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| Name: | Register No.: | Class: |

# CRESCENT GIRLS' SCHOOL
# SECONDARY THREE
# END OF YEAR EXAMINATION 2016

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## MATHEMATICS

4048

**05 October 2016**

**2 hours 30 minutes**

For Section A, candidates answer on the Question Paper.
For Section B, candidates answer on the writing paper and graph paper given.

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### READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

At the end of the examination, submit section A and B separately.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for Section A is 40.
The total of the marks for Section B is 60.

| For Examiner's Use | |
|---|---|
| **Section A** | 40 |
| **Section B** | 60 |
| **Total** | 100 |
| | |

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This paper consists of **15** printed pages, including this cover page.

## Mathematical Formulae

*Compound Interest*

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

## SECTION A

Answer all the questions.

1 Solve the inequality $\frac{3x}{2}-1 \le \frac{2x-3}{3} < -\frac{3-x}{4}$.

*Answer* .................................. [2]

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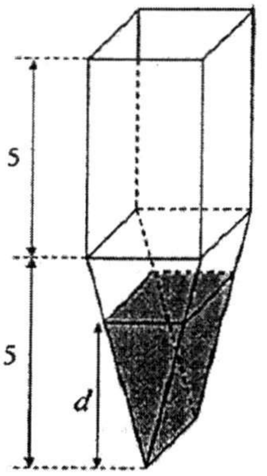
2 The force, $F$, between two particles is inversely proportional to the square of the distance between them.
The force is 36 units when the distance between the two particles is $r$ metres.
Find the force when the distance is $3r$ metres.

*Answer* ...................... units [2]

3



The container shown in the diagram is a prism.
The cross-section consists of a rectangle and a triangle.
The heights of both the rectangle and the triangle are 5 cm.
Water is poured into the empty container at a constant rate and filled it in 6 minutes.
How many minute(s) will it take to fully fill the triangular prism?

*Answer* ........... minute(s) [2]

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4 Simplify the following and leave your answer in positive index.

$$\frac{9m^{-3}n^{-4}}{81\left(m^{3}n\right)^{-1}}\times\frac{162\left(mn^{3}\right)^{-1}}{27m^{-3}n}$$

*Answer* ................................ [3]

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5 The diameter of a strand of hair with a circular cross-sectional area is 1000 pico-metres.

(a) Express 1000 pico-metres, in metres, in standard form.

*Answer* …………………… m [1]

(b) Assuming that the strand of hair has a uniform cross-sectional area, calculate the length of the hair given that the volume of the strand of the hair is $3.2 \times 10^{-20}$ m$^3$, leaving your answer in standard form.

*Answer* …………………… m [2]

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6 Let $j$ and $k$ be the roots of the equation $x^2 + ax + 5 = 0$.

(a) Find the value of $jk$.

*Answer* $jk =$ …………………… [1]

(b) If $j + \frac{1}{k}$ and $k + \frac{1}{j}$ are the roots of the equation $x^2 + gx + h$, find the value of $h$.

*Answer* $h =$ …………………… [2]

7 $A$ is an obtuse angle and $\sin A = \frac{12}{13}$.

(a) Find angle $A$.

*Answer* .............................. ° [1]

(b) It is given that $A$ and $B$ are supplementary angles.
Without using a calculator, find the exact values of $\sin B$ and $\cos A$.

*Answer* $\sin B =$ ........................ [2]

$\cos A =$ ........................

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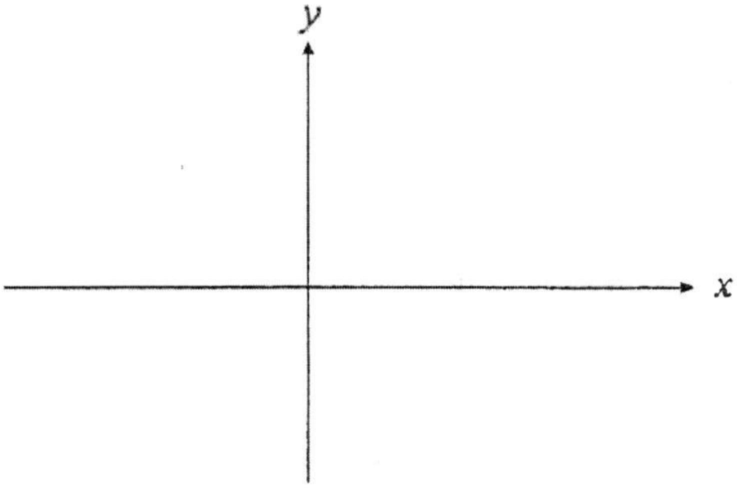
8 (a) Express $x^2 - 4x + 1$ in the form $(x-a)^2 + b$.

*Answer* .................................... [2]

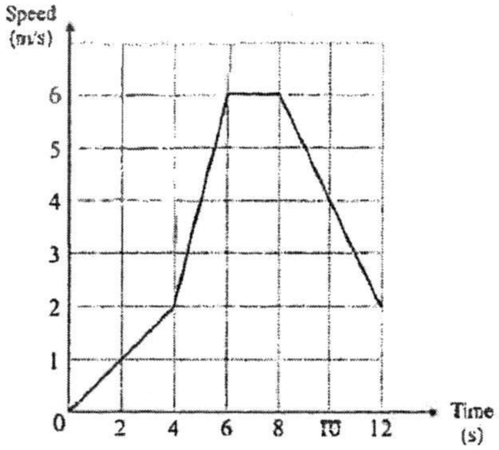
(b) Sketch the graph of $y = x^2 - 4x + 1$.

*Answer*



[2]

9 The diagram represents the speed-time graph of a particle.



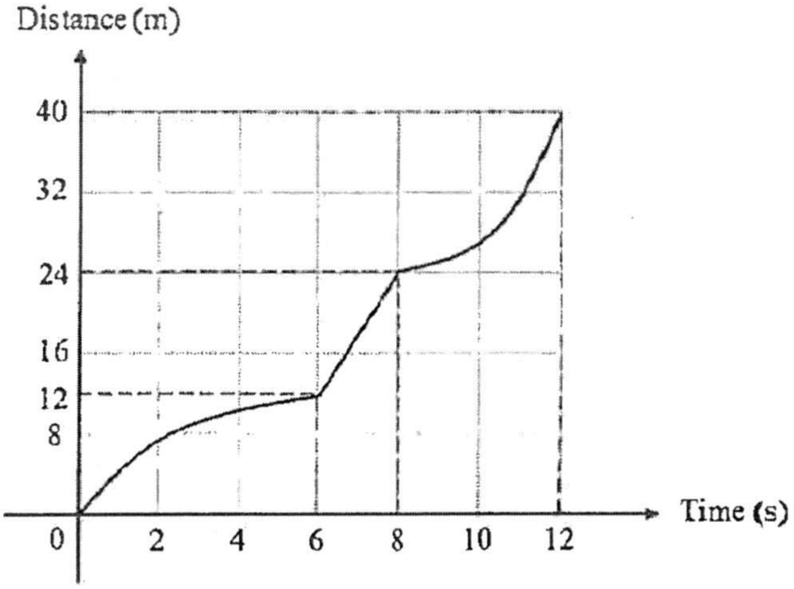
(a) Showing your working clearly, find the speed of the particle when time = 8.2 seconds.

*Answer* ........................ m/s [2]

(b) Calculate the time taken for the particle to travel the first 21 metres.

*Answer* ...................... seconds [2]

**(c)** Alan drew the following distance-time graph.



Barry claimed that Alan's graph is wrong.
Who is correct?
Explain your answer with appropriate mathematical knowledge.

*Answer* [2]

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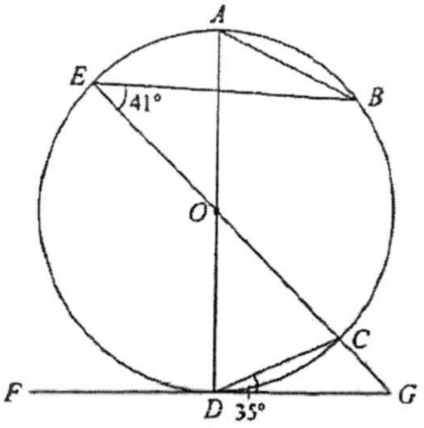


*A*, *B*, *C*, *D* and *E* are points on the circumference of the circle with centre *O*.
*AD* intersects *CE* at *O* and *FD* is a tangent to the circle at *D*.
*EC* produced meets *FD* produced at the point *G*.
Angle *BEC* = 41° and angle *CDG* = 35°.

(a) Find $C\hat{G}D$.

*Answer* .............................. ° [2]

(b) Find reflex $A\hat{O}C$.

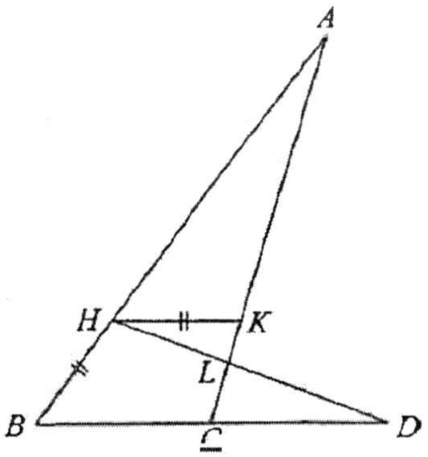
*Answer* .............................. ° [2]

(c) Find $A\hat{C}B$.

*Answer* .............................. ° [2]

[Turn over

11



In the diagram, $AHB$, $AKC$, $BCD$ and $HLD$ are straight lines.
$AH = BD$, $HK = HB$ and $HK$ is parallel to $BD$.

**(a)** Show that triangle $AHK$ is congruent to triangle $DBH$.

*Answer*

[2]

**(b)** Show that triangles $AHL$ and $DCL$ are similar.

*Answer*

[2]

**(c)** Given that $AL = 11.3$ cm, $HL = 3$ cm and $CL = 2$ cm, calculate $DH$.

*Answer* .............................. cm [2]

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**END OF SECTION A**

## 2016 CGS SEC 3 EM EOY
## SECTION B
Answer all the questions.

1 Tap *A* can fill a rectangular tank in $x$ hours.
Tap *B* can fill the same tank in $(x+5)$hours.
If both taps are turned on at the same time, the tank can be filled in 10 hours.

(a) Write down, an expression in terms of $x$, the amount of water in the tank that is filled by Tap *A* in an hour. [1]

(b) Write down, an expression in terms of $x$, the amount of water in the tank that is filled by Tap *B* in an hour. [1]

(c) Form an equation in $x$ and show that it reduces to $x^2-15x-50=0$. [2]

(d) Solve the equation $x^2-15x-50=0$ leaving your answer(s) to 2 decimal places. [2]

(e) Tap *A* is turned on to fill the rectangular tank. [2]
When the tank is one-third filled, Tap *B* is also turned on until the tank is completely filled.
How long will it take for the empty tank to be completely filled?
Give your answers in hours and minutes (correct to the nearest minute).

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2 (a) Triangle *ABC* has points $A(-2,-3)$ and $C(2,4)$.
*AB* is parallel to the $x$-axis and *B* has a positive $x$-coordinate.

(i) Find the equation of the line *BC*, given that the length of *AB* is 6 units. [3]

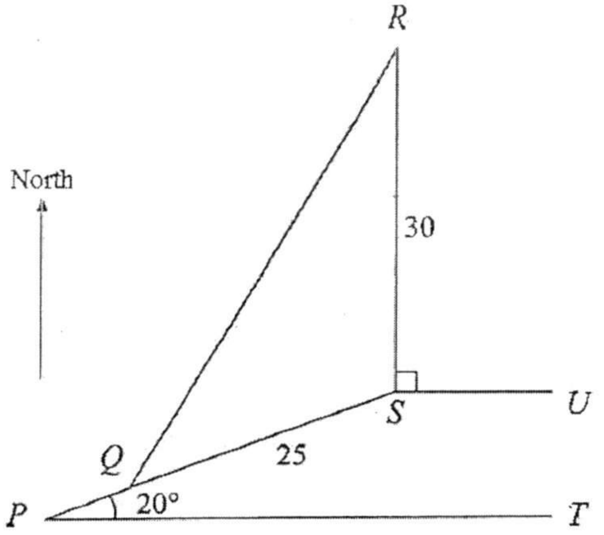
(ii) Calculate *BC*. [2]

(iii) Calculate the shortest distance from *A* to *BC*, given that the area of triangle *ABC* is 20 units$^2$. [2]

(b) Paul has $9000 to invest in either Company *A* or Company *B*. [4]
Company *A* offers 5.9% per annum simple interest.
Company *B* offers 5.8% per annum compound interest, compounded half-yearly.
Paul wishes to invest the money for a period of 5 years.

Calculate the difference in interest earned after 5 years and suggest which company Paul should invest in.

3



In the diagram, $P$, $Q$, $R$, $S$, and $T$ are five points on a horizontal field.
$QS = 25$ m and $RS = 30$ m.
Angle $RSU = 90°$ and angle $SPT = 20°$.
$SU$ is parallel to $PT$ and $R$ is north of $S$.

**(a)** Calculate $QR$. [4]

**(b)** Calculate the bearing of $Q$ from $R$. [3]

**(c)** Given that the area of the field enclosed by $\Delta PRS$ is 416 $\text{m}^2$, find the length of $PQ$. [2]

**(d)** A radio mast of height 1000 cm stands at $S$. [3]
Calculate the greatest angle of elevation of the top of the radio mast when viewed by a man walking along $RQ$.

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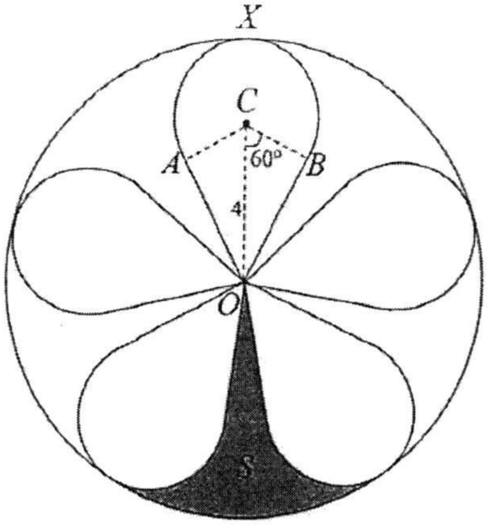
4 The diagram shows a toy made of wire.



*AXB* is an arc of circle with centre *C*.
*O* is the centre of a large circle which touches the arc *AXB* at *X*.
The lines *OA* and *OB* are tangents to the circle with centre *C* and together with the arc *AXB* they form a flap of *OAXBO*.
The other four flaps are identical to *OAXBO* and placed equidistant from each other.
Given that $OC = 4$ cm and angle $BCO = 60°$,

(a) Show that the radius of the large circle = 6 cm. [2]

(b) Find the total length of the wire needed to make the toy. [3]

(c) Calculate the area of the shaded region, S, in the diagram. [4]

[Turn over

5 **Answer the whole of this question on a sheet of graph paper.**

The variables $x$ and $y$ are connected by the equation

$$y = 3x - 2 + \frac{10}{x+3}.$$

Some corresponding values of $x$ and $y$ are given in the table below.

The values of $y$ are corrected to 2 decimal places where appropriate.

| $x$ | −2.25 | −2 | −1.5 | −1 | −0.5 | 0 | 0.5 | 1 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 4.58 | 2 | 0.17 | 0 | 0.5 | 1.33 | $p$ | 3.5 |

(a) Calculate the value of $p$. [1]

(b) Use a scale of 4 cm to represent 1 unit, draw a horizontal $x$-axis for $-2.5 \le x \le 1$. [3]
Use a scale of 4 cm to represent 1 unit, draw a vertical $y$-axis for $-0.5 \le y \le 5$.

On your axes, plot the points given in the table and join them with a smooth curve.

(c) By drawing the tangent, find the gradient of the curve when $x = -1.65$. [2]

(d) Using your graph, find the range of values of $x$ in the range $-2.5 \le x \le 1$ for which

(i) $3x + \frac{10}{x+3} < 3$ [2]

(ii) $(x+3)(8-5x) = 20$ [2]

6 A supplier received an order to customise a gold trophy for a competition.
A sample of the gold trophy is shown in Diagram I.

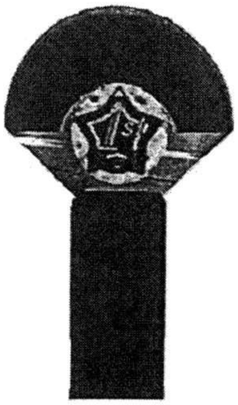


Diagram I

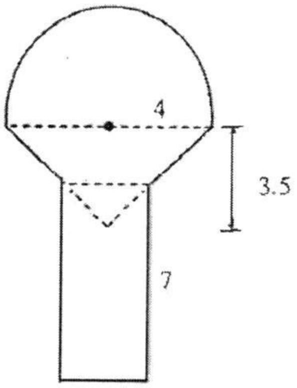


Diagram II

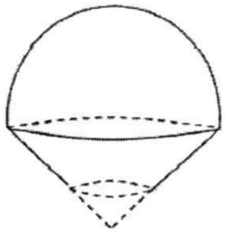


Diagram III

Diagram II shows the cross-section of the gold trophy.
The trophy consists of a hemisphere with radius 4 cm joined to a cone with a height of 3.5 cm, partly embedded into a cylinder with height 7 cm.
The total height of the trophy is 13.1 cm.

**(a)** Show that the part of the cone embedded into the cylinder has a height of 1.4 cm. [1]

**(b)** Find the total volume of the trophy. [3]

**(c)** The surface of the trophy is painted in different colours. [3]
The top part of the trophy, as shown in Diagram III is painted with gold paint.
A litre of gold paint cost \$3.60 and can be used to paint 100 $cm^2$ of area.
Find the cost of paint used for 20 gold trophies.

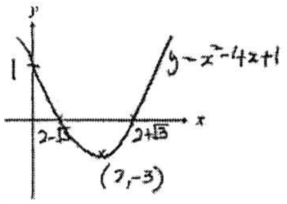
**(d)** A silver trophy for the same competition is geometrically similar to the gold trophy. [3]
The height of the silver trophy is 10% shorter than the gold trophy.
Given that the mass of the silver trophy is 540 g, find the mass of the gold trophy and the density of the material used to make the trophies, in $g/cm^3$.

**End of Section B**

## Answer Key

### Section A

1. $x \le 0$
2. 4
3. 2
4. $\frac{2m^2}{3n^7}$
5. (a) $1\times10^{-9}$ (b) $4.07\times10^{-2}$
6. (a) 5 (b) 7.2
7. (a) 112.6° (b) $\sin B=\frac{12}{13}$ $\cos A=-\frac{5}{13}$
8. (a) $(x-2)^2-3$ (b)



9. (a) 5.8 (b) 7.5 (c) Barry is correct
10. (a) 20° (b) 250° (c) 14°
11. (c) 10.5

### Section B

1. (a) $\frac{1}{x}$ (b) $\frac{1}{x+5}$ (d) $x=17.81$ or $-2.81$ (e) 12 hours 37 minutes
2. (a)(i) $y=-\frac{7}{2}+11$ (ii) 7.28 units (iii) 5.49 units (b) Company B
3. (a) 45.1 m (b) 211.4° (c) 4.51 (d) 32.6°
4. (a) 6 cm (b) 114 cm (c) 7.31 $\text{cm}^2$
5. (a) 2.36 (c) $-2.46(\pm0.5)$ (d)(i) $-1.82<x<-0.18(\pm0.1)$

   (ii) $x=-1.84$ or $x=0.44(\pm0.1)$
6. (b) 245 $\text{cm}^3$ (c) $112.77 (d) 3.02 $\text{g/cm}^3$