| Class | Index Number | Name |
|---|---|---|
| | | |

新加坡海星中学

**MARIS STELLA HIGH SCHOOL**

**SEMESTRAL ASSESSMENT ONE**

**SECONDARY THREE**

**MATHEMATICS** **06 May 2016**

**2 hours 30 minutes**

*Additional Materials:*
Writing paper (3 sheets)
Graph paper (1 sheet)

**INSTRUCTIONS TO CANDIDATES**

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

At the end of the examination, submit Section A and B **separately**.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

| For Examiner's Use | | | |
|---|---|---|---|
| | Section A | Section B | |
| Subtotal | | | |
| Presentation | | | |
| Unit | | | |
| Rounding off | | | 100 |

This document consists of **14** printed pages.

*Mathematical Formulae*

*Compound Interest*

$$\text{Total amount} = p\left(1+\frac{r}{100}\right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

**SECTION A (56 MARKS)**
Answer **ALL** the questions in the spaces provided.

**1.** Given that $-1 \le a < 3$ and $-5 \le b \le -1$, where $a$ and $b$ are integers, find

(a) the greatest value of $ab$,

*Answer* (*a*) .............................. [1]

**(b)** the smallest possible value of $\frac{a}{b^2}$, given that $\frac{a}{b^2}$ is a perfect cube.

*Answer* (*b*) .............................. [1]

---

**2.** Simplify $\frac{(p^{-1}q^2)^5 \div (2pr^{-4})}{(3qr)^3(2p^{-2}q^3)^0}$, leaving your answer in positive index notation.

*Answer* .............................. [3]

---

3. **(a)** Factorise $169+52q+4qr-r^2$ completely.

*Answer (a)* ........................ [2]

**(b)** Make $x$ the subject of the equation $2x^2=\frac{y-x^2}{2y}+1$.

*Answer (b)* $x=$........................ [3]

4. The curve $y=2(x+2)(x+k)$ cuts the $y$-axis at $(0,-8)$.

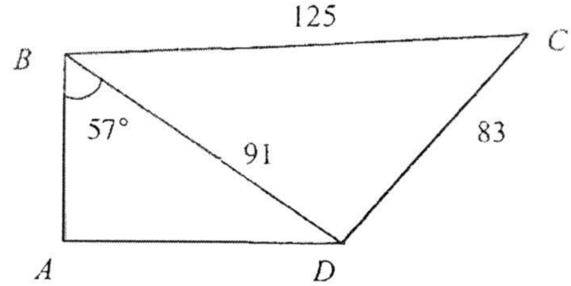
**(a)** Find the value of $k$.

*Answer (a)* $k=$ ........................ [2]

**(b)** Using the value of $k$ found in part (a), find the coordinates of the points where the curve cuts the $x$-axis.

*Answer (b)* ........................... [2]

**5.** The figure below represents a plot of land *ABCD*. *B* is due north of *A*, *D* is due east of *A*. $BC = 125$ m, $CD = 83$ m and $BD = 91$ m. Angle $ABD = 57°$.




**(a)** Find angle *BDC*.

*Answer* (*a*) .........................° [3]

**(b)** Find the bearing of *D* from *C*.

*Answer* (*b*) .........................° [2]

6. **(a)** Solve the inequality $\frac{x-35}{3} < 5-3x \leq 2(4-x)$. Represent your answer on the number line below.


*Answer* (*a*) ............................ [4]

**(b)** Write down the largest prime number which satisfies

$$\frac{x-35}{3} < 5-3x \leq 2(4-x).$$

*Answer* (*b*) $x =$........................ [1]

7. A hard disk has a memory capacity of 2 terabytes.

(a) If a low-resolution photograph takes up about 250 kilobytes, how many million photographs can be stored in the hard disk?

*Answer* (*a*) .................. million [2]

(b) If the hard disk is to store 25 video clips with capacity of 273 megabytes each, how much capacity is left in the disk? Give your answer in standard form, correct to 3 significant figures.

*Answer* (*b*) ..................... bytes [3]

8. **(a)** Express $\frac{5}{12x+9}-\frac{2x+1}{9-16x^2}$ as a single fraction.

*Answer* (*a*) .......................... [3]

**(b)** **Hence**, solve $\frac{5}{12x+9}-\frac{2x+1}{9-16x^2}=1$.

*Answer* (*b*) $x=$ ........ or .......... [3]

9. **(a)** Solve $\frac{25^x}{5} - \sqrt{\frac{1}{625}} = 0$.

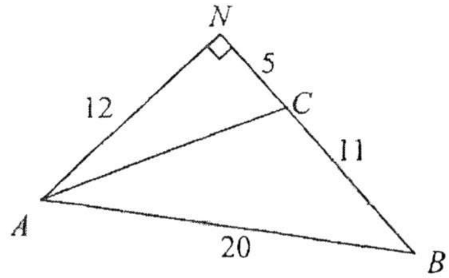
*Answer* (*a*) $x =$ ....................... [3]

**(b)** Find the value of $m$ for which $\sqrt{a^2 \sqrt{a^{\frac{1}{3}}}} = a^m$.

*Answer* (*b*) $m =$ ....................... [3]

**10.** In the diagram, $ABN$ is a right-angled triangle, and $BCN$ is a straight line.

$AN = 12$ cm, $CN = 5$ cm, $BC = 11$ cm and $AB = 20$ cm.




Calculate

**(a)** the length of $AC$,

*Answer* (*a*) .......................... cm [1]

**(b)** $\cos\angle ACB$,

*Answer* (*b*) .......................... [2]

**(c)** angle $BAC$.

*Answer* (*c*) .......................... ° [3]

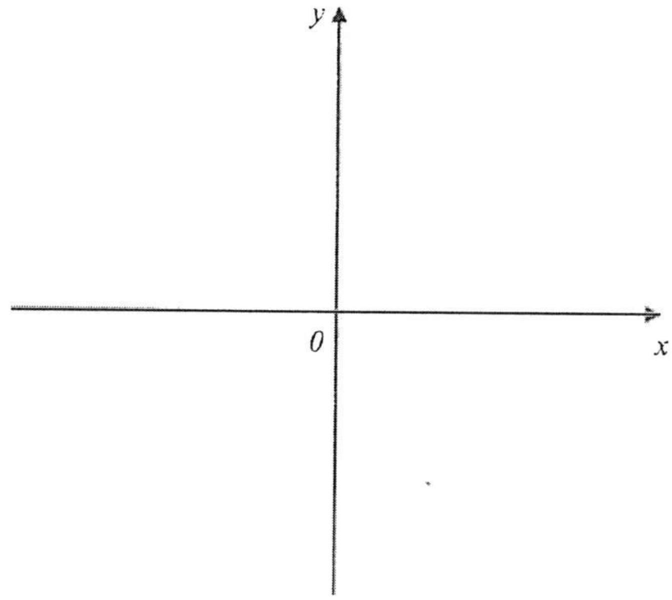
11. (a) Express $-x^2 + 6x - 3$ in the form $-(x+b)^2 + c$.

*Answer* (*a*) .......................... [3]

(b) **Hence**, solve the equation $x^2 + 3 = 6x$.

*Answer* (*b*) x = ........ or ........... [3]

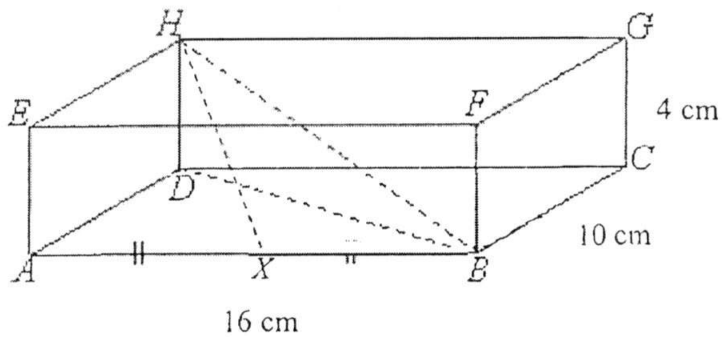
(c) Sketch the graph $y = -x^2 + 6x - 3$ on the axes provided, labeling your turning point, $x$-intercepts and $y$-intercept clearly. [3]




**END OF SECTION A**

SECTION B (44 MARKS)
Answer **ALL** the questions on the writing papers provided.

12. The diagram below shows a cuboid of dimensions 16 cm by 10 cm by 4 cm. Point $X$ lies on $AB$ such that $AX = XB$ and angle $HBX = 33.9°$.




Calculate

(a) the length of $BH$, [3]

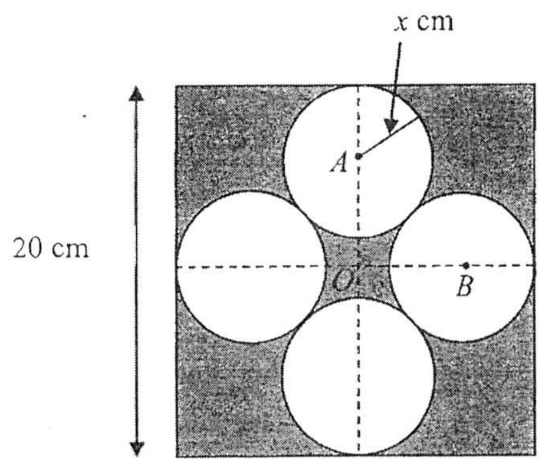
(b) angle $HBD$, [2]

(c) $HX$, [2]

(d) angle $DHX$. [2]

---

13.




The diagram shows a square of length 20 cm. $O$ is the intersection of the diagonals of the square. Four smaller identical circles, touching each other, with radius $x$ cm are drawn as shown. $A$ and $B$ are the centres of two of the smaller circles.

(a) Express the length of $OA$ and of $AB$ in terms of $x$. [2]

(b) Use Pythagoras' Theorem to form an equation in $x$ and show that it can be simplified to $x^2 + 20x - 100 = 0$. [3]

(c) Solve the equation $x^2 + 20x - 100 = 0$, giving your answers correct to 2 decimal places. [3]

(d) Calculate the area of the shaded region. [2]

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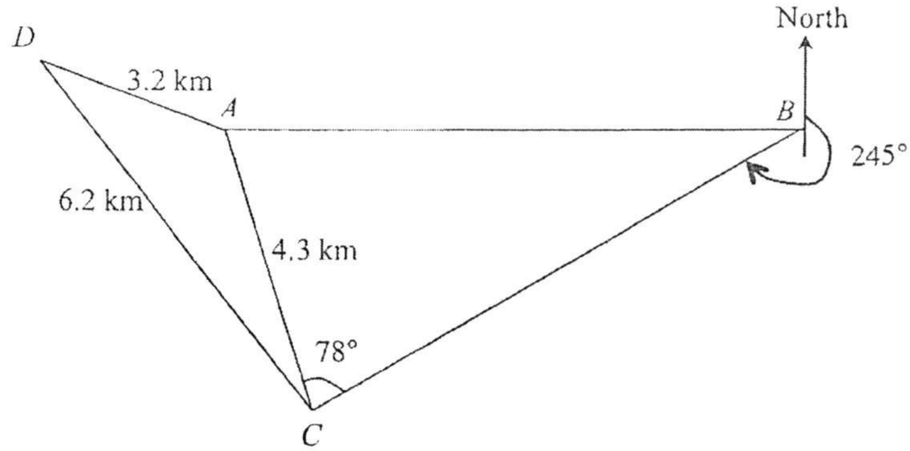
**14.**




In the diagram, $A$, $B$, $C$ and $D$ are four points on level ground. It is given that $AC = 4.3$ km, $AD = 3.2$ km, $CD = 6.2$ km, angle $ACB = 78°$, the bearing of $C$ from $B$ is $245°$ and $B$ is due east of $A$. Calculate

**(a)** angle $ACD$, [3]

**(b)** the bearing of $B$ from $C$, [2]

**(c)** the length of $AB$, [3]

**(d)** the area of triangle $ADC$. [2]

Alex walked from $D$ to $C$ along the path $DC$.

**(e)** He stopped at point $X$ when he is at the shortest distance to point $A$. Calculate $AX$. [2]

**(f)** At point $X$, Alex saw a building standing vertically at point $A$. Given that the height of the building is 980 m tall, calculate the angle of elevation of the top of building when viewed by Alex. [2]

**15.** **Answer the whole of this question on a piece of graph paper.**

The variables $x$ and $y$ are connected by the equation $y = 2x^2 - 5x - 3$. Some corresponding values are given in the following table.

| $x$ | $-2$ | $-1$ | $0$ | $0.5$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | $a$ | $4$ | $-3$ | $-5$ | $-6$ | $-5$ | $0$ | $9$ |

**(a)** Calculate the value of $a$. [1]

**(b)** Taking 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 5 units on the $y$-axis, draw the graph of $y = 2x^2 - 5x - 3$ for the range $-2 \leq x \leq 4$. [3]

**(c)** From your graph, find

**(i)** the value(s) of $x$ when $y = 5$, [2]

**(ii)** the minimum value of $y$. [1]

**(d)** By adding suitable line(s) on the same graph paper, find

**(i)** the values of $x$ for which $2x^2 - 5x - 3 = 0$, [1]

**(ii)** the solutions for the equation $2x^2 - 7x = 0$. [3]

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**END OF PAPER**

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新加坡海星中学

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**SEMESTRAL ASSESSMENT ONE**

**SECONDARY THREE**

**MATHEMATICS** **06 May 2016**

**[Solution]**

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## *Mathematical Formulae*

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**SECTION A (56 MARKS)**
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**1.** Given that $-1 \le a < 3$ and $-5 \le b \le -1$, where $a$ and $b$ are integers, find

**(a)** the greatest value of $ab$,

**[Solution]**
the greatest possible value of $ab = (-1)(-5)$
$= 5$

*Answer (a)* ........5.................... [1]

**(b)** the smallest possible value of $\frac{a}{b^2}$, given that $\frac{a}{b^2}$ is a perfect cube.

**[Solution]**
the smallest possible value of $\frac{a}{b^2} = \frac{-1}{(-1)^2}$
$= -1$

*Answer (b)* ............-1................ [1]

---

**2.** Simplify , leaving your answer in positive index notation.

**[Solution]**

$$\frac{\left(p^{-1}q^{2}\right)^{5} \div \left(2pr^{-4}\right)}{\left(3qr\right)^{3}\left(2p^{-2}q^{3}\right)^{0}}$$

$$= \frac{p^{-5}q^{10}}{\left(27q^{3}r^{3}\right)\left(2pr^{-4}\right)}$$

$$= \frac{q^{7}r}{54p^{6}}$$

*Answer* ............ $\frac{q^{7}r}{54p^{6}}$ ............ [3]

---

**3.** **(a)** Factorise $169+52q+4qr-r^2$ completely.

**[Solution]**

$169+52q+4qr-r^2$

$=169-r^2+52q+4qr$

$=(13-r)(13+r)+4q(13+r)$

$=(13+r)(13-r+4q)$

*Answer* $(13+r)(13-r+4q)$ .......................... [2]

**(b)** Make $x$ the subject of the equation $2x^2=\dfrac{y-x^2}{2y}+1$.

**[Solution]**

$2x^2=\dfrac{y-x^2}{2y}+1$

$4x^2y=y-x^2+2y$

$4x^2y+x^2=y+2y$

$x^2(4y+1)=3y$

$x^2=\dfrac{3y}{4y+1}$

$x=\pm\sqrt{\dfrac{3y}{4y+1}}$

*Answer* $x=\pm\sqrt{\dfrac{3y}{4y+1}}$ .......................... [3]

---

**4.** The curve $y=2(x+2)(x+k)$ cuts the $y$-axis at $(0,-8)$.

**(a)** Find the value of $k$.

**[Solution]**

At $(0,\ -8)$,

$-8=2(0+2)(0+k)$

$4k=-8$

$k=-2$

*Answer (a)* $k=$ -2 .................. [2]

**(b)** Using the value of $k$ found in part (a), find the coordinates of the points where the curve cuts the $x$-axis.

**[Solution]**

When $y=0$,

$0=2(x+2)(x-2)$

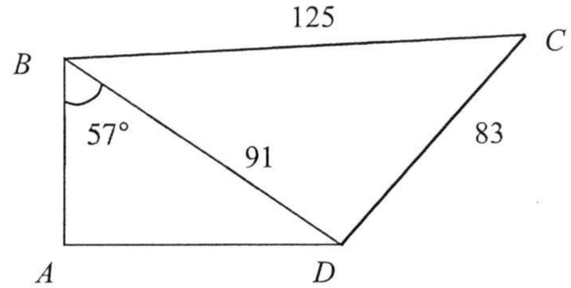
$x=-2$ or $2$

$(-2,0)$ and $(2,0)$

*Answer (b)* (-2,0) and (2,0) ........................ [2]

---

**5.** The figure below represents a plot of land $ABCD$. $B$ is due north of $A$, $D$ is due east of $A$. $BC = 125$ m, $CD = 83$ m and $BD = 91$ m. Angle $ABD = 57°$.




**(a)** Find angle $BDC$.

**[Solution]**

By cosine rule,

$$125^2 = 91^2 + 83^2 - 2(91)(83)\cos\angle BDC$$

$$\cos\angle BDC = \frac{125^2 - 91^2 - 83^2}{-2(91)(83)}$$

$$\angle BDC = \cos^{-1}\left(-\frac{5}{166}\right)$$

$$\approx 91.726°$$

$$= 91.7° \text{ (to 1d.p.)}$$

*Answer (a)* ......91.7...... ° [3]

**(b)** Find the bearing of $D$ from $C$.

**[Solution]**

Bearing from D from C

$= (91.726° - 57°) + 180°$

$\approx 214.726°$

$= 214.7°$ (to 1 d.p.)

*Answer (b)* ......214.7...... ° [2]

6. **(a)** Solve the inequality $\frac{x-35}{3} < 5-3x \le 2(4-x)$. Represent your answer on the number line below.

**[Solution]**

$$\frac{x-35}{3} < 5-3x \le 2(4-x)$$

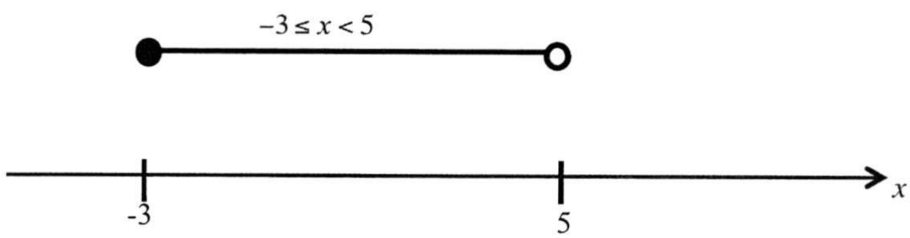
$$\frac{x-35}{3} < 5-3x \quad \text{and} \quad 5-3x \le 2(4-x)$$

$x-35 < 15-9x \qquad 5-3x \le 8-2x$

$10x < 50 \qquad -x \le 3$

$x < 5 \qquad x \ge -3$

$\therefore -3 \le x < 5$




Answer (a) $-3 \le x < 5$ [4]

**(b)** Write down the largest prime number which satisfies

$$\frac{x-35}{3} < 5-3x \le 2(4-x).$$

Answer (b) $x =$ 3 [1]

**7.** A hard disk has a memory capacity of 2 terabytes.

**(a)** If a low-resolution photograph takes up about 250 kilobytes, how many million photographs can be stored in the hard disk?

**[Solution]**

no. of photographs

$= \dfrac{2 \times 10^{12}}{250 \times 10^{3}}$

$= \dfrac{2 \times 10^{12}}{2.5 \times 10^{5}}$

$= 0.8 \times 10^{7}$

$= 8 \times 10^{6}$

$= 8$ million

*Answer (a)* ...........8........... million [2]

**(b)** If the hard disk is to store 25 video clips with capacity of 273 megabytes each, how much capacity is left in the disk? Give your answer in standard form, correct to 3 significant figures.

**[Solution]**

capacity left

$= 2 \times 10^{12} - (25 \times 273 \times 10^{6})$

$= 2 \times 10^{12} - 6825 \times 10^{6}$

$= 2 \times 10^{12} - 6.825 \times 10^{9}$

$= 2000 \times 10^{9} - 6.825 \times 10^{9}$

$= 1993.175 \times 10^{9}$ bytes

$= 1.993175 \times 10^{12}$ bytes

$= 1.99 \times 10^{12}$ bytes (to 3 s.f.)

*Answer (b)* .....$1.99 \times 10^{12}$..... bytes [3]

**8.** **(a)** Express $\frac{5}{12x+9}-\frac{2x+1}{9-16x^2}$ as a single fraction.

**[Solution]**

$$\frac{5}{12x+9}-\frac{2x+1}{9-16x^2}$$
$$=\frac{5}{3(4x+3)}-\frac{2x+1}{(3+4x)(3-4x)}$$
$$=\frac{5(3-4x)-(2x+1)(3)}{3(3+4x)(3-4x)}$$
$$=\frac{15-20x-6x-3}{3(3+4x)(3-4x)}$$
$$=\frac{-26x+12}{3(3+4x)(3-4x)}$$

*Answer (a)* $\frac{-26x+12}{3(3+4x)(3-4x)}$ [3]

**(b)** **Hence**, solve $\frac{5}{12x+9}-\frac{2x+1}{9-16x^2}=1$.

**[Solution]**

$$\frac{5}{12x+9}-\frac{2x+1}{9-16x^2}=1$$
$$\frac{-26x+12}{3(3+4x)(3-4x)}=1$$
$$-26x+12=3(9-16x^2)$$
$$48x^2-26x-15=0$$
$$x=\frac{-(-26)\pm\sqrt{(-26)^2-4(48)(-15)}}{2(48)}$$
$$=\frac{26\pm\sqrt{3556}}{96}$$
$x=0.892$ or $x=-0.350$ (to 3 s.f.)

*Answer (b)* $x=$ 0.892 or −0.350 [3]

**9.** **(a)** Solve $\frac{25^x}{5} - \sqrt{\frac{1}{625}} = 0$.

**[Solution]**

$$\frac{25^x}{5} - \sqrt{\frac{1}{625}} = 0$$

$$\frac{5^{2x}}{5} - \frac{1}{25} = 0$$

$$5^{2x-1} = 5^{-2}$$

By comparing indices,

$$2x - 1 = -2$$

$$x = -\frac{1}{2}$$

*Answer* *(a)* $x =$ $-\frac{1}{2}$ .................. [3]

**(b)** Find the value of $m$ for which $\sqrt{a^2 \sqrt{a^{\frac{1}{3}}}} = a^m$.

**[Solution]**

$$\sqrt{a^2 \sqrt{a^{\frac{1}{3}}}} = a^m$$

$$\sqrt{a^2 \, a^{\frac{1}{6}}} = a^m$$

$$\sqrt{a^{\frac{13}{6}}} = a^m$$

$$a^{\frac{13}{12}} = a^m$$

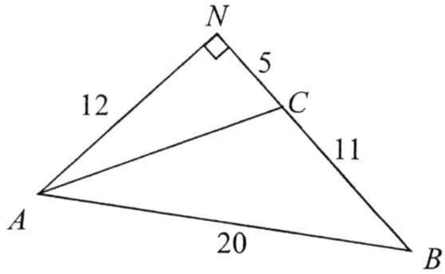
By comparing indices,

$$m = 1\frac{1}{12}$$

*Answer* *(b)* $m =$ $1\frac{1}{12}$ ..................... [3]

**10.** In the diagram, $ABN$ is a right-angled triangle, and $BCN$ is a straight line.

$AN = 12$ cm, $CN = 5$ cm, $BC = 11$ cm and $AB = 20$ cm.




Calculate

**(a)** the length of $AC$,

**[Solution]**

By Pythagoras' Theorem,

$$AC = \sqrt{12^2 + 5^2}$$
$$= 13 \text{ cm}$$

*Answer (a)* .......13........ cm [1]

**(b)** $\cos \angle ACB$,

**[Solution]**

$$\cos \angle ACN = \frac{5}{13}$$
$$\cos \angle ACB = -\cos \angle ACN$$
$$= -\frac{5}{13}$$

*Answer (b)* ......$-\frac{5}{13}$...... [2]

**(c)** angle $BAC$.

**[Solution]**

$$\frac{\sin \angle BAC}{11} = \frac{\sin \angle ACB}{20}$$
$$\sin \angle BAC = \frac{11 \times \frac{12}{13}}{20}$$
$$= \frac{33}{65}$$
$$\angle BAC = \sin^{-1} \frac{33}{65}$$
$$\approx 30.51024°$$
$$= 30.5° \text{ (to 1 d.p.)}$$

*Answer (c)* ......30.5............ ° [3]

**11.** **(a)** Express $-x^2+6x-3$ in the form $a(x+b)^2+c$.

**[Solution]**

$$-x^2+6x-3$$
$$=-\left(x^2-6x\right)-3$$
$$=-\left(x^2-6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2\right)-3$$
$$=-\left((x-3)^2-9\right)-3$$
$$=-(x-3)^2+6$$

*Answer (a)* $-(x-3)^2+6$ [3]

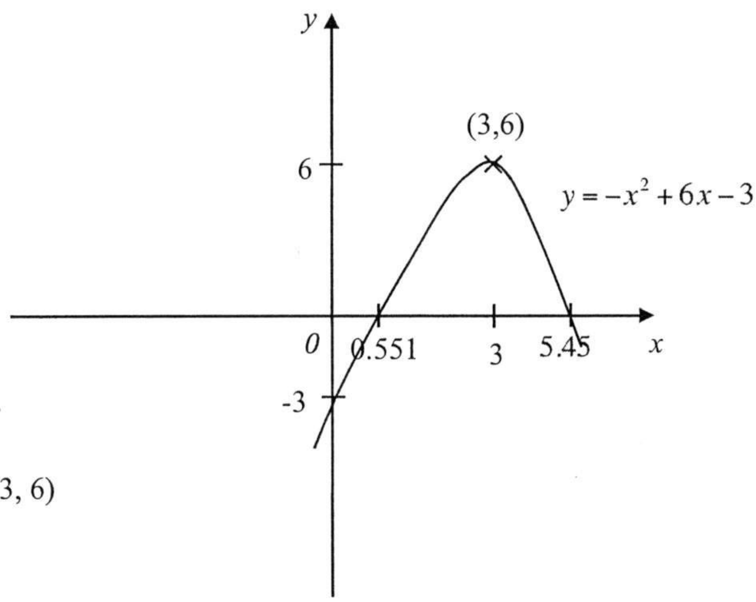
**(b)** **Hence**, solve the equation $x^2+3=6x$.

**[Solution]**

$$x^2+3=6x$$
$$-x^2+6x-3=0$$
$$-(x-3)^2+6=0$$
$$(x-3)^2=6$$
$$x-3=\pm\sqrt{6}$$
$$x=3\pm\sqrt{6}$$
$$\approx 0.55051 \text{ or } 5.4495$$
$$=0.551 \text{ or } 5.45$$

*Answer (b)* $x=$ 0.551 or 5.45 [3]

**(c)** Sketch the graph $y=-x^2+6x-3$ on the axis provided, labeling your turning point and $x$-intercepts and $y$-intercept clearly. [3]




When $x=0$,
$y=-3$
max. point (3, 6)

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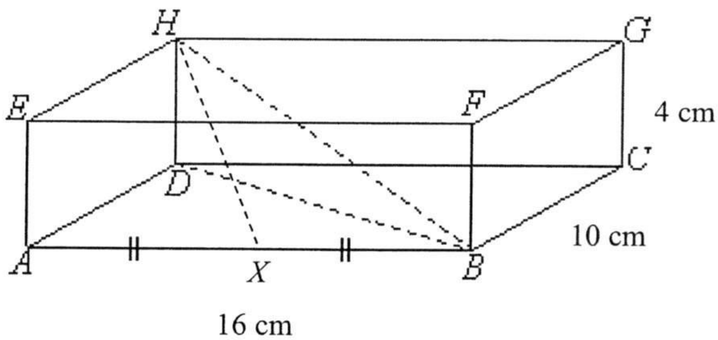
Answer **ALL** the questions on the writing papers provided.

**12.** The diagram below shows a cuboid of dimensions 16 cm by 10 cm by 4 cm. Point $X$ lies on $AB$ such that $AX = XB$ and angle $HBX = 33.9°$.




Calculate

**(a)** the length of $BH$, [3]

**(b)** angle $HBD$, [2]

**(c)** $HX$, [2]

**(d)** angle $DHX$. [2]

**[Solution]**

(a) Length of $BD = \sqrt{16^2 + 10^2}$

$= \sqrt{356}$

$\approx 18.868$ cm

Length of $BH = \sqrt{4^2 + 18.868^2}$

$= \sqrt{16 + 356}$

$= \sqrt{372}$

$\approx 19.287$

$= 19.3$ cm (to 3 s.f.)

(b) $\tan \angle HBD = \dfrac{4}{\sqrt{356}}$

$$\angle HBD = \tan^{-1} \frac{4}{\sqrt{356}}$$
$$\approx 11.969°$$
$$= 12.0° \text{ (to 1 d.p.)}$$

(c) By cosine rule,

$$HX^2 = BH^2 + XB^2 - 2(BH)(XB)\cos \angle HBX$$
$$HX^2 = \left(\sqrt{372}\right)^2 + 8^2 - (\sqrt{372})(8)\cos 33.9°$$
$$HX \approx 13.411$$
$$= 13.4 \text{ cm (to 3 s.f.)}$$

(d) $\cos \angle DHX = \dfrac{4}{13.411}$

$$\angle DHX = \cos^{-1} \frac{4}{13.411}$$
$$\approx 72.647°$$
$$= 72.6° \text{ (to 1 d.p.)}$$

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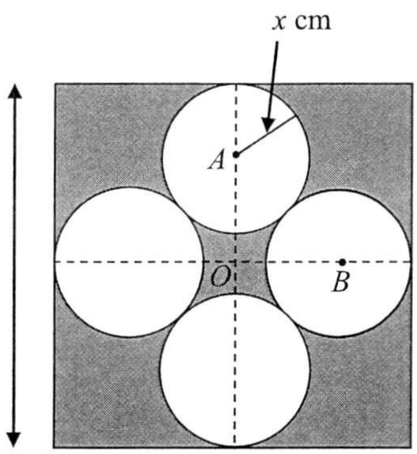
**13.**




The diagram shows a square of length 20 cm. $O$ is the intersection of the diagonals of the square. Four smaller identical circles, touching each other, with radius $x$ cm are drawn as shown. $A$ and $B$ are the centres of two of the smaller circles.

**(a)** Express the length of $OA$ and of $AB$ in terms of $x$. [2]

**(b)** Use Pythagoras' Theorem to form an equation in $x$ and show that it can be simplified to $x^2 + 20x - 100 = 0$. [3]

**(c)** Solve the equation $x^2 + 20x - 100 = 0$, giving your answers correct to 2 decimal places. [3]

**(d)** Calculate the area of the shaded region. [2]

**[Solution]**

(a) $OA = \dfrac{20}{2} - x$

$= (10 - x)$ cm

$AB = 2x$ cm

(b) By Pythagoras' Theorem,

$(10-x)^2 + (10-x)^2 = (2x)^2$

$2(100 - 20x + x^2) = 4x^2$

$2x^2 + 40x - 200 = 0$

$x^2 + 20x - 100 = 0$ (shown)

(c) $x = \dfrac{-20 \pm \sqrt{(20)^2 - 4(1)(-100)}}{2}$

$= \dfrac{-20 \pm \sqrt{800}}{2}$

$\approx -24.142$ or $4.1421$

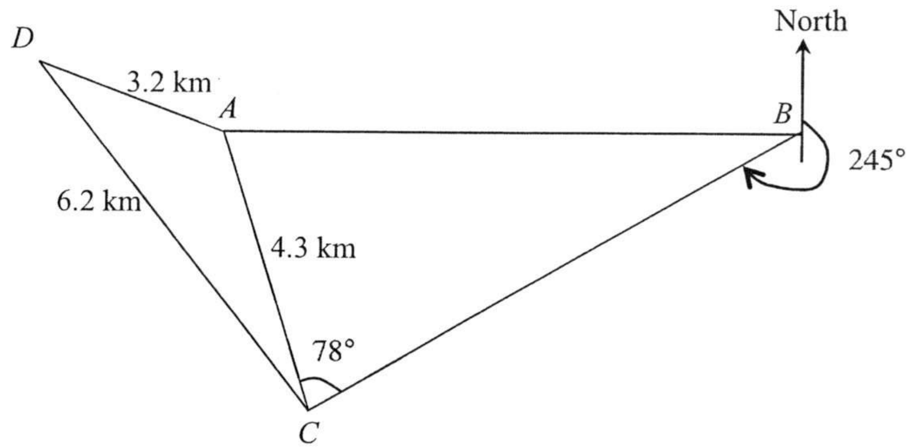
$= -24.14$ or $4.14$ (to 2 d.p.)

(d) $x = -24.1$ rejected as $x > 0$

Area of shaded region $= 20^2 - 4\pi(4.1421)^2$

$= 184 \text{ cm}^2$ (to 3 s.f.)

**14.**




In the diagram, $A$, $B$, $C$ and $D$ are four points on level ground. It is given that $AC = 4.3$ km, $AD = 3.2$ km, $CD = 6.2$ km, angle $ACB = 78°$, the bearing of $C$ from $B$ is $245°$ and $B$ is due east of $A$. Calculate

**(a)** angle $ACD$, [3]

**(b)** the bearing of $B$ from $C$, [2]

**(c)** the length of $AB$, [3]

**(d)** the area of triangle $ADC$. [2]

Alex walked from $D$ to $C$ along the path $DC$.

**(e)** He stopped at point $X$ when he is at the shortest distance to point $A$. Calculate $AX$. [2]

**(f)** At point $X$, Alex saw a building standing vertically at point $A$. Given that the height of the building is 980 m tall, calculate the angle of elevation of the top of building when viewed by Alex. [2]

**[Solution]**

(a) By cosine rule,

$$3.2^2 = 6.2^2 + 4.3^2 - 2(6.2)(4.3)\cos\angle ACD$$

$$\cos\angle ACD = \frac{3.2^2 - 6.2^2 - 4.3^2}{-2(6.2)(4.3)}$$

$$\angle ACD = \cos^{-1}\left(\frac{4669}{5332}\right)$$

$$\approx 28.877°$$

$$= 28.9° \text{ (to 1 d.p.)}$$

(b) Bearing from $B$ to $C$

$= 245° - 180°$

$= 065°$

(c) $\angle ABC = 270° - 245°$

$= 25°$

By sine rule,

$$\frac{\sin 25°}{4.3} = \frac{\sin 78°}{AB}$$

$$AB = \frac{4.3 \sin 78°}{\sin 25°}$$

$\approx 9.9523$

$= 9.95$ km (to 3 s.f.)

(d) Area of $\Delta ADC = \frac{1}{2}(4.3)(6.2)\sin \angle ACD$

$= \frac{1}{2}(4.3)(6.2)\sin 28.877°$

$\approx 6.4375$

$= 6.44 \text{ km}^2$ (to 3 s.f.)

(e) $\sin \angle ACD = \frac{AX}{4.3}$

$AX = (\sin 28.877°)(4.3)$

$\approx 2.0766$

$= 2.08$ km (to 3 s.f.)

(f) $\tan \angle TXA = \frac{0.98}{2.0766}$

$$\angle TXA = \tan^{-1}\left(\frac{0.98}{2.0766}\right)$$

$\approx 25.264°$

$= 25.3°$ (to 1 d.p.)

**15.** **Answer the whole of this question on a piece of graph paper.**

The variables $x$ and $y$ are connected by the equation $y = 2x^2 - 5x - 3$. Some corresponding values are given in the following table.

| $x$ | $-2$ | $-1$ | $0$ | $0.5$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | $a$ | $4$ | $-3$ | $-5$ | $-6$ | $-5$ | $0$ | $9$ |

**(a)** Calculate the value of $a$. [1]

**(b)** Taking 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 5 units on the $y$-axis, draw the graph of $y = 2x^2 - 5x - 3$ for the range $-2 \le x \le 4$. [3]

**(c)** From your graph, find

**(i)** the value(s) of $x$ when $y = 5$, [2]

**(ii)** the minimum value of $y$. [1]

**(d)** By adding suitable line(s) on the same graph paper, find

**(i)** the value(s) of $x$ for which $2x^2 - 5x - 3 = 0$, [1]

**(ii)** the solutions for the equation $2x^2 - 7x = 0$. [3]

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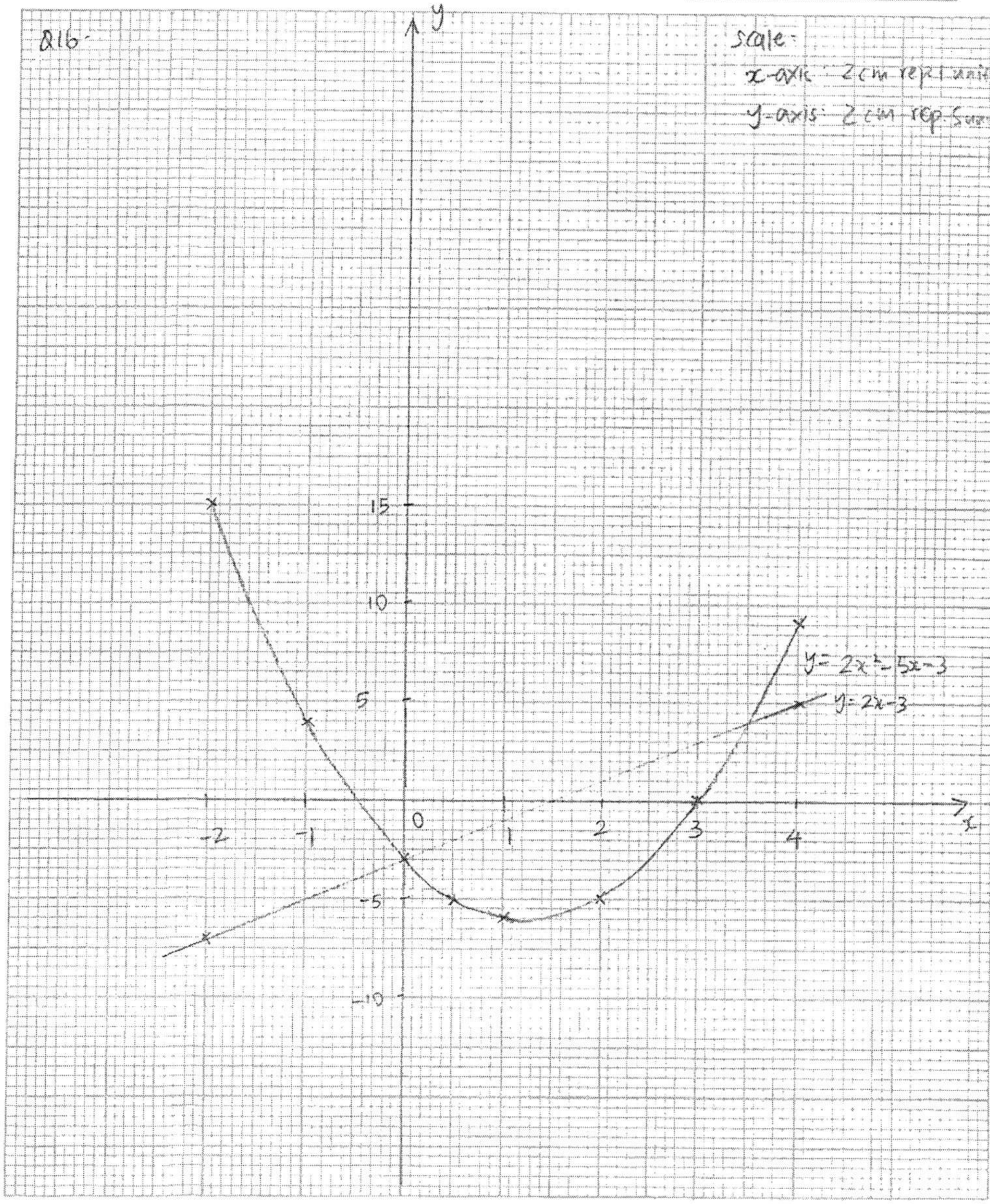
**END OF PAPER**

B1 : smoothness & scale
B1 : All pts plot correctly
B1 : labelling

姓 名 / Name: ______ ( )
科目 / Subject: ______
班级 / Class: ______



(a) $a = 15$ — B1

(c)(i) when $y = 5$, $x = -1.1$ or $x = 3.6$ ($\pm$ 0.05) — B1, B1

(ii) min. value of $y = -6$ ($\pm$ 0.25) — B1

(d)(i) $2x^2 - 5x - 3 = 0$

$x = -0.5$ or $x = 3$ — B1

(ii) $2x^2 - 7x = 0$

$2x^2 - 5x - 2x - 3 = -3$

$2x^2 - 5x - 3 = 2x - 3$ — M1

Plot $y = 2x - 3$ — B1

From the graph, $x = 0$ or $x = 3.5$ — A1

For Examiner's Use

**1** **(a)** Calculate $\frac{22.7 \times 2016}{1956 + \sqrt{60}}$.

Write down the first five digits on your calculator.

*Answer*.................................. [1]

**(b)** Write your answer to part **(a)** correct to 2 decimal places.

*Answer*.................................. [1]

**2** The Land Transport Authority announced in a recent news that there were 14 major breakdowns on the MRT network in 2015 which was an increase of 40 percent from the previous year.
How many major breakdowns were there in 2014?

*Answer*.................................. [2]

**3** These are the first four terms in a sequence.

60 57 54 51

**(a)** Write down the next two terms.

*Answer*.............. , .............. [1]

**(b)** Write down an expression, in terms of $n$, for the $n^{th}$ term in the sequence.

*Answer*.................................. [1]

**[Turn over]**

*For Examiner's Use*

*For Examiner's Use*

**4** John thought of five positive numbers.
The median is 11 and the mode is 15.
The biggest number is five times of the smallest number.
The mean of the five numbers is 10.4.
Find the five numbers.

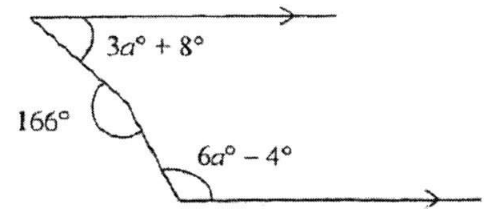
*Answer* ..........., ..........., ..........., ..........., ........... [2]

**5** If $y$ is inversely proportional to $\sqrt{x}$ and $y = 0.8$ when $x = 16$,
find the value of $y$ when $x = 100$.

*Answer* $y =$ .............................. [2]

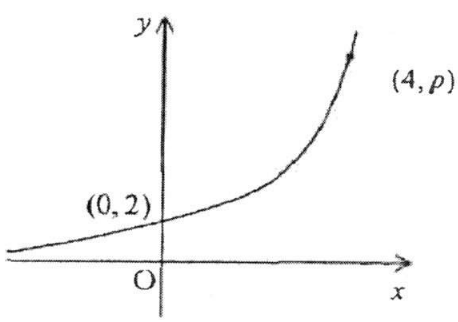
For Examiner's Use

**6** Calculate the value of $a$.




*Answer* $a =$ .............................. [2]

**7** The sketch shows the graph of $y = k(3^x)$. The points (0, 2) and (4, $p$) lie on the graph. Find the values of $k$ and of $p$.




*Answer* $k =$ .............................. [1]

$p =$ .............................. [1]

For Examiner's Use

**8** The numbers 60 and 2016, written as the product of their prime factors, are
$60 = 2^2 \times 3 \times 5$ and $2016 = 2^5 \times 3^2 \times 7$.

**(a)** Find the highest common factor and lowest common multiple of 60 and 2016.

*Answer* HCF = ............ LCM = .................. [2]

**(b)** Given that $60k$ is a perfect cube, write down the smallest possible integer value of $k$.

*Answer* $k$ = .............................. [1]

**(c)** Find the smallest positive integer value of $n$ for which $60n$ is a multiple of 2016.

*Answer* $n$ = .............................. [1]

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**9** $\xi$ = {first 17 natural numbers}
$A$ = {1, 4, 9, 16}
$B$ = {5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17}

**(a)** Describe the set $A$ in words.

*Answer* .................................................................................. [1]

**(b)** List the elements in $B'$.

*Answer*.............................. [1]

**(c)** Express {9, 16} in terms of $A$ and $B$ in set notation.

*Answer*.............................. [1]

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For Examiner's Use

10 There are 22 boys and $x$ girls in a group. The probability of selecting a girl from the group is $\frac{3}{14}$.

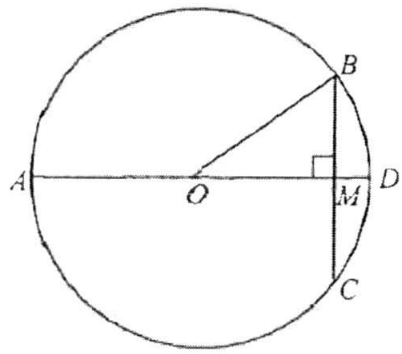
(i) Find the value of $x$.

*Answer* $x$ = ................................ [1]

(ii) Using your value of $x$ found in part (i), find the extra number of boys that have to join the group so that the probability of selecting a boy from the group will be $\frac{5}{6}$.

*Answer* ................................ [2]

11 The diagram shows a circle with centre $O$ and radius 41 cm.
$BC = 18$ cm and $\angle BMO = 90°$. $AOMD$ is a straight line.




(a) Find $AM$.

*Answer* ................................cm [2]

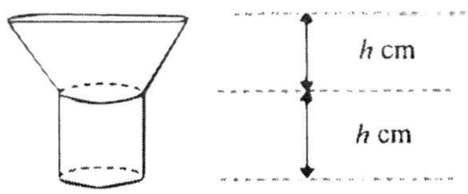
(b) Write down the exact value of $\cos A\hat{O}B$.

*Answer* ................................ [1]

[Turn over]

For Examiner's Use

**12** The diagram shows a container which is made up of a frustum and a cylinder of the same height. It is initially full of water.
The volumes of the frustum and the cylinder are in the ratio of 3 : 1.
Water is leaking through a hole at the bottom of the container at a constant rate.
The container is completely empty in 16 minutes.

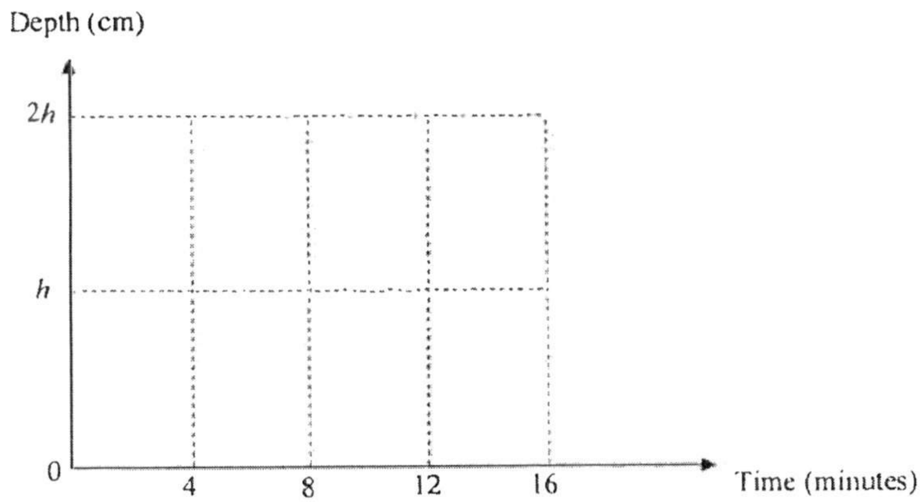



(a) Find the time taken for the depth of the water to be $h$ cm.

*Answer*.....................minutes [1]

(b) On the axes in the answer space, sketch the graph showing how the depth of the water, $h$ cm, in the container varies over the 16 minutes.

*Answer*




[2]

For Examiner's Use

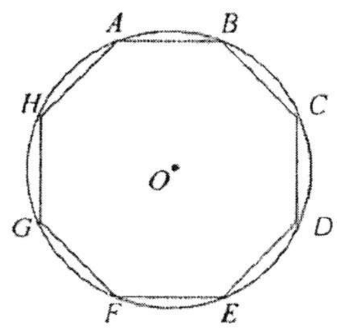
**13** (a) Express $x^2 - 10x - 13$ in the form $(x-a)^2 + b$.

*Answer* ................................ [1]

(b) **Hence** solve the equation $x^2 - 10x - 13 = 0$.

*Answer* $x =$ ............or .............. [2]

**14** A regular octagon *ABCDEFGH* fits exactly inside a circle of centre *O* and radius 6 cm.




Find the area of the circle not covered by the octagon.

*Answer* ......................... $cm^2$ [4]

[Turn over]

For Examiner's Use

**15** **(a)** Factorise $a^2 - 2ab + b^2 - 4b^2c^2$ completely.

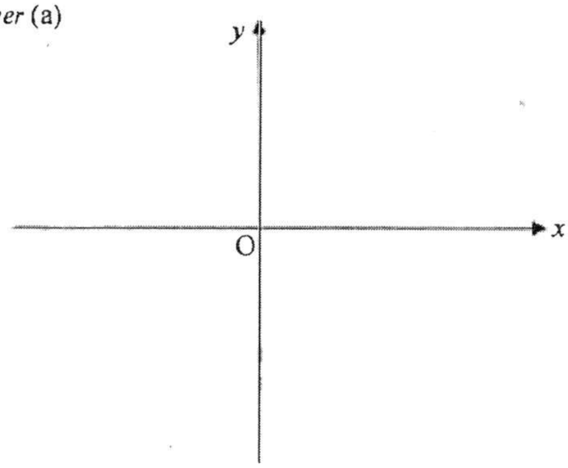
*Answer* .............................................. [2]

**(b)** Given that $T = 2\pi\sqrt{\frac{L}{g}}$, make $L$ the subject of the formula.

*Answer*.............................. [2]

**16** **(a)** Sketch the graph of $y = 2(x-3)(x+1)$.

*Answer* (a)




[2]

**(b)** Write down the equation of the line of symmetry of $y = 2(x-3)(x+1)$.

*Answer*.............................. [1]

**(c)** Write down the coordinates of the turning point.

*Answer*( .........., ..........) [1]

**(d)** Write down another quadratic equation, other than $y = 2(x-3)(x+1)$ that has the same roots.

*Answer*.............................. [1]

For Examiner's Use

**17** Solve the following equations.

**(a)** $16^x \div 4^{3x} = 0.25$

*Answer* $x =$ .................................... [2]

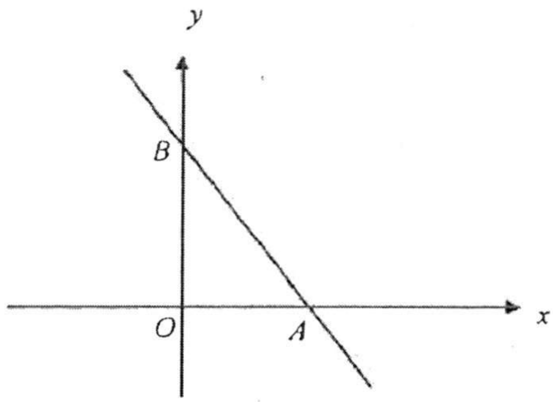
**(b)** $\sqrt[p]{25} = 125$

*Answer* $p =$ .................................... [2]

**[Turn over]**

For Examiner's Use

**18** The diagram shows a straight line $4y + 5x = 20$ passing through the points $A$ and $B$.




**(a)** Find the coordinates of $A$ and of $B$.

*Answer* $A$ (.......... , ..................) [1]

$B$ (.......... , ..................) [1]

**(b)** Find the area of $\Delta OAB$ and **hence** find the shortest distance from $O$ to the line $AB$.

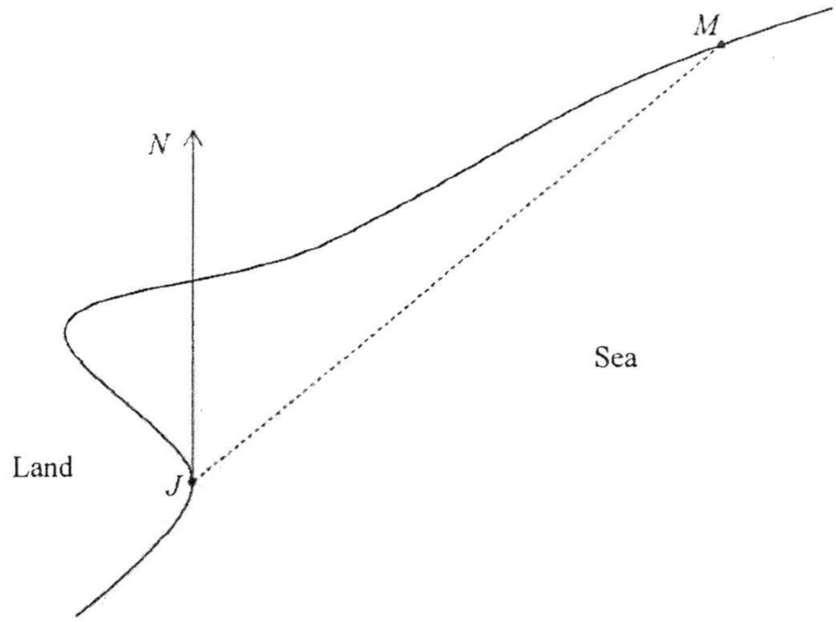
*Answer* *Area* = .......................... units$^2$ [1]

*Shortest distance* = ...................... units [2]

**[Turn over]**

For Examiner's Use

**19** The scale drawing shows a jetty, $J$, and a man, $M$.
The scale is 1 cm to 10 km.




**(a)** Measure the bearing of $M$ from $J$.

*Answer*..............................° [1]

**(b)** A boat is 85 km from $J$ on a bearing of 110°.
Mark and label on the diagram the position, $B$, of the boat. [1]

State the bearing of $J$ from $B$.

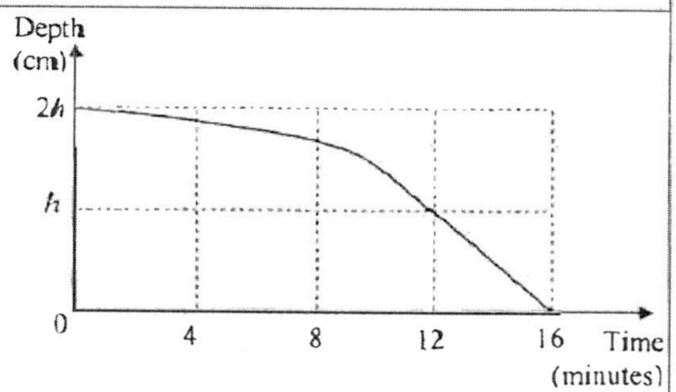
*Answer*..............................° [1]

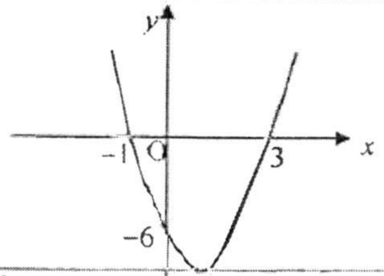
**(c)** The boat travels in a straight line towards $J$ at an average speed of 35 km/h.
Calculate the travelling time of the boat.
Give your answer in hours and minutes, to the nearest minute.

*Answer*..........h ............mins [2]

**End of paper**

Answers

| | | |
|---|---|---|
| 1 | (a) | 23.304 |
| | (b) | 23.30 |
| 2 | | 10 |
| 3 | (a) | 48, 45 |
| | (b) | $63 - 3n$ or $60 - 3(n - 1)$ |
| 4 | | 5, 8, 11, 15, 15 |
| 5 | | 0.32 |
| 6 | | 18 |
| 7 | | $k = 2$; $p = 162$ |
| 8 | (a) | HCF = 12<br>LCM = 10080 |
| | (b) | $k = 450$ |
| | (c) | $n = 168$ |
| 9 | (a) | Squares numbers /Perfect squares |
| | (b) | 1, 2, 3, 4, 6, 12 |
| | (c) | $A \cap B$ |
| 10 | (i) | 6 |
| | (ii) | 8 |
| 11 | (a) | 81cm |
| | (b) | $-\frac{40}{41}$ |
| 12 | (a) | 12 |
| | (b) |  |
| 13 | (a) | $(x-5)^2 - 38$ |
| | (b) | $x = 11.2$ or $1.16$ |
| 14 | | $11.3 \text{ cm}^2$ |
| 15 | (a) | $(a-b+2bc)(a-b-2bc)$ |
| | (b) | $L = \frac{gT^2}{4\pi^2}$ |

| | | |
|---|---|---|
| 16 | (a) |  |
| 16 | (b) | $x = 1$ |
| | (c) | $(1, -8)$ |
| | (d) | $y = (x+1)(x-3)$ / $y = -(x+1)(x-3)$ / $y = 5(x+1)(x-3)$ |
| 17 | (a) | 1 |
| | (b) | $\frac{2}{3}$ |
| 18 | (a) | $A$ (4, 0)  $B$ (0, 5) |
| | (b) | 10 units$^2$<br>Shortest distance = 3.12 units |
| 19 | (a) | 050° (± 1°) |
| | (b) | 290° |
| | (c) | 2 h 26 mins |