SAM

TEMASEK SECONDARY SCHOOL
Mid-Year Examination 2017
Secondary Three Express

ADDITIONAL MATHEMATICS

2 hours

Question Booklet

Additional Material: Writing paper (8 sheets) and Cover page (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open the booklet until you are told to do so.

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

This document consists of **5** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

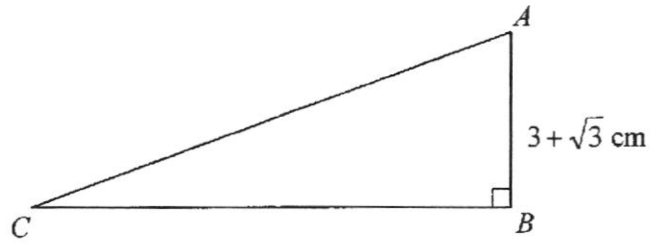
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1



The diagram shows a triangle ABC where $AB = 3+\sqrt{3}$ cm. Given that the area of the triangle is $\dfrac{27+19\sqrt{3}}{2}$ cm^2, find the length of BC in the form $a+b\sqrt{3}$ cm where a and b are integers. [4]

2 Solve the equation $4^{x+6} = \dfrac{16}{\left(3^{x+1}\right)^2}$. [5]

3 Express $\dfrac{2x^3+x-7}{(2x-1)(x^2+1)}$ in partial fractions. [5]

4 Given that $2x^4+3x^3+7x^2+6x+a = Q(x)(x^2+1)+3x+4$ for all values of x, where $Q(x)$ is a polynomial, find

(i) the value of the constant a, [3]

(ii) $Q(x)$ [1]

5 Find the coefficient of $\dfrac{1}{x^{11}}$ in the expansion of $\left(\dfrac{2x^2}{3}-\dfrac{1}{4x^3}\right)^7$. [4]

6 Given that $Ax^2-20x+5 \equiv B(3x-1)^2+3(Cx+1)+4x$ for all real values of x, find the values of A, B and C. [5]

7 The roots of the quadratic equation $3x^2-2x+4=0$ are α and β.

Find the quadratic equation whose roots are α^3 and β^3. [6]

8 (i) Solve the equation $3x^3-26x^2+33x+14=0$. [5]

(ii) Hence solve the equation $24y^3-104y^2+66y+14=0$. [2]

9 (i) A curve has the equation $y=x^2+kx+3+k$, where k is a constant.

Find the range of values of k for which the curve lies completely above the $x-$axis . [4]

(ii) Find the set of values of a for which the line $y=2ax+3$ intersects the curve $y=ax^2+4x+4$ at two distinct points. [4]

10 The expression $p^2x^3+52x^2+qx+9$, where p and q are positive integers, is exactly divisible by $4x+1$ and leaves a remainder of $p^2+27p+1$ when divided by $x-1$.

Find

(i) the value of p and of q, [5]

(ii) the remainder when the expression is divided by $x+2$. [2]

11 Solve the equations

(i) $\log_2 y-\log_y 16=3$, [4]

(ii) $\log_x 128=2+\log_x 2$. [3]

12 (i) Expand $(a-3x)^n$, in ascending power of x, up to and including the term in x^2 in terms of a and n where $0<a<n$ and $n>2$. [3]

(ii) Given that the ratio of the constant term to the coefficient of x^2 in the expansion of $(a-3x)^n$ is 4 : 189, show that $a^2=\dfrac{2n(n-1)}{21}$. [3]

(iii) Given that the difference of a and n is 5, find the values of a and n. [5]

13 The premium, \$$A$, per month for an insurance can be modelled using the formula $A = 25 + p(2.1)^{qt}$ where p and q are constants, t is the age of the insurance policy holder **round down to the nearest years**.

(i) Given that the premium per month for a newborn baby is \$25.20, state the value of p. [1]

The premium per month for a 73 years old policy holder is \$70.

(ii) Find the value of q correct to 1 significant figure. [3]

Using of the values of p and q found in part **(i)** and **(ii)** respectively, find

(iii) the premium when the policy holder is 67 years old, [1]

(iv) the age of the policy holder when the premium paid per month is \$34. [2]

End of Paper

Temasek SA1

Sec 3 E A Math Marking Scheme

1.	Area of Triangle $ABC = \frac{1}{2}(3+\sqrt{3})(BC)$ $\frac{27+19\sqrt{3}}{2} = \frac{1}{2}(3+\sqrt{3})(BC)$ — M1 for correct values used $BC = \frac{27+19\sqrt{3}}{3+\sqrt{3}}$ $BC = \frac{(27+19\sqrt{3})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$ — M1 for rationalise $BC = \frac{81-27\sqrt{3}+57\sqrt{3}-57}{9-3}$ — A1 for correct expansion $BC = \frac{24+30\sqrt{3}}{6}$ $BC = 4+5\sqrt{3}$ cm — A1
2.	$4^{x+6} = \frac{16}{(3^{x+1})^2}$ $4^x(4^6) = \frac{16}{3^{2x}(3^2)}$ — M1 for splitting the power (with the correct indices law applied) $(4^x)(3^{2x}) = \frac{16}{4^6(3^2)}$ — M1 for shifting all terms with variable to one side $4^x(9^x) = \frac{1}{2304}$ $36^x = \frac{1}{2304}$ — M1 for getting 36^x or 6^{2x} $x\ln 36 = \ln\left(\frac{1}{2304}\right)$ — M1 taking ln or log both side $x = -2.16\ (3\ sf)$ — A1

3.

$$\frac{2x^3+x-7}{(2x-1)(x^2+1)}=1+\frac{A}{(2x-1)}+\frac{Bx+C}{(x^2+1)}$$

M1 for correct partial fraction

M1 for obtaining the quotient 1

Zero marks if did not recognize that it is an improper fraction

$$2x^3+x-7=(2x-1)(x^2+1)+A(x^2+1)+(Bx+C)(2x-1)$$

let $x=\frac{1}{2}$

$$2\left(\frac{1}{2}\right)^3+\frac{1}{2}-7=A\left[\left(\frac{1}{2}\right)^2+1\right]$$

$$\frac{1}{4}+\frac{1}{2}-7=A\left(\frac{5}{4}\right)$$

$$A=-5$$

let $x=0$

$$-7=(-1)(1)+A+C(-1)$$

$$-7=-1-5-C$$

$$C=1$$

let $x=1$

$$-4=(1)(2)+2A+(B+C)(1)$$

$$-4=2-10+b+1$$

$$B=3$$

Values of A, B and C

A2, A1, A0

$$\frac{2x^3+x-7}{(2x-1)(x^2+1)}=1-\frac{5}{(2x-1)}+\frac{3x+1}{(x^2+1)}$$

A1 for partial fractions

4 (i)

$$\begin{array}{r} 2x^2+3x+5 \\ x^2+1\,\overline{)\,2x^4+3x^3+7x^2+6x+a} \\ \underline{2x^4 \qquad +2x^2} \\ 3x^3+5x^2+6x \\ \underline{3x^3 \qquad +3x} \\ 5x^2+3x+a \\ \underline{5x^2 \qquad +5} \\ 3x+(a-5) \end{array}$$

M1 for using long division

A1 for correct remainder in terms of a

$$a=9$$

A1 for a value

4 (ii)	$Q(x) = 2x^2 + 3x + 5$ B1
5	$T_{r+1} = \binom{7}{r}\left(\frac{2x^2}{3}\right)^{7-r}\left(-\frac{1}{4x^3}\right)^r$ $= \binom{7}{r}\left(\frac{2}{3}\right)^{7-r}(x^2)^{7-r}\left(-\frac{1}{4}\right)^r(x^{-3})^r$ $= \binom{7}{r}\left(\frac{2}{3}\right)^{7-r}\left(-\frac{1}{4}\right)^r x^{14-2r-3r}$ $= \binom{7}{r}\left(\frac{2}{3}\right)^{7-r}\left(-\frac{1}{4}\right)^r x^{14-5r}$ A1 for x with correct power If use $\frac{1}{4}$ instead of $-\frac{1}{4}$ then only award the first 2 marks If expand all the terms and simplified but without stating which term to focus on then 2 marks For x^4 term, $14-5r=-11$ A1 for correct valve of r $r=5$ Coefficient of $\frac{1}{x^{11}}$ term is $\binom{7}{5}\left(\frac{2}{3}\right)^2\left(-\frac{1}{4}\right)^5 = -\frac{7}{768}$ A1 A1 for sub in for coefficient
6	Comparing constant term $5 = B+3$ $B = 2$ A1 for B value Comparing x^2 term $A = 9B$ M1 $A = 18$ A1 for A value Sub $x=1$ M1 for method $A - 20 + 5 = B(2)^2 + 3(C+1) + 4$ $18 - 20 + 5 = 2(2)^2 + 3c + 3 + 4$ A1 for C value $C = -4$

7.	$\alpha+\beta=\frac{2}{3}$, $\alpha\beta=\frac{4}{3}$ — B1 for both values $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ — M1 for cubic factorisation $=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$ — M1 for changing $\alpha^2+\beta^2$ to $(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{2}{3}\right)\left[\left(\frac{2}{3}\right)^2-3\left(\frac{4}{3}\right)\right]$ $=\frac{2}{3}\left(\frac{4}{9}-4\right)$ $=-2\frac{10}{27}$ $\left(\text{Accept}-\frac{64}{27}\right)$ — A1 $\alpha^3\beta^3=(\alpha\beta)^3$ $=\left(\frac{4}{3}\right)^3$ $=2\frac{10}{27}$ $\left(\text{Accept}\frac{64}{27}\right)$ — A1 — A1 must =0 Required Equation is $27x^2+64x+64=0$ $\left(\text{Accept } x^2+\frac{64}{27}x+\frac{64}{27}=0\right)$
8(i)	Let $f(x)=3x^3-26x^2+33x+14$ $f(2)=3(2)^3-26(2)^2+33(2)+14$ $=0$ $\therefore x-2$ is a factor of $f(x)$ — M1 must show substitution and equals to 0 M1 for either following method $f(x)=(x-2)(Ax^2+Bx+C)$ — OR Long Division done correctly $3x^3-26x^2+33x+14=(x-2)(Ax^2+Bx+C)$ By Observation, $A=3, C=-7$ Sub $x=1$ $3-26+33+14=(-1)(3+B-7)$ $24=4-B$ $B=-20$ $f(x)=(x-2)(3x^2-20x-7)$ — A1 for quadratic factor $=(x-2)(3x+1)(x-7)$ — M1 correct factorisation $f(x)=0$ $(x-2)(3x+1)(x-7)=0$ $x=2$ or $x=-\frac{1}{3}$ or $x=7$ — A1

8(ii)	$3(2y)^3-26(2y)^2+33(2y)+14=0$ $2y=2$ or $2y=-\frac{1}{3}$ or $2y=7$ $y=1$ or $y=-\frac{1}{6}$ or $y=\frac{7}{2}$	M1 for able to identify to replace x by $2y$ A1 Zero if did not use part (i)
9 (i)	Discriminant <0 $k^2-4(1)(3+k)<0$ $k^2-4k-12<0$ $(k-6)(k+2)<0$ $-2<k<6$	M1 for knowing discriminant <0 M1 for correct substitution of values M1 for factorisation A1 Zero if D>0 or D=0
9 (ii)	$2ax+3=ax^2+4x+4$ $ax^2+(4-2a)x+1=0$ Discriminant >0 $(4-2a)^2-4(a)(1)>0$ $16-16a+4a^2-4a>0$ $a^2-5a+4>0$ $(a-4)(a-1)>0$ $a<1$ or $a>4$	M1 for equating the two equations M1 for getting discriminant >0 and sub in the correct values M1 for simplify to quadratic equation A1 (must show factorisation)

10 (i)	let $f(x)=p^2x^3+52x^2+qx+9$ $f\left(-\frac{1}{4}\right)=0$ $-\frac{p^2}{64}+\frac{13}{4}-\frac{q}{4}+9=0$ $-p^2+208-16q+576=0$ $-p^2-16q+784=0 \quad -(1)$ $f(1)=p^2+27p+1$ $p^2+52+q+9=p^2+27p+1$ $q=27p-60 \quad -(2)$ Sub (2) into (1) $-p^2-16(27p-60)+784=0$ $-p^2-432p+960+784=0$ $p^2+432p-1744=0$ $(p-4)(p+436)=0$ $p=4$ or $p=-436$ (reject) Sub $p=4$ into (2) $q=27(4)-60$ $q=48$ A1	M1 for factor theorem (sub in values and =0) M1 for remainder throrem (sub in values) M1 for substitution A1 for $p=4$ and reject negative value
10 (ii)	$\therefore f(x)=16x^3+52x^2+48x+9$ $f(-2)=16(-2)^3+52(-2)^2+48(-2)+9$ $=-7$ A1	M1 sub in $x=-2$ regardless of correct p and q

11 (i)	$\log_2 y - \log_y 16 = 3$ $\log_2 y - \frac{\log_2 16}{\log_2 y} = 3$ $(\log_2 y)^2 - 4 = 3\log_2 y$ M1 for change of base with a purpose. (either base 2 or y) $(\log_2 y)^2 - 3\log_2 y - 4 = 0$ $(\log_2 y - 4)(\log_2 y + 1) = 0$ M1 for factorise $\log_2 y - 4 = 0$ or $\log_2 y + 1 = 0$ $\log_2 y = 4$ or $\log_2 y = -1$ $y = 2^4$ or $y = 2^{-1}$ $y = 16$ or $y = \frac{1}{2}$ A2 (1 for each box)
11 (ii)	$\log_x 128 = 2 + \log_x 2$ $\log_x 128 - \log_x 2 = 2$ $\log_x 64 = 2$ M1 apply law of log $x^2 = 64$ $x = 8$ (reject -8) M1 for changing from log form to index A1 (must reject negative value)

12 (i)	$(a-3x)^n = a^n + \binom{n}{1}(a)^{n-1}(-3x) + \binom{n}{2}(a)^{n-2}(-3x)^2 + \ldots$ $= a^n - 3na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}(9x^2) + \ldots$ $= a^n - 3na^{n-1}x + \frac{9n(n-1)a^{n-2}}{2}x^2 + \ldots$ A1 (for $a^n - 3na^{n-1}x$) A1 (for $\frac{9n(n-1)a^{n-2}}{2}x^2$)	M1 for $\binom{n}{1} = n$ and $\binom{n}{2} = \frac{n(n-1)}{2!}$
12 (ii)	$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{4}{189}$ $\frac{a^n}{\left(\frac{9n(n-1)a^{n-2}}{2}\right)} = \frac{4}{189}$ M1 for correction ratio $\frac{2a^n}{9n(n-1)a^{n-2}} = \frac{4}{189}$ M1 for simplifying to get a^2 $a^2 = \left(\frac{4}{189}\right)\left(\frac{9n(n-1)}{2}\right)$ M1 for making a^2 the subject $a^2 = \frac{2n(n-1)}{21}$ (Shown)	
12 (iii)	$n - a = 5$ M1 for getting showing n is bigger than a $n = 5 + a \quad -(1)$ $a^2 = \frac{2n(n-1)}{21} \quad -(2)$ Sub (1) into (2) $a^2 = \frac{2(5+a)(4+a)}{21}$ M1 for substitution $21a^2 = 2(20 + 9a + a^2)$ $21a^2 - 2a^2 - 18a - 40 = 0$ $19a^2 - 18a - 40 = 0$ M1 for quadratic equation $(19a+20)(a-2) = 0$ $a = -\frac{20}{19}$ or $a = 2$ A1 Must reject negative value (reject) Sub $a = 2$ into (1) $n = 7$ A1	Alternative Method: solve for n first Quadratic equation: $19n^2 - 208n + 525 = 0$ $(19n-75)(n-7) = 0$ $n = \frac{75}{19}$ *or* 7

13 (i)	$p = 0.20$	B1
13 (ii)	$70 = 25 + 0.2(2.1)^{73q}$ $45 = 0.2(2.1)^{73q}$ $(2.1)^{73q} = 225$ $\lg(2.1)^{73q} = \lg 225$ $q = \frac{\lg 225}{73 \lg 2.1}$ $q = 0.1\,(1\ sf)$	M1 for sub in values correctly M1 for taking log both side (or ln) A1
13 (iii)	$A = 53.83\,(2\ dp)$	B1 no deduct if $ is written
13 (iv)	$34 = 25 + 0.2(2.1)^{0.1t}$ $2.1^{0.1t} = \frac{9}{0.2}$ $t = \frac{\lg 45}{(0.1)\lg 2.1}$ $t = 51.31$ years old $t = 51$ years old	M1 for making t the subject A1