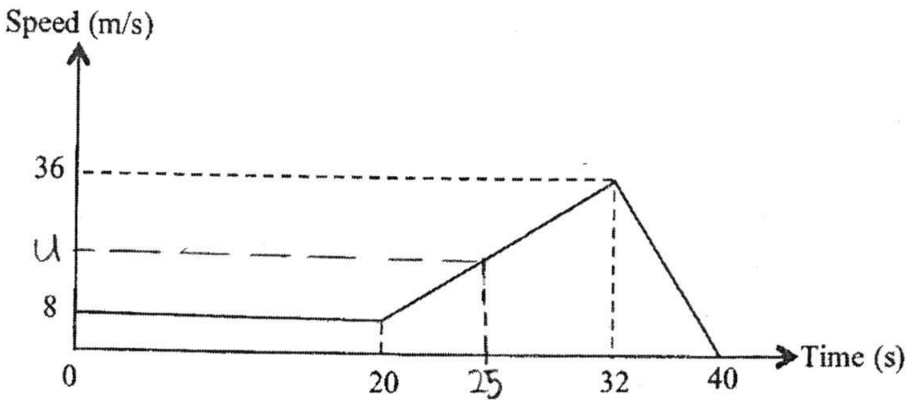
(*c*) n = ____________________ [2]

17. The graph below shows the speed of a car during a period of 40 seconds.

(a) Calculate

(i) the speed of the car after 25 seconds,

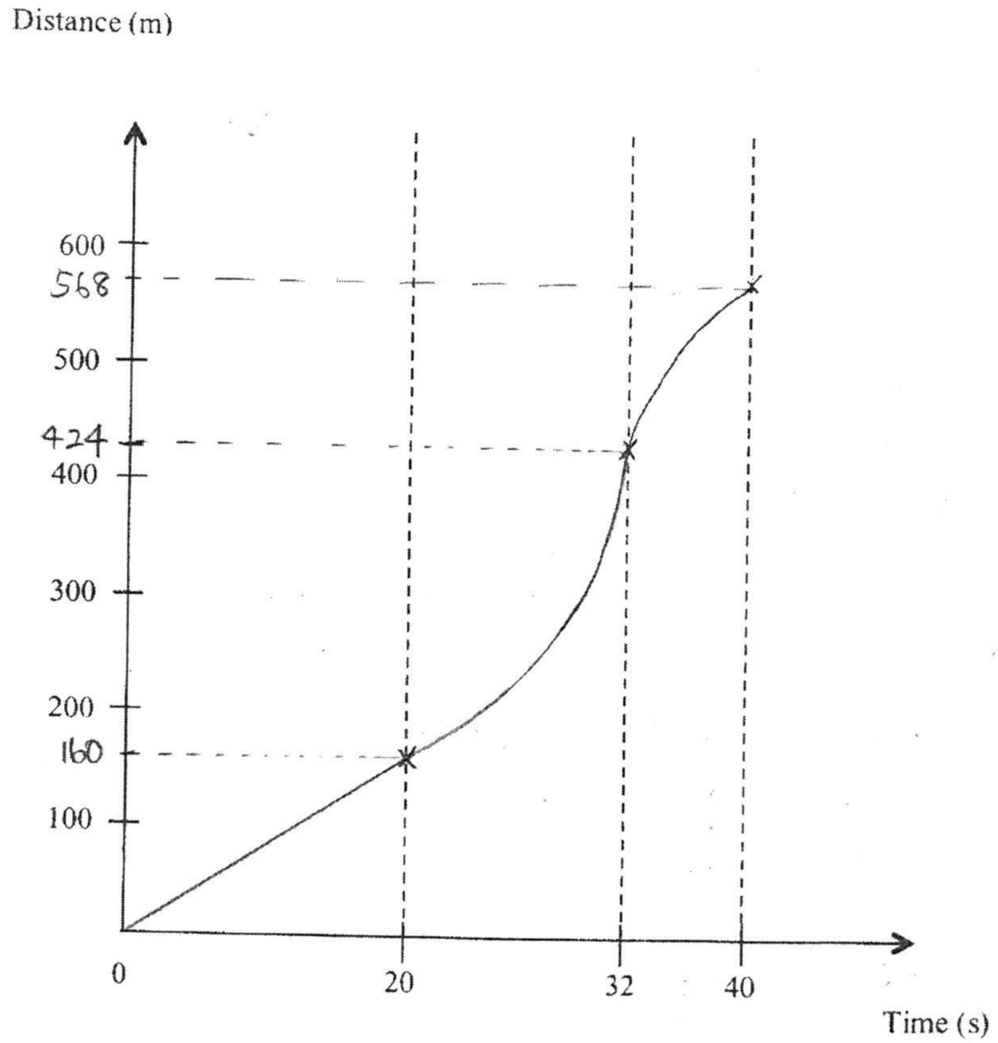
(ii) the deceleration of the car during the last five seconds.



Answer (a) (i) ________________ m/s [2]

(ii) ________________ m/s^2 [2]

(b) On the axes given below, sketch the distance-time graph for the whole journey. [3]



18. In triangle ABC, $AB = 12$ cm, $BC = 13$ cm and $AC = 5$ cm.
AC is produced to D and $CD = 30$ cm.

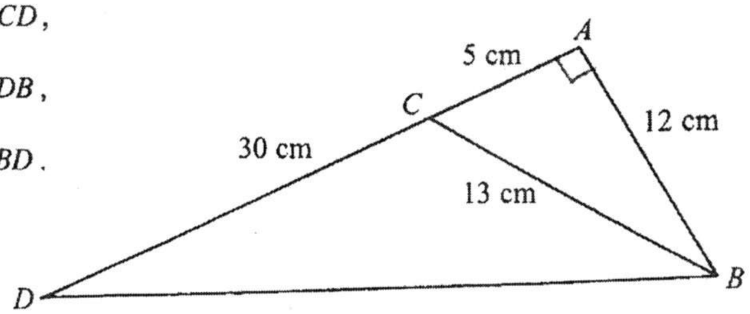
(a) Explain why angle BAC is a right angle. [2]

(b) Express each of the following as a fraction in its exact form.

(i) $\cos\angle BCD$,

(ii) $\tan\angle ADB$,

(iii) $\sin\angle CBD$.



Answer (*b*) (i)____________________[1]

(ii)____________________[1]

(iii)____________________[3]

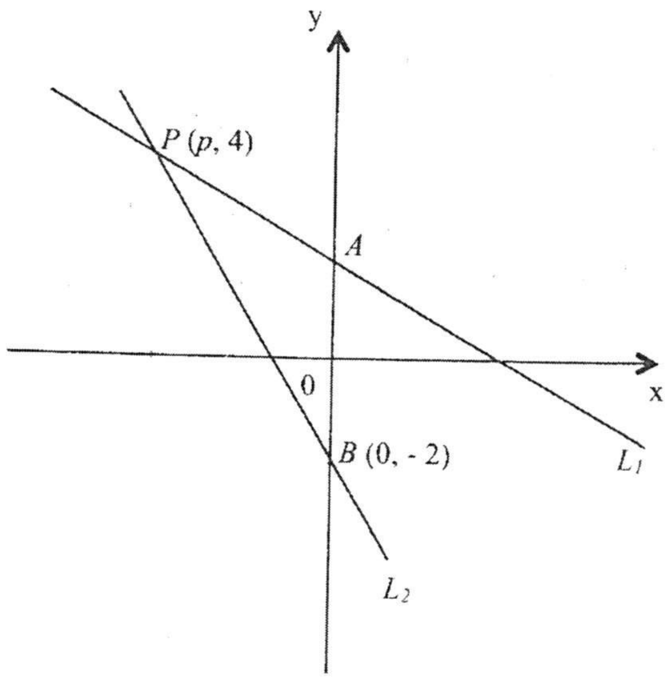
19. The diagram below, which is not drawn to scale, shows two lines, L_1 and L_2, intersecting at the point $P(p, 4)$ and cutting the y – axis at the points A and $B(0, -2)$ respectively. The equation of L_1 is $2y + x - 6 = 0$.

(a) State the equation of the line passing through A and is parallel to the x – axis.

(b) Show that $p = -2$, and hence find the equation of line L_2.

(c) Find the length of PB.

(d) A trapezium $PABC$, with AB parallel to PC, has an area of 12 units^2. Find the coordinates of C.



Answer (a) ____________________ [1]

(b) ____________________ [3]

(c) ____________________ [1]

(d) (______,______) [2]

END OF PAPER

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SECONDARY THREE

MATHEMATICS **4016/01**

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Subtotal	
P	
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For Examiner's Use
80

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

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Answer **all** the questions.

1. Factorise the expression $16-9m^2-6mn-n^2$.

$16-9m^2-6mn-n^2 = 16-(9m^2+6mn+n^2)$

$= (4)^2-(3m+n)^2$

$= [4-(3m+n)][4+(3m+n)]$

$= (4-3m-n)(4+3m+n)$ //

Answer $(4-3m-n)(4+3m+n)$ [2]

2. Given that $7^x=3$ and $7^y=5$, find the value of 7^{3x+y}.

$7^{3x+y} = 7^{3x} \times 7^y$

$= (7^x)^3 \times 7^y$

$= (3)^3 \times 5$

$= 135$ //

Answer 135 [2]

3. Given that $m^2+\frac{1}{m^2}=11$, find the values of $\frac{1}{4}\left(m-\frac{1}{m}\right)$.

$(m-\frac{1}{m})^2 = m^2 - 2(m)(\frac{1}{m}) + \frac{1}{m^2}$

$= m^2+\frac{1}{m^2}-2$

$= 11-2$

$= 9$

$\therefore m-\frac{1}{m} = \pm\sqrt{9}$

$= 3$ or -3

Hence, $\frac{1}{4}(m-\frac{1}{m}) = \frac{1}{4}(3)$ or $\frac{1}{4}(-3)$

$= \frac{3}{4}$ or $-\frac{3}{4}$

Answer $\frac{3}{4}$ or $-\frac{3}{4}$ [3]

For Examiner's Use

For Examiner's Use

4. y is inversely proportional to the square root of x, where $x > 0$. It is given that $y = 12$ for a particular value of x. Find the decrease in the value of y when this value of x is increased by 800%.

$$y = \frac{k}{\sqrt{x}}$$

Let $x = a$ when $y = 12$. $12 = \frac{k}{\sqrt{a}}$

$$k = 12\sqrt{a}$$

Let the new y-value be y_1.

$$y_1 = \frac{12\sqrt{a}}{\sqrt{900\% \times a}} \quad] * \frac{12\sqrt{a}}{\sqrt{9a}}$$

$$= \frac{12\sqrt{a}}{3\sqrt{a}}$$

$$= 4$$

∴ Decrease in Value $= 12 - 4$

$= 8$ //

Answer ## 8 [3]

5. Given that $p = 1 - \sqrt{\frac{m^2 + n}{m^2}}$, make m the subject of the formula.

$$p - 1 = -\sqrt{\frac{m^2+n}{m^2}}$$

$$1 - p = \sqrt{\frac{m^2+n}{m^2}}$$

$$(1-p)^2 = \frac{m^2+n}{m^2}$$

$$m^2(1-p)^2 = m^2 + n$$

$$m^2(1-p)^2 - m^2 = n$$

$$m^2[(1-p)^2 - 1] = n$$

$$m^2 = \frac{n}{[(1-p)^2-1]}$$

$$m = \pm\sqrt{\frac{n}{[(1-p)^2-1]}}$$

$$= \pm\sqrt{\frac{n}{[(1-p)+1][(1-p)-1]}}$$

$$= \pm\sqrt{\frac{n}{[2-p][-p]}}$$

$$= \pm\sqrt{\frac{n}{p(p-2)}} \text{ //}$$

Answer $m = \pm\sqrt{\frac{n}{p(p-2)}}$ [3]

6. If 2 men can make 50 tables in 7 days, how long will 14 men take to make 225 tables?

Men	tables	days
2	50	7
2x7=14	50	1
14	225	$\frac{225}{50} = 4\frac{1}{2}$ //

Answer $4\frac{1}{2}$ days [3]

For Examiner's Use

For Examiner's Use

7. Simplify $\left(\frac{2x^2y^2}{54x^5y^{-4}}\right)^{-\frac{1}{3}}$, expressing your answer in the positive index form.

$\left(\frac{54x^5y^{-4}}{2x^2y^2}\right)^{\frac{1}{3}} = \left(27x^{5-2}y^{-4-2}\right)^{\frac{1}{3}}$

$= \left(27x^3y^{-6}\right)^{\frac{1}{3}}$

$= 27^{\frac{1}{3}}x\,y^{-2}$

$= \frac{3x}{y^2}$ //

Answer $\frac{3x}{y^2}$ [3]

8. (a) A polygon has n sides. Three of its interior angles are 148°, 157° and 175°.

The remaining interior angles are 155° each. Find the value of n.

$180° - 148° = 32°$
$180° - 157° = 23°$ (adj. ∠s on st. line)
$180° - 175° = 5°$

Three of the exterior angles are 32°, 23° and 5°

$180° - 155° = 25°$ (adj. ∠s on st. line)

There are $(n-3)$ exterior angles that are 25°.

$32° + 23° + 5° + (n-3)(25)° = 360°$

$25n - 75° = 300°$

$25n° = 375°$

$\therefore n = 15$ //

Answer $n =$ 15 [2]

(b) Explain why the interior angle of a regular polygon cannot be 130°.

Let the no. of sides of the polygon be n.

Exterior ∠ $= 180° - 130° = 50°$

$n = \frac{360°}{50°} = 7\frac{1}{5}$ //

Since n is not an integer, the interior angle of the regular polygon cannot be 130° [1]

9. Express $\frac{5x+2}{3x^2-12}+\frac{1}{2-x}$ as a single fraction in its simplest form.

$$\frac{5x+2}{3(x^2-4)}+\frac{1}{2-x} = \frac{5x+2}{3(x-2)(x+2)} - \frac{1}{x-2}$$

$$= \frac{(5x+2)-3(x+2)}{3(x-2)(x+2)}$$

$$= \frac{5x+2-3x-6}{3(x-2)(x+2)}$$

$$= \frac{2x-4}{3(x-2)(x+2)}$$

$$= \frac{2(x-2)}{3(x-2)(x+2)}$$

$$= \frac{2}{3(x+2)}$$ //

Answer $\frac{2}{3(x+2)}$ [3]

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10. It is given that Cylinder A has a volume of 300 cm^3. Calculate the volume of

(a) Cylinder B with base radius $\frac{2}{5}$ that of Cylinder A and a height thrice that of Cylinder A.

(b) Cylinder C which is geometrically similar to Cylinder A but has a curved surface area 16 times that of Cylinder A.

(a) Let base radius of cylinder A be r cm and its height be h cm

Volume of A $= \pi r^2 h = 300 cm^3$

Volume of B $= \pi (\frac{2}{5}r)^2(3h)$

$= \frac{12}{25}\pi r^2 h$

$= \frac{12}{25}(300)$

$= 144 cm^3$ //

(b) $\frac{\text{Height of C}}{\text{Height of A}} = \sqrt{\left(\frac{16}{1}\right)}$

$= \frac{4}{1}$

$\frac{\text{Volume of C}}{\text{Volume of A}} = \left(\frac{4}{1}\right)^3$

Volume of C $= \frac{64}{1} \times 300 cm^3$

$= 19200 cm^3$ //

Answer (a) 144 cm^3 [2]

(b) 19200 cm^3 [2]

For Examiner's Use

For Examiner's Use

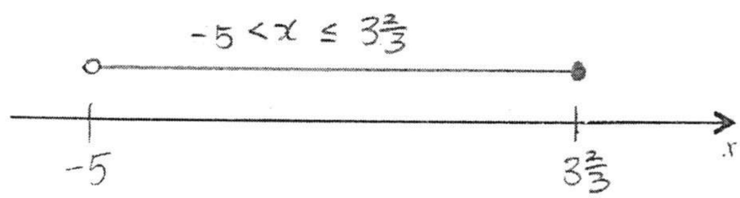
11. (a) Solve the inequality $-\frac{1}{3}+x \le \frac{x+3}{2} < x+4$. Represent your answer on the number line below.

$-\frac{1}{3}+x \le \frac{x+3}{2}$ AND $\frac{x+3}{2} < x+4$

$-\frac{2}{3}+2x \le x+3$ $\qquad$ $x+3 < 2x+8$

$x \le 3\frac{2}{3}$ $\qquad$ $-5 < x$

$\therefore -5 < x \le 3\frac{2}{3}$ //



Answer (a) $-5 < x \le 3\frac{2}{3}$ [4]

(b) Write down all the integers that satisfy $-\frac{1}{3}+x \le \frac{x+3}{2} < x+4$.

Answer (b) −4, −3, −2, −1, 0, 1, 2, 3. [1]

For Examiner's Use

For Examiner's Use

12. The radius of a spherical particle is approximately 5 picometres.
Find, leaving your answer in standard form,

(a) the diameter of one such particle in centimetres,

(b) the number of particles that must be placed side by side in order to make a length of 30 millimetres,

(c) the total volume, in cubic centimetres, of 1 million of such particles. Give your answer correct to 3 significant figures.

Radius of particle $= 5 \times 10^{-12}$ m

(a) Diameter of one particle $= 2 \times 5 \times 10^{-12}$ m
$= 10 \times 10^{-12}$ m
$= 100 \times 10 \times 10^{-12}$ m
$= 1 \times 10^{-9}$ cm //

(b) $(30 \times 10^{-3}\text{ m}) \div (10 \times 10^{-12}\text{ m}) = 3 \times 10^{9}$ //

(c) Volume of 1 particle $= \frac{4}{3}\pi \left(\frac{1 \times 10^{-9}}{2}\right)^3$
$= \frac{\pi}{6} \times 10^{-27}$ cm^3

Volume of 1 million particles $= \left(\frac{\pi}{6} \times 10^{-27}\right) \times 10^{6}$
$= \frac{\pi}{6} \times 10^{-21}$
$\approx 0.523599 \times 10^{-21}$
$= 5.24 \times 10^{-22}$ (3 s.f.) //

Answer (a) 1×10^{-9} cm [2]

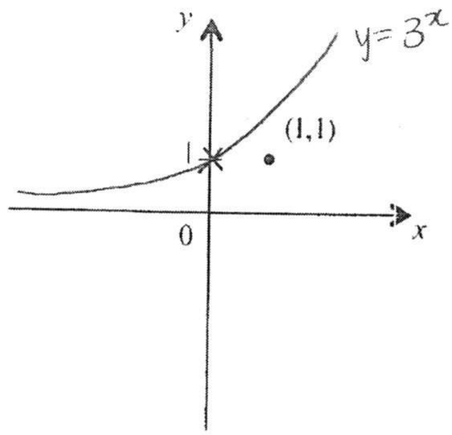
(b) 3×10^{9} [1]

(c) 5.24×10^{-22} cm^3 [2]

For Examiner's Use

For Examiner's Use

13. The point $(1,1)$ is marked on the diagram below. Sketch the graph of $y = 3^x$.



When $x = 0$,
$y = 3^0$
$= 1$
When $x = 1$,
$y = 3^1$
$= 3$

[1]

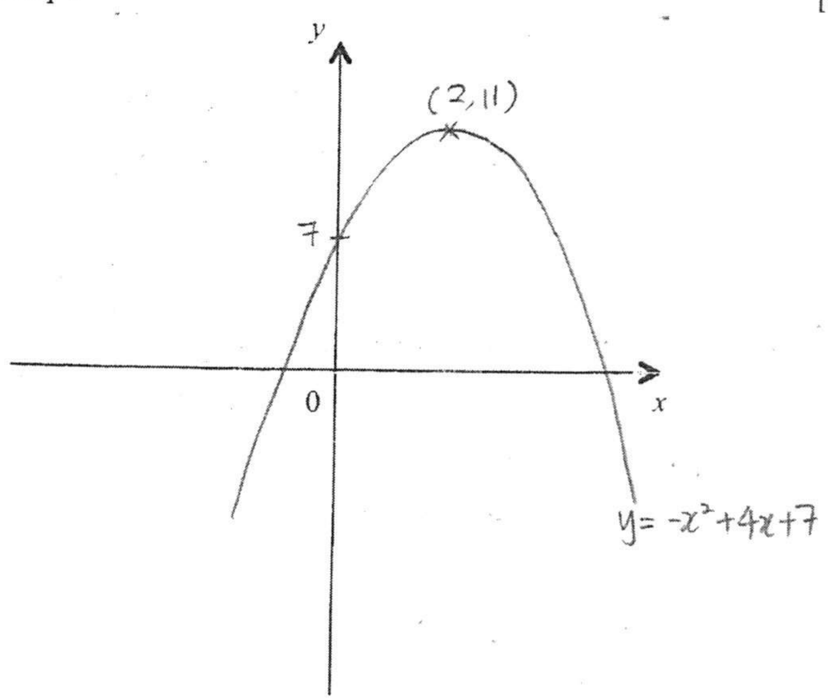
14. (a) Express $-x^2 + 4x + 7$ in the form of $-(x+h)^2 + k$.

$$-x^2+4x+7 = -(x^2-4x-7)$$
$$= -[x^2-4x+(-\tfrac{4}{2})^2-(-\tfrac{4}{2})^2-7]$$
$$= -[(x-2)^2-11]$$
$$= -(x-2)^2+11$$

Answer (a) $-(x-2)^2+11$ [3]

(b) Hence, sketch the graph of $y = -x^2 + 4x + 7$ on the axes below, indicating the turning point and the y – intercept. [2]

① ∩
② y-intercept: (0,7)
③ turning pt: (2, 11)

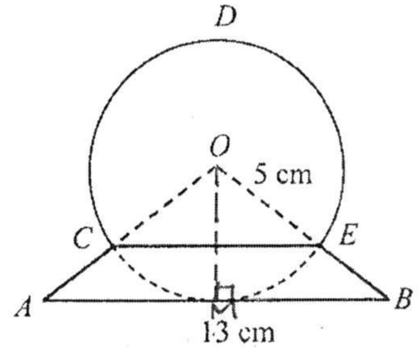


15. The diagram below shows the cross-section of a snowglobe with centre O, of radius 5 cm. The base makes an isosceles triangle OAB. AB is a tangent to the circle and is 13 cm long.

(a) Show that angle $AOB = 1.83$ radians, correct to 3 significant figures. [2]

(b) Calculate

(i) the area of major segment CDE,
(ii) the perimeter of the snowglobe $ABEDC$.



(a) Let the foot of the perpendicular from point O to AB be M.

$OM = 5\text{cm}$ and $MB = \frac{13}{2}$
$= 6\frac{1}{2}\text{cm}$

$\tan M\hat{O}B = \frac{6\frac{1}{2}}{5}$

$M\hat{O}B \approx 0.9151007$

$A\hat{O}B = 2 \times M\hat{O}B$
$= 2 \times 0.9151007$

$\therefore A\hat{O}B = 1.83$ (3sf). (shown)

(bi) Area of major sector OCDE $= \frac{1}{2}(5)^2(2\pi - 1.8302014)$
$\approx 55.662299 \text{ cm}^2$

Area of $\triangle OCE = \frac{1}{2}(5)(5)\sin(1.8302014)$
$\approx 12.081784 \text{ cm}^2$

$\therefore$ Area of major segment CDE $= 55.662299 + 12.081784$
$= 67.7 \text{ cm}^2$ (3s.f.)

(ii) Length of major arc CDE $= (5)(2\pi - 1.8302014)$
≈ 22.2649195 cm

By Pythagoras' Theorem,

$OA = \sqrt{5^2 + (6\frac{1}{2})^2}$
$= \sqrt{\frac{269}{4}}$ cm.

$\therefore CA = (\sqrt{\frac{269}{4}} - 5)$ cm

Perimeter of ABEDC $= 22.2649195 + 2(\sqrt{\frac{269}{4}} - 5) + 13$
$= 41.7$ cm (3 s.f.)

Answer (b) (i) 67.7 cm² [3]

(ii) 41.7 cm [3]

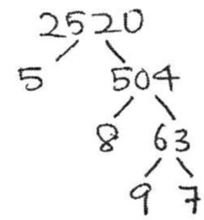
16. (a) Express the numbers 66 and 2520 as products of their prime factors.

(b) Find the smallest positive integer, k, such that $2520k$ is a perfect cube.

(c) Find the smallest positive integer, n, such that $66n$ is a multiple of 2520.

(a) $66 = 2 \times 3 \times 11$ //

$2520 = 2^3 \times 3^2 \times 5 \times 7$ //



(b) $2520k = 2^3 \times 3^2 \times 5 \times 7 \times k$

For 2520k to be a perfect cube,

$k = 3 \times 5^2 \times 7^2$

$= 3675$ //

(c) $66n = 2 \times 3 \times 11 \times n$

For 66n to be a multiple of 2520, } *66n must have 2520 as its factor

$n = 2^2 \times 3 \times 5 \times 7$

$= 420$ //

Answer (*a*) 66 = $2 \times 3 \times 11$ [1]

2520 = $2^3 \times 3^2 \times 5 \times 7$ [1]

(*b*) k = 3675 [2]

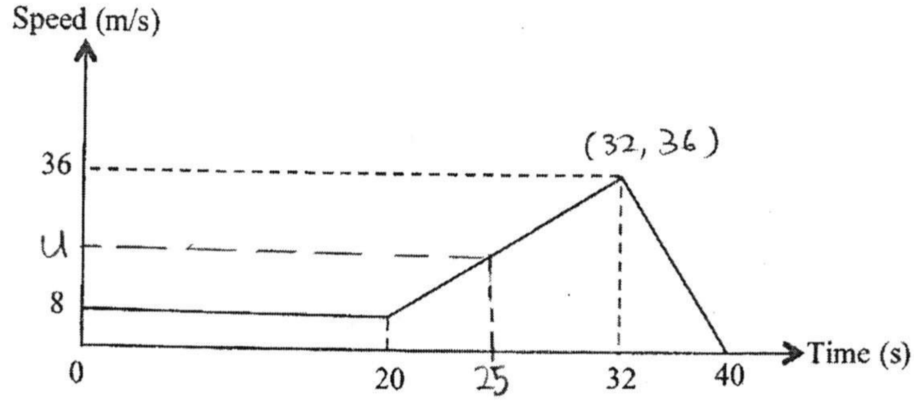
(*c*) n = 420 [2]

17. The graph below shows the speed of a car during a period of 40 seconds.

(a) Calculate

(i) the speed of the car after 25 seconds,

(ii) the deceleration of the car during the last five seconds.



17(ai) Let speed of the car at 25th second be u m/s.

$$\frac{u-8}{25-20} = \frac{36-8}{32-20} \quad \text{(equal gradients)}$$

$$\frac{u-8}{5} = \frac{7}{3}$$

$$u = \left(\frac{7}{3}\times 5\right)+8$$

$$= 19\tfrac{2}{3}$$

∴ Speed of car at 25th second is $19\frac{2}{3}$ m/s.

(ii) Acceleration $= \frac{36-0}{32-40}$

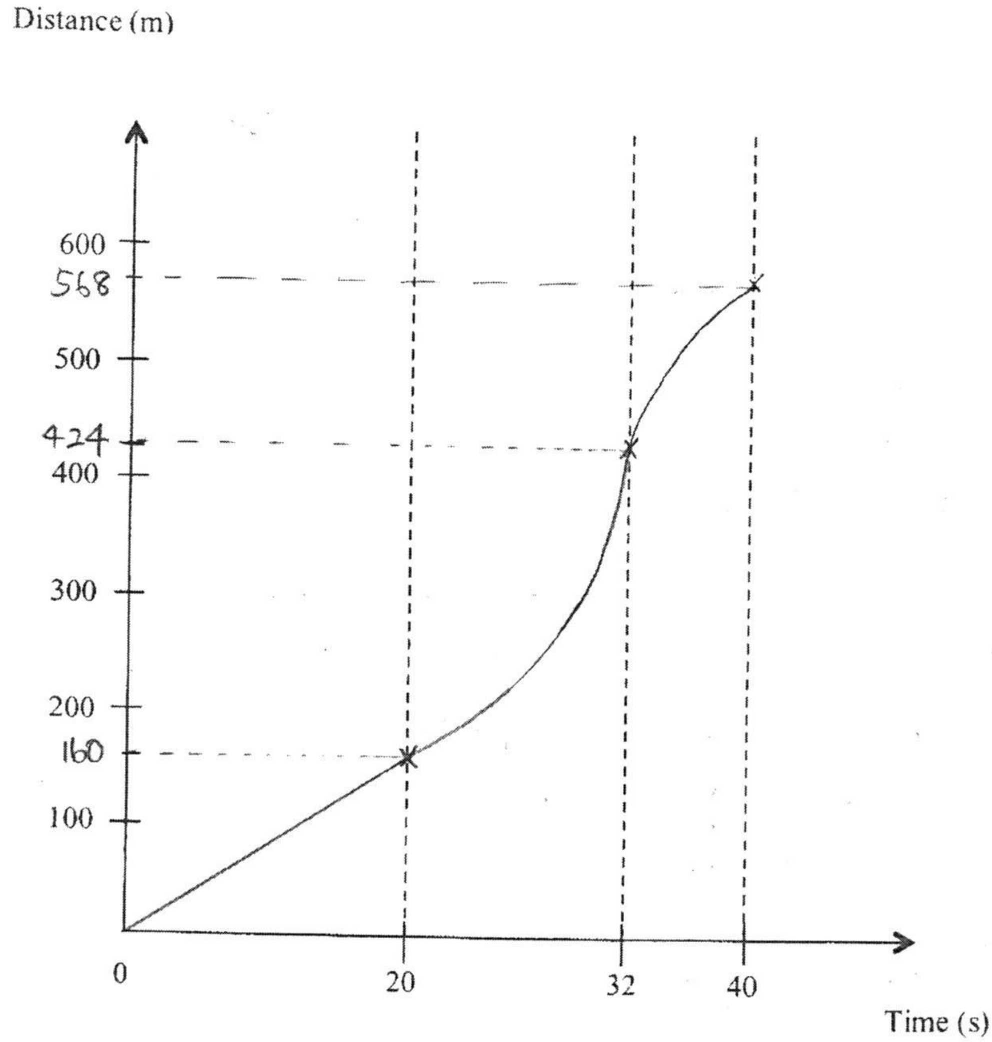
$$= -4\tfrac{1}{2} \text{ m/s}^2$$

∴ Deceleration is $4\frac{1}{2}$ m/s²

Answer (a) (i) $19\frac{2}{3}$ m/s [2]

(ii) $4\frac{1}{2}$ m/s² [2]

(b) On the axes given below, sketch the distance-time graph for the whole journey. [3]



Distance covered in first 20s = 20 × 8
= 160m

Distance covered from 20s to 32s = $\frac{1}{2}(8+36)(32-20)$
= 264m.

Distance covered from 32s to 40s = $\frac{1}{2} \times (40-32)(36)$
= 144m.

18. In triangle ABC, $AB = 12$ cm, $BC = 13$ cm and $AC = 5$ cm.
AC is produced to D and $CD = 30$ cm.

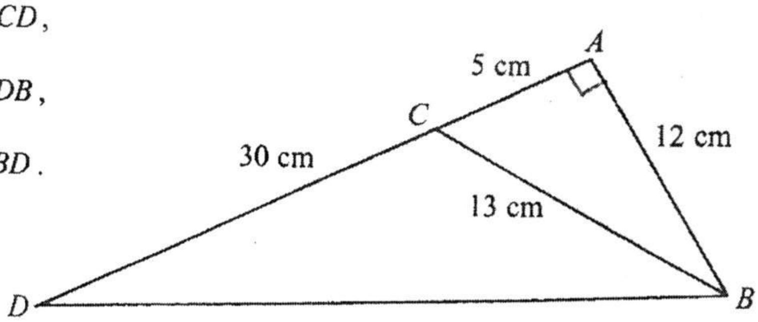
(a) Explain why angle BAC is a right angle. [2]

(b) Express each of the following as a fraction in its exact form.

(i) $\cos\angle BCD$,

(ii) $\tan\angle ADB$,

(iii) $\sin\angle CBD$.



(a) $BC^2 = 13^2$
$= 169$
$AC^2 + AB^2 = 5^2 + 12^2$
$= 169$
Since $BC^2 = AC^2 + AB^2$, by converse of Pythagoras' Theorem, $\triangle ABC$ is a right-angle triangle and $\angle BAC = 90°$.

(b i) $\cos\angle BCD = -\cos(180° - \angle BCD)$
$= -\cos(\angle BCA)$
$= -\frac{5}{13}$ //

(ii) $\tan\angle ADB = \frac{12}{30+5}$
$= \frac{12}{35}$ //

(iii) Area of $\triangle BCD = \frac{1}{2} \times 30\text{cm} \times 12\text{cm}$
$= 180\text{cm}^2$

$BD^2 = 35^2 + 12^2$
$BD = \sqrt{1369}$
$= 37$ cm

$\frac{1}{2}(37)(13)\sin\angle CBD = 180$
$\sin\angle CBD = \frac{180}{\frac{1}{2}(37)(13)}$
$= \frac{360}{481}$ //

Answer (*b*) (i) $-\frac{5}{13}$ [1]

(ii) $\frac{12}{35}$ [1]

(iii) $\frac{360}{481}$ [3]

For Examiner's Use

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19. The diagram below, which is not drawn to scale, shows two lines, L_1 and L_2, intersecting at the point $P(p, 4)$ and cutting the y – axis at the points A and $B(0, -2)$ respectively. The equation of L_1 is $2y + x - 6 = 0$. → $2y = -x + 6 \Rightarrow y = -\frac{1}{2}x + 3$

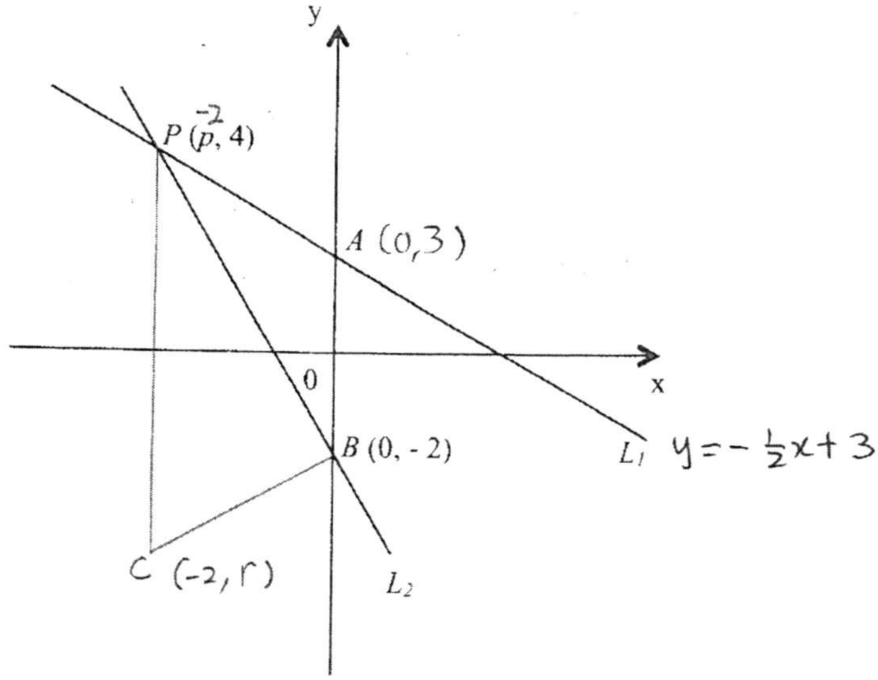
(a) State the equation of the line passing through A and is parallel to the x – axis.

(b) Show that $p = -2$, and hence find the equation of line L_2.

(c) Find the length of PB.

→ * C is vertically below P.

(d) A trapezium $PABC$, with AB parallel to PC, has an area of 12 units². Find the coordinates of C.



(a) When $x = 0$,

$y = -\frac{1}{2}(0) + 3$

$y = 3$

Coordinates of A are (0, 3).

∴ Equation of line passing through A and parallel to x-axis is $y = 3$ //

(b) When $x = p$ and $y = 4$,

$4 = -\frac{1}{2}p + 3$

$\frac{1}{2}p = -1$

$p = -2$ (shown) //

Gradient of $L_2 = \frac{4 - (-2)}{-2 - 0}$

$= -3$

∴ Equation of L_2 is $y = -3x - 2$ //

For Examiner's Use

For Examiner's Use

(c) Length of PB $= \sqrt{(-2-0)^2 + [4-(-2)]^2}$

$= \sqrt{4+36}$

$= \sqrt{40}$

$= 6.32$ units (3 s.f.)

(d) Let coordinates of C be $(-2, r)$

$\frac{1}{2}\{[3-(-2)] + (4-r)\} \times 2 = 12$

$\{5 + 4 - r\} = 12$

$9 - r = 12$

$-3 = r$

$\therefore$ Coordinates of C are $(-2, -3)$.

Answer (a) $y=3$ [1]

(b) $y=-3x-2$ [3]

(c) 6.32 units [1]

(d) (-2 , -3) [2]

END OF PAPER

Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

SEMESTRAL ASSESSMENT TWO

SECONDARY THREE

MATHEMATICS **12 Oct 2016**

2 hours

Additional Materials:
Writing paper (4 sheets)
Graph paper (1 sheet)

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of **10** printed pages.

1. A small pond can be filled by two taps A and B in 3 hours. Tap A can fill up the pond in x hours while Tap B takes $(2x+3)$ hours to fill.

(a) Find the fraction of the pond that can be filled up in 1 hour by

(i) Tap A, [1]

(ii) Tap B. [1]

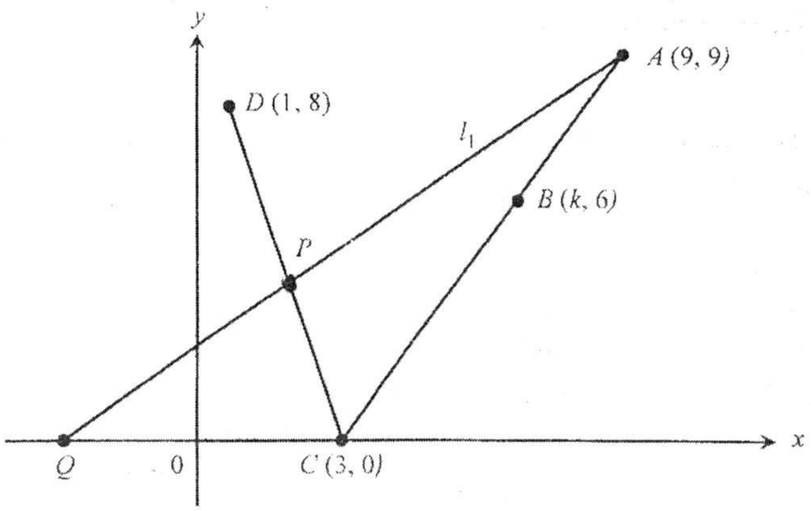
(b) Form an equation in x and show that it reduces to $2x^2-6x-9=0$. [3]

(c) Solve the equation $2x^2-6x-9=0$, giving your answers correct to 2 decimal places. [2]

(d) Explain why one of the solutions in (c) is rejected. [1]

(e) How much longer does it take for Tap B to fill up the pond than Tap A? Give your answer correct to the nearest minute. [2]

2. The coordinates of points A, B, C and D are (9, 9), (k, 6), (3, 0) and (1, 8) respectively.



(a) Find the length of AC. [2]

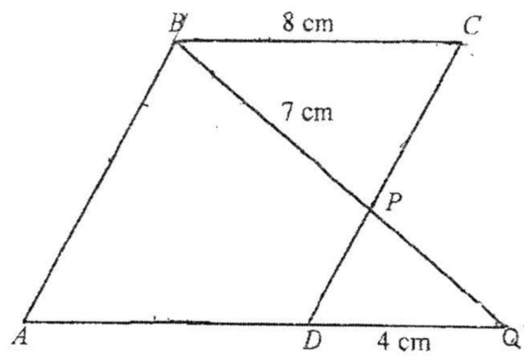
(b) Given that the point B lies on AC, find the value of k. [2]

(c) Find the equation of CD. [2]

(d) A line l_1 with equation $7y-5x-18=0$ intersects CD at the point P. Find the coordinates of P. [2]

(e) Find the coordinates of the point Q where l_1 cuts the x-axis. [2]

3. **(a)** In the diagram, $ABCD$ is a parallelogram. The point Q lies on AD produced. The line BQ intersects CD at point P. It is given that $BP = 7$ cm, $BC = 8$ cm and $DQ = 4$ cm.



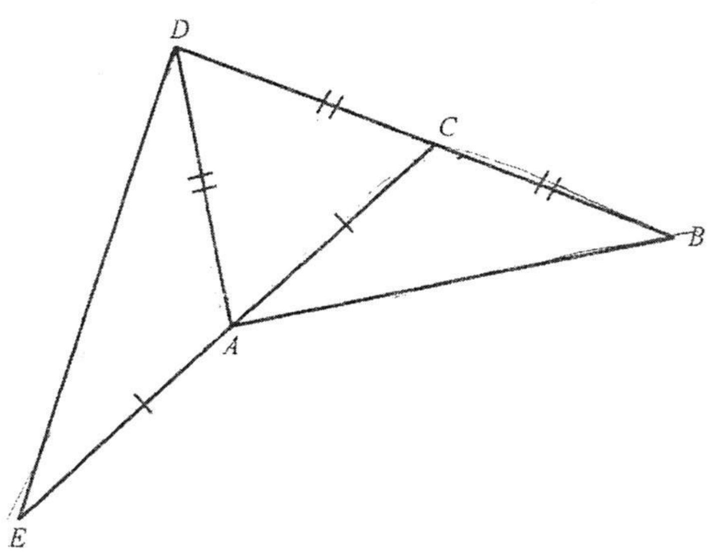
(i) Prove that triangles BCP and QAB are similar. [2]

Find

(ii) PQ, [2]

(iii) $\dfrac{\text{Area of } \Delta BPC}{\text{Area of quadrilateral } ABPD}$. [2]

(b) In the diagram below, $BC = CD = DA$ and $AC = AE$. Show that triangles ABC and EDA are congruent. [3]



4. A series of diagrams of shaded and unshaded small triangles is shown below.

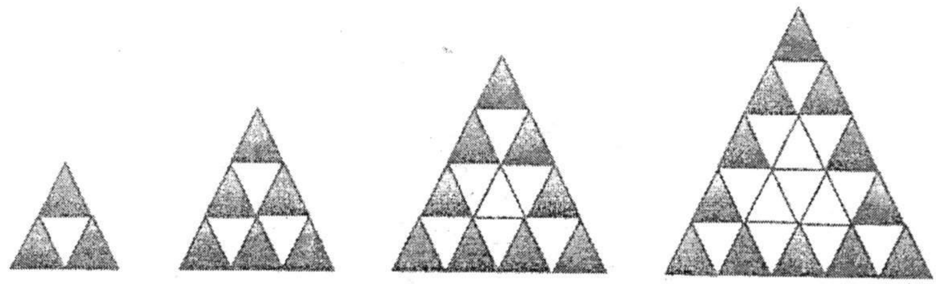


Diagram 1 *Diagram 2* *Diagram 3* *Diagram 4*

The shaded triangles are those which have at least one side on the edge of the big triangle. All of the other small triangles are unshaded.

The following table shows numbers of small triangles.

Diagram	1	2	3	4	5	...	n
Number of shaded triangles	3	6	9	12			x
Total number of triangles	4	9	16	25			y
Number of unshaded triangles	1	3	7	13			z

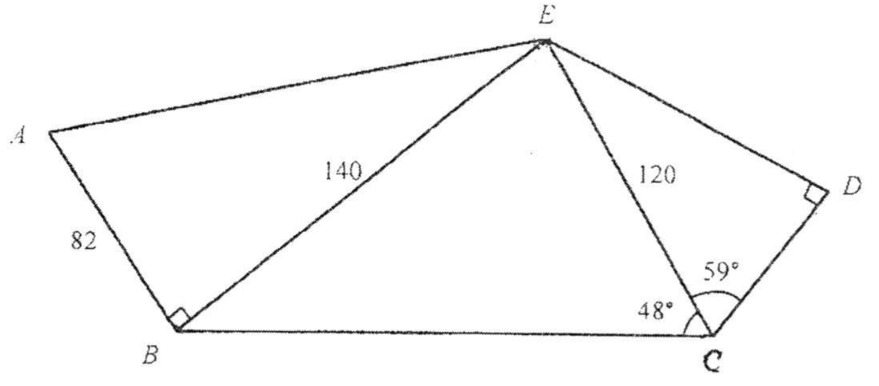
By considering the number patterns, without drawing further diagrams,

(i) write down the number of shaded triangles, the total number of triangles and the number of unshaded triangles in Diagram 5, [2]

(ii) find, in terms of n, expressions for x, y and z, [3]

(iii) find the number of unshaded triangles when $n = 2016$. [1]

5. The diagram shows footpaths BE and CE in a park $ABCDE$. There are Pokestops at locations A, B, C, D and E. Given that $AB = 82$ m, $BE = 140$ m, $CE = 120$ m, $B\hat{C}E = 48^\circ$, $D\hat{C}E = 59^\circ$ and $A\hat{B}E = C\hat{D}E = 90^\circ$.



(a) Calculate

(i) the distance between Pokestops A and E. [2]

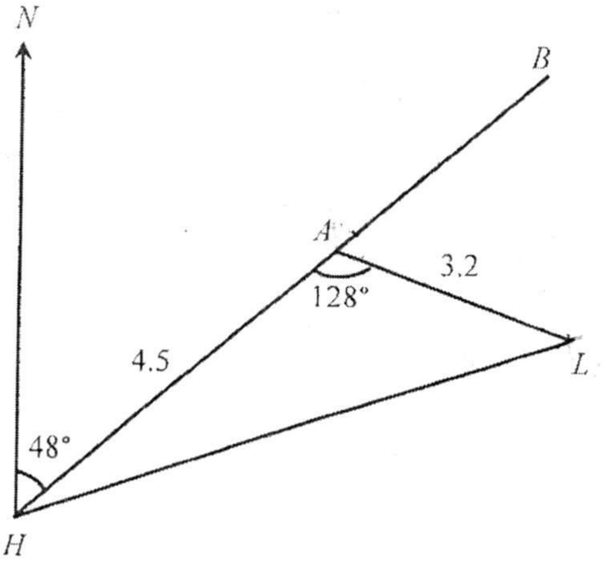
(ii) the distance between Pokestops C and D, [2]

(iii) $C\hat{E}B$. [3]

(b) Given that there is a lure set to attract pokemons at Pokestops B, C and E, find the area of the triangle formed by these three pokestops. [2]

(c) Using the radar map, a rare pokemon, Snorlax, is sighted at C. Given that Snorlax will disappear in 15 minutes, determine if a trainer will be able to catch the Snorlax if he runs from B at 10 km/h. [3]

6. The diagram shows the positions of Tanah Merah Habour H, a lighthouse L and two buoys A and B. HAB forms a straight line. The bearing of A from H is 048°. It is given that $HA = 4.5$ km, $AL = 3.2$ km and $H\hat{A}L = 128^\circ$.



(a) Calculate the

(i) bearing of L from A, [2]

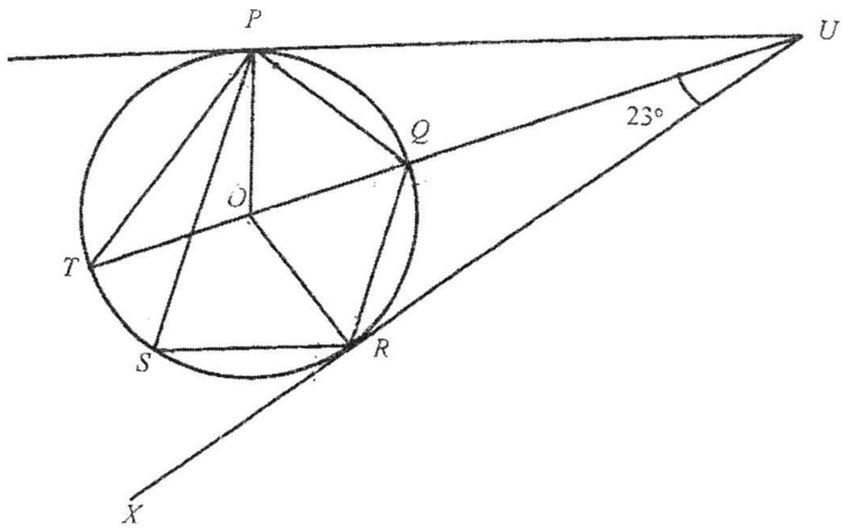
(ii) bearing of H from L. [3]

(b) A boat sailed from the habour along the route HAB.

(i) The boat sailed at a constant speed of 5 m/s. Given that the boat reached A at 09 45, find the time it left the habour. [2]

(ii) Given that the height of the lighthouse is 130 m, calculate the greatest angle of elevation of the top of the lighthouse when viewed from the boat along its path from H to B. [3]

7. In the diagram, $PQRST$ are points on a circle with centre O. UP and UR are tangents to the circle. $TOQU$ is a straight line and $O\hat{U}R = 23^\circ$.



Find, stating your reasons clearly,

(a) $\angle ORU$, [1]

(b) $\angle TPQ$, [1]

(c) reflex $\angle POR$, [2]

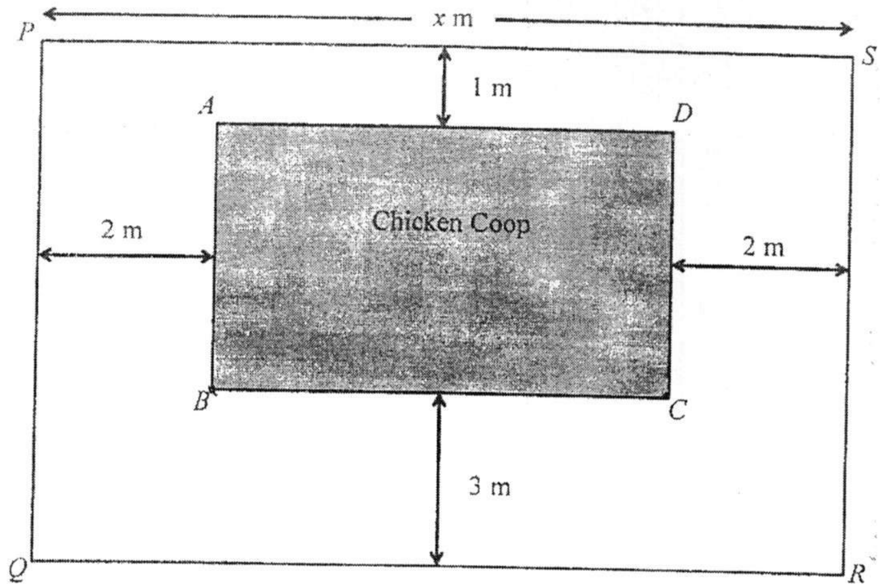
(d) $\angle PQR$, [2]

(e) $\angle PSR$, [2]

(f) $\angle QRX$. [2]

8. **Answer the whole of this question on a sheet of graph paper.**

The area of a rectangular plot of land *PQRS* in a primary school is 180 m^2.



(a) Given that the length of the plot of land is x m, write down expressions, in terms of x, for

(i) BC, [1]

(ii) AB. [2]

(b) Hence, show that the area, y m^2, of the chicken coop, *ABCD*, is given by

$$y = 196 - 4x - \frac{720}{x}.$$ [2]

The table below shows some values of x and the corresponding values of y, correct to 1 decimal place, where $y = 196 - 4x - \frac{720}{x}$.

x	5	10	15	20	25	30	35	40
y	32	84	88	a	67.2	52	35.4	18

(c) Find the value of a. [1]

(d) Using a scale of 2 cm to represent 5 m on the x-axis ~~for $11 \le x \le 18$~~ and 2 cm to represent 10 m^2 on the y-axis ~~for $56 \le y \le 64$~~, draw the graph of

$$y = 196 - 4x - \frac{720}{x}.$$ [3]

(e) By drawing a tangent, find the gradient of the curve where $x = 20$. [2]

(f) Use your graph to find

(i) the range of values of x for which the area of the chicken coop is at least 60 m^2, [1]

(ii) the value of x for which the area of the chicken coop is greatest. [1]

- End of Paper -

新加坡海星中學
MARIS STELLA HIGH SCHOOL

Marks: ______

Subject: ______ Paper No: ______ Date: ______

Name: ______ Class: ______ No: ______

For Examiner's Use

1(a)(i) Fraction of pond filled by A in 1h = $\frac{1}{x}$ //

(ii) Fraction of pond filled by B in 1h = $\frac{1}{2x+3}$ //

(b) Fraction of pond filled by BOTH A and B in 1h = $\frac{1}{3}$

$$\frac{1}{x} + \frac{1}{2x+3} = \frac{1}{3}$$

$$3(2x+3) + 3x = x(2x+3)$$

$$6x+9+3x = 2x^2+3x$$

$$2x^2-6x-9=0 \quad \text{(shown)}$$ //

(c)
$$x = \frac{-(-6) \pm \sqrt{(-6)^2-4(2)(-9)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{108}}{4}$$

$$= 4.10 \text{ or } -1.10 \quad (2\text{ d.p.})$$ //

(d) As the measurement of time is positive, ∴ x cannot be -1.10.

(e) $2x+3-x = x+3$

When $x \approx 4.098076$,

∴ Time difference = $4.098076+3$

= 7.098076 h

= 4:26 min (nearest min.) //

For Examiner's Use

2(a) Length of AC $= \sqrt{(9-3)^2 + (9-0)^2}$

$= \sqrt{117}$

≈ 10.8 units (3 s.f.)

(b) Gradient of AB = Gradient of AC

$\frac{9-6}{9-k} = \frac{9-0}{9-3}$

$\frac{3}{9-k} = \frac{9}{6}$

$18 = 9(9-k)$

$2 = 9-k$

$-7 = -k$

$\therefore k = 7$

(c) Gradient of CD $= \frac{8-0}{1-3}$

$= -4$

$\frac{y-0}{x-3} = -4$

$y = -4x+12$

Equation of CD is $y = -4x+12$.

(d) $7y - 5x - 18 = 0$ — ①

$y = -4x+12$ — ②

Sub. ② into ①,

$7(-4x+12) - 5x - 18 = 0$

$-28x + 84 - 5x - 18 = 0$

$-33x + 66 = 0$

$-33x = -66$

$x = 2$

Sub. $x=2$ into ②,

$y = -4(2)+12$

$= 4$

$\therefore$ Coordinates of P are (2, 4).

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For Examiner's Use

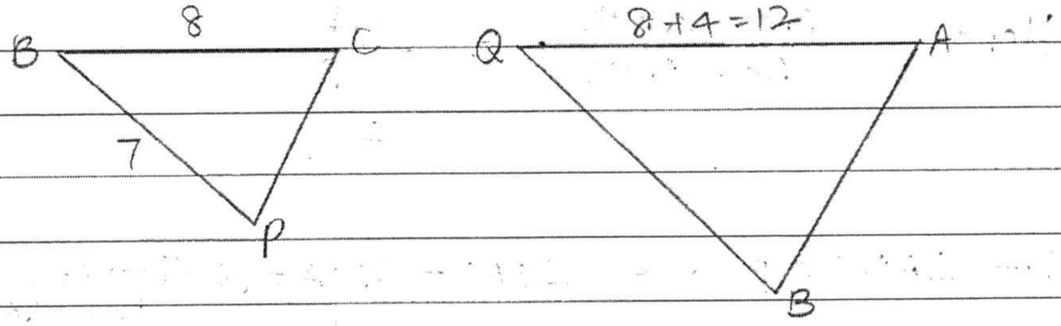
(cont'd)

2(e) When $y = 0$,

$$7(0) - 5x - 18 = 0$$
$$-5x = 18$$
$$x = -3\frac{3}{5}$$

$\therefore$ coordinates of Q are $(-3\frac{3}{5}, 0)$ //

3(a)(i)



① $\angle BCP = \angle QAB$ (opposite $\angle$s in a parallelogram are equal).

② $\angle CBP = \angle AQB$ (alt. $\angle$s, $BC // QA$)

$\therefore \triangle BCP$ is similar to $\triangle QAB$. (*AA-similarity)

(ii)

$$\frac{BQ}{7} = \frac{12}{8}$$
$$BQ = 10\frac{1}{2} \text{ cm}$$
$$\therefore PQ = 10\frac{1}{2} - 7$$
$$= 3\frac{1}{2} \text{ cm} //$$

(iii)

$$\frac{\text{Area } \triangle BPC}{\text{Area } \triangle QAB} = \left(\frac{8}{12}\right)^2$$
$$= \frac{4}{9}$$

→

(cont'd)

For Examiner's Use

3(aiii) * observe that $\triangle QDP$ is similar to $\triangle QAB$.

$$\frac{\text{Area } \triangle QDP}{\text{Area } \triangle QAB} = \left(\frac{3\frac{1}{2}}{10\frac{1}{2}}\right)^2$$

$$= \frac{1}{9}$$

$$\therefore \frac{\text{Area of } \triangle QAB}{\text{Area of } ABPD} = \frac{9}{9-1}$$

$$= \frac{9}{8}$$

Area $\triangle BPC$: Area $\triangle QAB$	Area $\triangle QAB$: Area $ABPD$
4 : 9	9 : 8

Hence, $\frac{\text{Area } \triangle BPC}{\text{Area } ABPD} = \frac{4}{8}$

$$= \frac{1}{2} \,//$$

(b) ① Let $\angle ADC = \beta$ and $\angle DAC = \angle DCA = \alpha$ (base $\angle$s of isos. $\triangle ADC$)

$\angle DAE = \alpha + \beta$ (ext. $\angle$ of $\triangle$)

$\angle BCA = \alpha + \beta$ (ext. $\angle$ of $\triangle$)

$\therefore \angle DAE = \angle BCA$

② Given $BC = DA$ and $AC = EA$,

$\therefore \triangle ABC \equiv \triangle EDA$ (SAS) //

*Remember: $\angle$ used must be an included $\angle$!

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For Examiner's Use

4(i) Number of shaded triangles $= 3 \times 5$
$= 15$ //

Total number of triangles $= 6^2$
$= 36$ //

Number of unshaded triangles $= 36 - 15$
$= 21$ //

(ii) $x = 3n$ //
$y = (n+1)^2$ // (OR) n^2+2n+1 //
$z = (n+1)^2 - 3n$ // (OR) $n^2 - n + 1$ //

5(ai) By Pythagoras' Theorem,
$AE^2 = 82^2 + 140^2$
$= 26324$
$AE = \sqrt{26324}$
$= 162$ m (3s.f.) //

(ii) $\cos 59° = \frac{CD}{120}$
$CD = 120 \cos 59°$
$= 61.8$ m (3s.f.) //

(iii) Using sine rule,
$\frac{\sin E\hat{B}C}{120} = \frac{\sin 48°}{140}$
$\sin E\hat{B}C = \frac{\sin 48°}{140} \times 120$
$E\hat{B}C \approx 39.56709°$
$B\hat{E}C = 180° - 39.56709° - 48°$ (< sum of $\triangle$)
$= 92.43291°$
$= 92.4°$ (1d.p.) //

For Examiner's Use

5 (b) Area of $\triangle BEC = \frac{1}{2}(140)(120)\sin(92.43291°)$

$\approx 8392.4283 \text{ m}^2$

$= 8390 \text{ m}^2$ (3 s.f.)

(c) Using sine rule,

$$\frac{BC}{\sin 92.43291°} = \frac{140}{\sin 48°}$$

$$BC = \frac{140}{\sin 48°} \times \sin 92.43291°$$

≈ 188.21877 m

$= 0.18821877$ km

Time taken by trainer to reach C from B $= \frac{0.18821877}{10}$ h

$= 0.018821877$ h

$= 1.13$ min (3 s.f.)

∴ The trainer will reach C in time to try and catch the Snorlax.

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Marks: ________

Subject: ________ Paper No: ________ Date: ________

Name: ________ Class: ________ No: ________

For Examiner's Use

6(a)(i) $\angle BAL = 180° - 128°$ (adj. ∠s on st. line)

$= 52°$

Bearing of L from A $= 48° + 52°$

$= 100°$

(ii) Using cosine rule,

$HL^2 = (4.5)^2 + (3.2)^2 - 2(4.5)(3.2)\cos 128°$

$= 30.49 - 28.8\cos 128°$

$HL = \sqrt{30.49 - 28.8\cos 128°}$

≈ 6.94414 km

Using sine rule,

$\frac{\sin A\hat{H}L}{3.2} = \frac{\sin 128°}{6.94414}$

$\sin A\hat{H}L = \frac{\sin 128°}{6.94414} \times 3.2$

$A\hat{H}L \approx 21.29263°$

Bearing of H from L $= 180° + (48° + 21.29263°)$ (alt. ∠s)

$= 249.3°$ (1d.p.)

b(i) 4.5 km = 4500 m

Time taken by boat to sail from H to A $= \frac{4500}{5}$

$= 900$ s

$= 15$ min

∴ The boat left the harbour at 0930

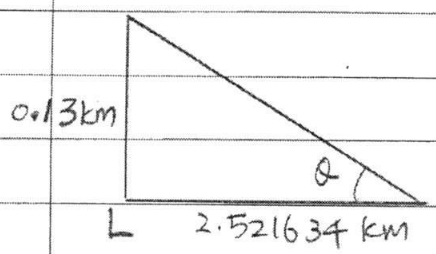
For Examiner's Use

6 b(ii) Let the perpendicular distance from L to HB be d.

$\sin 52° = \frac{d}{3.2}$

$d = 3.2 \times \sin 52°$

≈ 2.521634 km.



Let the greatest ∠ of elevation be θ.

$\tan \theta = \frac{0.13}{2.521634}$

$\theta = 3.0°$ (1 d.p.)

7(a) $\angle ORU = 90°$ // (tan ⊥ rad.)

(b) $\angle TPQ = 90°$ // (∠ in semicircle)

(c) $\angle PUO = \angle RUO = 23°$ (tan. from ext. pt.)

$\angle OPU = 90°$ (tan ⊥ rad.)

$\angle POR = 360° - 90° - 90° - (2 \times 23°)$

$= 134°$

reflex $\angle POR = 360° - 134°$ (∠s at a pt.)

$= 226°$ //

(d) $\angle PQR = 226° \div 2$ (∠ at centre = 2×∠ at circumference)

$= 113°$ //

(e) $\angle PSR = 134° \div 2$ (∠ at centre = 2×∠ at circumference)

$= 67°$

f) $\angle UOR = 180° - 90° - 23°$ (∠ sum of △UOR)

$= 67°$

$\angle ORQ = \frac{180° - 67°}{2}$ (base ∠s of isos. △OQR)

$= 56.5°$

→

For Examiner's Use

(cont'd)

7(f) $\angle QRX = 90^\circ + 56.5^\circ$

$= 146.5^\circ$ //

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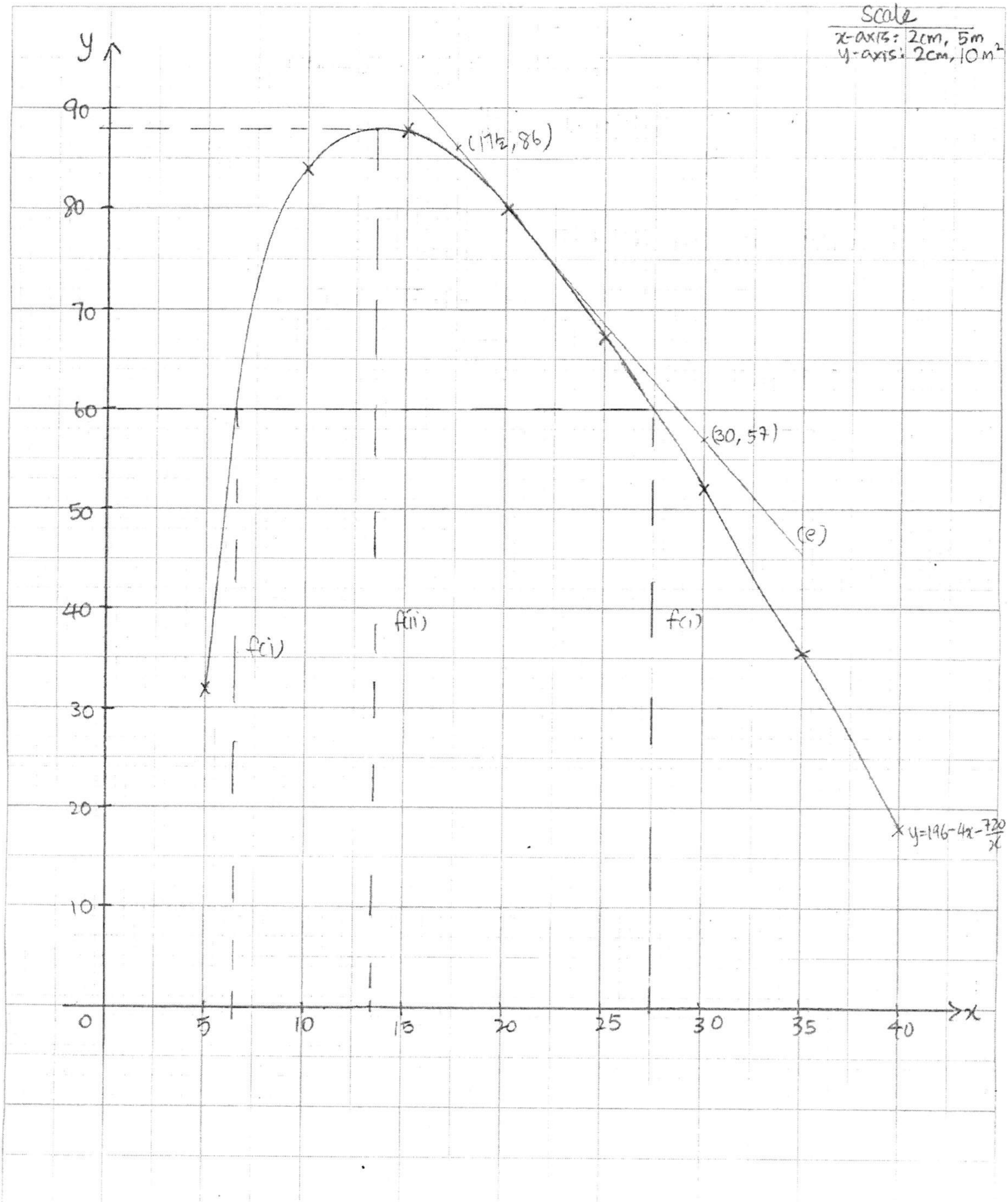

a(i) $BC = (x - 4)$ m

(ii) $SR = \frac{180}{x}$ m

$\therefore AB = \left(\frac{180}{x} - 4\right)$ m

(b) $y = (x-4)\left(\frac{180}{x} - 4\right)$

$y = 180 - 4x - \frac{720}{x} + 16$

$\therefore y = 196 - 4x - \frac{720}{x}$ (shown)

(c) $a = 196 - 4(20) - \frac{720}{20}$

$= 80$

(e) Gradient of curve (at $x = 20$) $= \frac{86 - 57}{17\frac{1}{2} - 30}$

≈ -2.32 (3 s.f.)

f(i) $6.5 \leq x \leq 27.5$

(ii) $x = 13\frac{1}{2}$