

SA2



TANJONG KATONG SECONDARY SCHOOL
Year-End Examination 2017
Secondary 3

CANDIDATE
NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

Paper 1

4047/01

Monday 9 October 2017

1 hour 30 minutes

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Write your answers on the writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 60.

2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

3

- 1 (a) Find the exact value of $\tan\left(\frac{2\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)$. [2]
- (b) If $\tan\theta = -\frac{1}{5}$ and $0^\circ < \theta < 180^\circ$, find the exact value of $\cos(360^\circ - \theta)$. [2]
- 2 (i) Sketch the graph of $y = \frac{1}{x^2}$. [1]
- (ii) On the same diagram sketch the graph of $x = y^3$. [2]
- (iii) Calculate the coordinates of the point of intersection of your graphs. [2]
- 3 The quadratic equation $2x^2 - 5x + 6 = 0$ has roots α and β .
- (i) Write down the value of $\alpha + \beta$ and $\alpha\beta$. [1]
- (ii) A second equation $4x^2 + px + q = 0$ has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
Find the value of p and of q . [3]
- 4 (i) Express $\frac{2x^3 + 5x^2 + 6x + 1}{(x+1)(x^2+1)}$ in the form $2 + \frac{ax^2 + bx + c}{(x+1)(x^2+1)}$ where a , b and c are unknown integers. [2]
- (ii) Hence, express $\frac{2x^3 + 5x^2 + 6x + 1}{(x+1)(x^2+1)}$ in partial fractions. [5]

4

- 5 Larry is given an injection which contains the drug Flugo. The concentration of Flugo, C , in the bloodstream is $4e^{-0.03t}$ units/cm at time t hours after an injection.

- (i) Determine the time when $C = 1$ unit/cm. [2]
- (ii) Sketch the graph of concentration against time for Flugo. [2]
- (iii) 15 hours later, Larry is given a second dose of the same injection. Calculate the concentration of Flugo in Larry's bloodstream another 5 hours later. [2]

A drug becomes ineffective when the concentration of the drug falls below 1 unit/cm.

- (iv) Another injection contains the drug SneezeFree and the concentration of this drug at time t hours after the injection is $2e^{-0.07t} + \frac{2}{t+1}$ units/cm.
Given that the concentration of the drug only decreases, show that SneezeFree becomes ineffective between 12 to 13 hours. [2]

- 6 (a) (i) Express $(3p)^2 p^{4x} + 7(p^{2x+1})^2$ in the form $16p^{ax+b}$, where a and b are unknown integers to be found. [2]
- (ii) Hence, show that $(3p)^2 p^{4x} + 7(p^{2x+1})^2$ is a perfect square. [2]
- (b) Solve $5^{2x+1} + 5^{x+1} = 10$. [5]

- 7 (a) Find the coefficient of x^{-1} in the expansion of $\left(\frac{x}{2} + \frac{1}{x}\right)^9$. [3]
- (b) Given that $(1+x)^p + (1+3x)^q = a + 23x + bx^2 + \dots$, where p and q are integers, and $p+q = 11$,
- (i) state the value of a , [1]
 - (ii) find the value of p , q and b . [5]
- (c) (i) Expand the first three terms of $(1-m)^7$, in ascending powers of m . [2]
- (ii) Hence, expand $(1-x-x^2)^7$ up to the term in x^2 . [2]

5

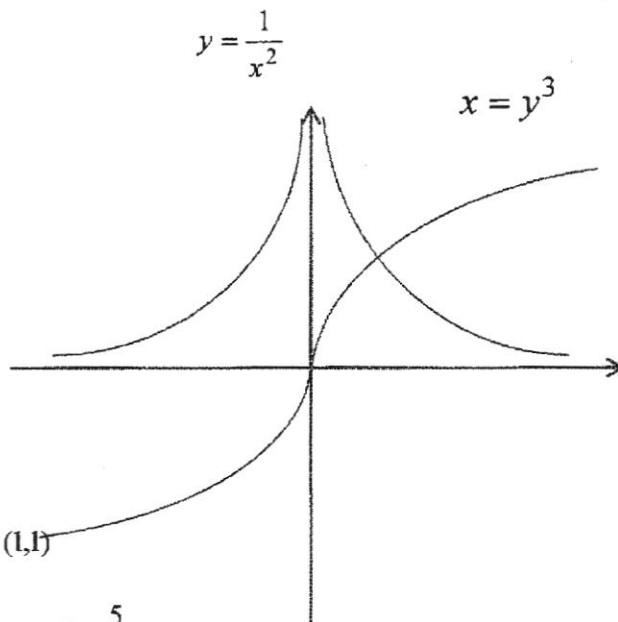
- 8 (a) Find the range of values of m for which the equation $2x^2 + mx + 2 = 0$ has no real roots for all real values of x . [4]
- (b) (i) Express $y = 4x - 2x^2$ in the form $y = a(x+m)^2 + n$, where a , m and n are constants. [2]
- (ii) Hence, explain why the highest value of the graph is 2. [1]
- (iii) Find the range of values of x for which the graph of $y = 4x - 2x^2$ is below the line $y = 3 - x$. [3]

End of Paper

1a $-\sqrt{3} + \frac{\sqrt{2}}{2}$

1b $\left(-\frac{5\sqrt{26}}{26}\right)$

2i, ii



2iii (1,1)

3i $\alpha + \beta = \frac{5}{2}$
 $\alpha\beta = 3$

$$3ii \quad p = -\frac{10}{3}$$

$$q = \frac{4}{3}$$

$$4i \quad 2 + \frac{3x^2 + 4x - 1}{(x+1)(x^2 + 1)}$$

$$4ii \quad 2 + \frac{3x^2 + 4x - 1}{(x+1)(x^2 + 1)} = 2 - \frac{1}{x+1} + \frac{4x}{x^2 + 1}$$

$$5i \quad 46.2 \text{ hours}$$

$$5iii \quad 5.64 \text{ units / cm}$$

$$6ai \quad 16p^{2+4x}$$

$$6b \quad x = 0$$

$$7a \quad 7.875$$

$$7bi \quad a = 2$$

$$7bii \quad p = 5$$

$$q = 6$$

$$b = 145$$

$$7ci \quad (1-m)^7 = 1 - 7m + 21m^2 + \dots$$

$$7cii \quad 1 - 7x + 14x^2 + \dots$$

$$8a \quad -4 < m < 4$$

$$8bi \quad 2(x-1)^2 + 2$$

$$8biii \quad x < 1 \text{ or } x > \frac{3}{2}$$

S12



TANJONG KATONG SECONDARY SCHOOL
Year-End Examination 2017
Secondary 3

CANDIDATE
NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Friday 6 Oct 2017

2 hours

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
 Write in dark blue or black pen.
 You may use an HB pencil for any diagrams or graphs.
 Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

2

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

3

- 1 A cylinder has a volume of $90\pi \text{ cm}^3$ and radius $(2 + \sqrt{3}) \text{ cm}$.
Find the height of the cylinder in the form $(a + b\sqrt{3}) \text{ cm}$. [5]
- 2 (a) Find the range of values of c for which the line $y = 2x + c$ does not intersect the curve $y = \frac{2}{1-x}$. [5]
- (b) Show that, for all real values of p , the equation $x^2 - 2(p+1)x + 2p = 0$ always has two real and distinct roots. [2]
- (c) A cubic equation $f(x) = 0$ has roots 0, -1 and $2k$.
Given the coefficient of the highest power is 2 and has a remainder of -72 when divided by $x - 3$. Find the value of k . [3]
- 3 (a) Solve the equation $\lg(5x - 6) + \lg(3x + 1) = 1 + 2\lg x$. [4]
- (b) Given that $4^a = 12^b$, prove that $\log_3 4 = \frac{b}{a-b}$. [4]
- (c) Given that $p = \log_2 x$ and $q = \log_2 y$, express the following in terms of p and/or q .
- (i) $\log_2 \frac{x}{y^3}$, [3]
- (ii) $\log_{16} x$. [2]
- 4 (a) Prove the identity $\frac{\sin x}{\sec x - 1} + \frac{\sin x}{\sec x + 1} \equiv 2 \cot x$. [4]
- (b) Solve the equation $2 \cos\left(y + \frac{\pi}{6}\right) = 1$ for $0 < y < 2\pi$.
Leave your answers in terms of π . [4]
- (c) Solve the equation $5 \sin A \cos A + 2 \sin A = 0$ for $0^\circ \leq A \leq 360^\circ$. [4]
- 5 (i) Sketch the graph of $y = \lg(x + 2)$. [2]
- (ii) In order to solve the equation $10^{2-x} = (x + 2)^4$, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = \lg(x + 2)$.
Find the equation of this straight line. [3]

- 6 (i) Find the coordinates of the centre, C and the radius of the circle which has the points $A(1, 4)$ and $B(7, 12)$ at the ends of a diameter.

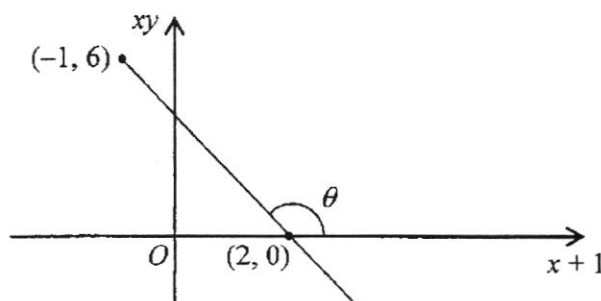
Hence write down the equation of the circle.

[4]

- (ii) Find the equation of the tangent to the circle at $P(8, 5)$.

[3]

- 7 (a) The diagram shows part of the straight line graph when xy is plotted against $(x + 1)$.



Find

- (i) the value of θ in degrees,

[2]

- (ii) y in terms of x .

[2]

- (b) The table shows experimental values of 2 variables x and y .

It is known that x and y are connected by the equation $y = ax^2 + bx$.

x	2	3	4	5	6
y	2.2	11.7	28.0	49.0	76.8

- (i) On graph paper, plot $\frac{y}{x}$ against x and draw a straight line graph.

[2]

- (ii) Use your graph to estimate the value of a and of b .

[3]

- (iii) The same equation $y = ax^2 + bx$ can also be represented by using a different vertical and horizontal axis.

Explain how the values of a and of b may be obtained.

[2]

- 8 Given that $y = p + q \cos 2x$, where p and q are integers.

5

- (i) state the period of y . [1]

Given that the maximum and minimum values of y are 2 and -4 respectively, find

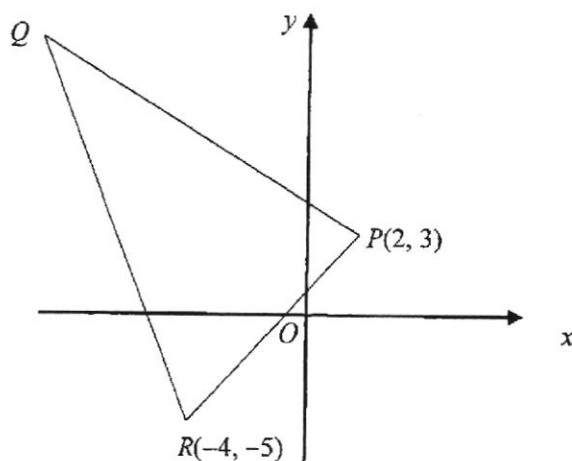
- (ii) the amplitude of y , [1]

- (iii) the value of p and of q . [2]

Using the values of p and q found in part (iii),

- (iv) sketch the graph of y for $0 \leq x \leq \pi$. [3]

- 9 The diagram drawn below is not to scale.



The diagram shows a triangle PQR in which the coordinates of points P and R are $(2, 3)$ and $(-4, -5)$ respectively and O is the origin.

The equation of PQ is $12y = -5x + 46$ and QR is parallel to the line $6y + 13x = 0$.

- (i) Find the coordinates of X which lies on RP such that $3RX = RP$. [2]
- (ii) Find the equation of QR and hence show that Q is $(-10, 8)$. [3]
- (iii) Find the area of triangle PQR . [3]
- (iv) Explain whether it is possible to draw a circle passing through the points P , Q and R with QR as the diameter. [2]

End of Paper

Answers

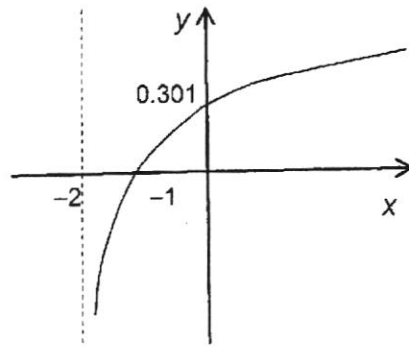
1 $(630 - 360\sqrt{3})\text{cm}$

2(a) $-6 < c < 2$

3(a) 3

4(b) $\frac{\pi}{6}, \frac{3\pi}{2}$

5(i)



2(c) 3

3(c)(i) $p - 3q$

3(c)(ii) $\frac{p}{4}$

4(c) $0^\circ, 180^\circ, 360^\circ, 113.6^\circ, 246.4^\circ$

(ii) $y = -\frac{1}{4}x + \frac{1}{2}$

6(i) $(x - 4)^2 + (y - 8)^2 = 25$

7(a)(i) 116.6°

(ii) $3y = 4x - 17$

(a)(ii) $y = -2 + \frac{2}{x}$

7(b)(i) (b)(ii) $b = -4.7$ [-4.8 to -4.6]

$a = \frac{9.8 - (-4.7)}{5 - 0} = 2.9$ [2.86-2.98]

7(b)(iii) $\frac{y}{x^2} = a + \frac{b}{x}$

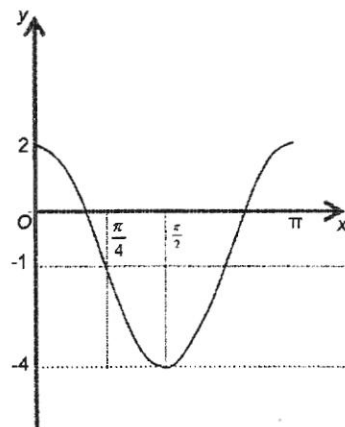
Plot $\frac{y}{x^2}$ against $\frac{1}{x}$, from the straight line obtained, the gradient = b and the vertical intercept = a

8(i) π

(ii) 3

(iii) $p = -1, q = 3$

(iv)

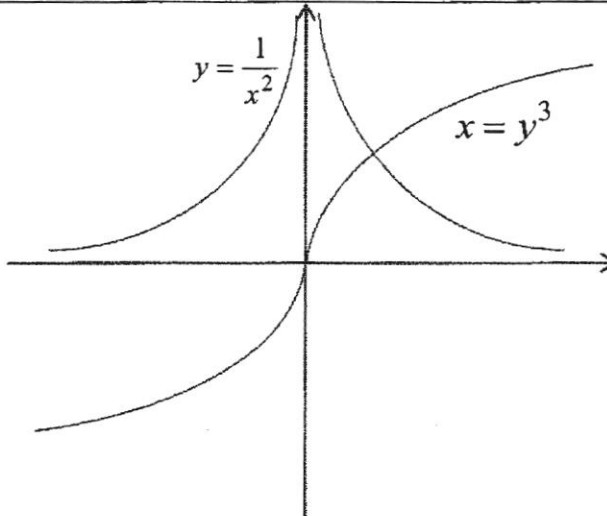


9(i) $X(-2, -2\frac{1}{3})$

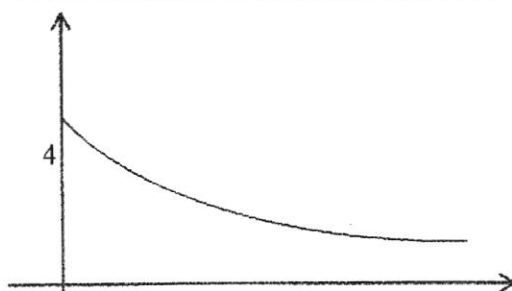
9(iii) 63 units²

SA2 Tanjung Katong

2017 Sec 3 Additional Mathematics Paper 1 Marking Scheme

1a	$\tan\left(\frac{2\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)$ $= -\sqrt{3} + \frac{\sqrt{2}}{2}$	B1, B1 isw
1b	$\cos(360^\circ - \theta) = -\frac{5}{\sqrt{26}}$ $\left(-\frac{5\sqrt{26}}{26}\right)$	B1 – negative B1 – ratio o.e.
2i, ii		(i) G1 – graph of $y = \frac{1}{x^2}$ (ii) 1 mark for 1 st quadrant, 1 mark for 3 rd quadrant
2iii	$y = \frac{1}{(y^3)^2}$ $y^7 = 1$ $y = 1$ $x = 1$ $(1,1)$	M1 – equate both graphs A1
3i	$\alpha + \beta = \frac{5}{2}$ $\alpha\beta = 3$	B1 for both

2017 Sec 3 Additional Mathematics Paper 1 Marking Scheme

3ii	$\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{p}{4}$ $\frac{\alpha + \beta}{\alpha\beta} = -\frac{p}{4}$ $\frac{5/2}{3} = -\frac{p}{4}$ $p = -\frac{10}{3}$ $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{q}{4}$ $\frac{1}{3} = \frac{q}{4}$ $q = \frac{4}{3}$	<p>M1 – find sum or product of new roots</p> <p>A1</p> <p>A1</p>
4i	$\frac{2x^3 + 5x^2 + 6x + 1}{(x+1)(x^2+1)} = 2 + \frac{3x^2 + 4x - 1}{(x+1)(x^2+1)}$	<p>M1 – long division or comparison</p> <p>A1</p>
4ii	$\frac{3x^2 + 4x - 1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ $3x^2 + 4x - 1 = A(x^2+1) + (Bx+C)(x+1)$ $x = -1$ $\therefore A = -1$ $B = 4$ $C = 0$ $2 + \frac{3x^2 + 4x - 1}{(x+1)(x^2+1)} = 2 - \frac{1}{x+1} + \frac{4x}{x^2+1}$	<p>B1</p> <p>(-1 mark if no brackets for $Bx + C$)</p> <p>M1 – substitution or comparing coefficient</p> <p>A2 – (minus 1 mark for each mistake)</p> <p>√B1 (follow through if split into correct cases)</p>
5i	$4e^{-0.03t} = 1$ $\ln e^{-0.03t} = \ln \frac{1}{4}$ $t = 46.2 \text{ hours}$	<p>M1 – ln/lg both sides</p> <p>A1 isw</p>
5ii		<p>G1 – shape (decreasing graph)</p> <p>If $t < 0$, -1 mark for shape</p> <p>G1 – intercept and asymptote</p>

2017 Sec 3 Additional Mathematics Paper 1 Marking Scheme

5iii	Concentration $= 4e^{-0.03(20)} + 4e^{-0.03(5)}$ $= 5.64 \text{ units / cm}$	M1 – find concentration at $t = 5$ or $t = 20$ A1
5iv	At $t = 12$, $C = 2e^{-0.07(12)} + \frac{2}{12+1}$ $= 1.02$ $C = 2e^{-0.07(13)} + \frac{2}{13+1}$ $= 0.948$ Hence, between $t = 12$ or $t = 13$, concentration must have been 1. So it becomes ineffective between $t = 12$ or $t = 13$.	M1 – attempt to show that concentration at one point is above 1 and at one point is less than 1. A1 – conclusion
6ai	$(3p)^2 p^{4x} + 7(p^{2x+1})^2$ $= 9p^2 p^{4x} + 7p^{4x+2}$ $= p^{2+4x}(16)$ $= 16p^{2+4x}$	B1 – expand $()^2$ (either bracket) B1 – factorise common term (in the given form)
6aii	$p^{2+4x}(16)$ $= (p^{1+2x})^2 4^2$	B1 – $(p^{1+2x})^2$ is a perfect square B1 – 16 is a perfect square
6b	$5^{2x+1} + 5^{x+1} = 10$ $(5^x)^2(5) + (5^x)(5) - 10 = 0$ let $5^x = a$ $5a^2 + 5a - 10 = 0$ $(a-1)(a+2) = 0$ $a = 1$ or $a = -2(\text{rej})$ $5^x = 1$ $x = 0$	M1 – split power $2x+1$ M1 – form quadratic equation M1 – factorise A1 (-2 rejected, either here or at final ans) A1
7a	General Term $= \binom{9}{r} \left(\frac{x}{2}\right)^r \left(\frac{1}{x}\right)^{9-r}$ Power of $x = 2r - 9$ $2r - 9 = -1$ $r = 4$ Coefficient $t = 7.875$	M1 Any method (general term, expansion) B1 – $r = 4$ A1

2017 Sec 3 Additional Mathematics Paper 1 Marking Scheme

7bi	$a = 2$	B1
7bii	$(1+x)^p = 1 + px + \dots$ $(1+3x)^q = 1 + 3qx + \dots$ $p + 3q = 23$ $p + q = 11$ $p = 5$ $q = 6$ $b = \binom{5}{2} + \binom{6}{2}(9)$ $= 145$	M1 – compare coeff of x M1 – solve simult. A1 A1 A1
7ci	$(1-m)^7 = 1 - 7m + 21m^2 + \dots$	B2 $1 - 7m, 21m^2$
7cii	Replace m with $x + x^2$ $1 - 7(x + x^2) + 21(x + x^2)^2 + \dots$ $= 1 - 7x - 7x^2 + 21x^2 + \dots$ $= 1 - 7x + 14x^2 + \dots$	M1 – substitute m with $x + x^2$ A1
8a	$m^2 - 4(2)(2) < 0$ $m^2 - 16 < 0$ $(m+4)(m-4) < 0$ $-4 < m < 4$	B1 – $b^2 - 4ac$ with substituitn B1 – Discriminant < 0 M1 – factorise (no factorisation, M0A0) A1
8bi	$4x - 2x^2$ $= -2(x^2 - 2x)$ $= -2(x^2 - 2x + 1 - 1)$ $= -2(x-1)^2 + 2$	M1 +1-1 A1
8bii	0 is the lowest value of $(x-1)^2$ 0 is the highest value of $-2(x-1)^2$ 2 is the highest value of $-2(x-1)^2 + 2$	B1
8biii	$4x - 2x^2 < 3 - x$ $-2x^2 + 5x - 3 < 0$ $(2x-3)(x-1) > 0$ $x < 1$ or $x > \frac{3}{2}$	B1 – form inequality/equate B1 roots 1, 1.5 seen B1

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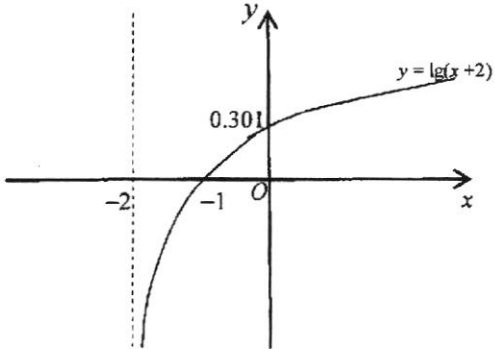
2017 Sec 3 End of Year Add Maths P2

No.	Solution	Remarks	
1	$\frac{90\pi}{\pi(2+\sqrt{3})^2}$ $\frac{90}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$ $\frac{90(7-4\sqrt{3})}{49-16(3)}$ $(630-360\sqrt{3})\text{cm}$	B1 Form eqn B1 Evaluate $(\text{rad})^2$ M1 Correct conjugate surd M1 Evaluate deno A1	
TOTAL			5
2(a)	$2x+c = \frac{2}{1-x}$ $(2x+c)(1-x) = 2$ $2x^2 + (c-2)x + 2 - c = 0$ $D < 0$ $(c-2)^2 - 4(2)(2-c) < 0$ $c^2 + 4c - 12 < 0$ $(c+6)(c-2) < 0$ $-6 < c < 2$	M1 Equate line & curve <u>and</u> get quad eqn B1 Inequality $D < 0$ M1 Correct sub of D M1 Factoring must be seen A1 [-1m poor presentation]	5
2(b)	$x^2 - 2(p+1)x + 2p = 0$ $D = [-2(p+1)]^2 - 4(2p)$ $= 4p^2 + 8p + 4 - 8p$ $= 4(p^2 + 1) > 0$ Real and distinct roots (shown)	B1 Correct sub of D B1 > 0 seen & statement	2
2(c)	$f(x) = 2x(x+1)(x-2k)$ $f(3) = -72$ $2(3)(4)(3-2k) = -72$ $48k = 144$ $k = 3$	B1 Form eqn M1 sub $x=3$, $R = -72$ A1	3
TOTAL			10
3(a)	$\lg(5x-6) + \lg(3x+1) = 1 + 2\lg x$ $\lg \frac{(5x-6)(3x+1)}{x^2} = 1$ $15x^2 - 13x - 6 = 10x^2$ $5x^2 - 13x - 6 = 0$ $(x-3)(5x+2) = 0$ $x = 3 \quad x = -\frac{2}{5} \text{ (rejected)}$	M1 Combine lg terms M1 Change to index form/ Remove lg M1 Solve quad eqn seen A1 Must reject -ve ans	4
3(b)	$4^a = 12^b$ $a \log_3 4 = b \log_3 (4 \times 3)$ $a \log_3 4 = b \log_3 4 + b \log_3 3$ $(a-b) \log_3 4 = b(1)$ $\log_3 4 = \frac{b}{a-b}$	M1 Bring down index M1 \log_3 both sides M1 Split $\log_3 12$ B1 Group $\log_3 4$ term & ans	4

2017 Sec 3 End of Year Add Maths P2

No.	Solution	Remarks	
3(c)(i)	$\log_2 \frac{x}{y^3} = \log_2 x - 3 \log_2 y$ $= p - 3q$	M1 Use $\lg \frac{a}{b} = \lg a - \lg b$ M1 Use $\lg a^m = m \lg a$ A1	3
3(c)(ii)	$\log_{16} x = \frac{\log_2 x}{\log_2 16}$ $= \frac{p}{4}$	B1 Change base B1	2
		TOTAL	13

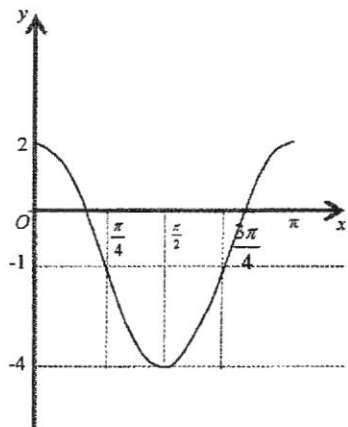
2017 Sec 3 End of Year Add Maths P2

No.	Solution	Remarks	
4(a)	$\frac{\sin x}{\sec x - 1} + \frac{\sin x}{\sec x + 1}$ $= \frac{\sin x(\sec x + 1) + \sin x(\sec x - 1)}{\sec^2 x - 1}$ $= \frac{2 \sin x \left(\frac{1}{\cos x} \right)}{\tan^2 x}$ $= \frac{2}{\tan x}$ $= 2 \cot x (\text{Proved})$	M1 Combine fractions M1 Use identity $\sec^2 x - 1 = \tan^2 x$ o.e. M1 Use $\sec x = \frac{1}{\cos x}$ M1 Use $\cot x = \frac{1}{\tan x}$ Prove LHS = RHS	4
4(b)	$2 \cos\left(y + \frac{\pi}{6}\right) = 1$ $y + \frac{\pi}{6} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ $y = \frac{\pi}{6}, \frac{3\pi}{2}$	B1 for $\frac{\pi}{3}$ B1 for $2\pi -$ (their Basic \angle) B2 do not accept deg equiv or not in π Accept if whole qn work in deg, then convert to ans in π	4
4(c)	$5 \sin A \cos A + 2 \sin A = 0$ $\sin A (5 \cos A + 2) = 0$ $\sin A = 0 \quad A = 0^\circ, 180^\circ, 360^\circ$ $\text{or } \cos A = -0.4 \quad A = 113.6^\circ, 246.4^\circ$	M1 Factorise A1 All 3 ans A2	4
TOTAL			12
5(i)		G1 Shape & y-int (lg 2 o.e.) G1 asymptote & x-int	2
5(ii)	$10^{-x+2} = (x+2)^4$ $\lg(x+2)^4 = -x+2$ $4 \lg(x+2) = -x+2$ $\lg(x+2) = -\frac{1}{4}x + \frac{1}{2}$ $y = -\frac{1}{4}x + \frac{1}{2}$	B1 Take lg on both sides B1 Bring down index B1 o.e.	3
TOTAL			5

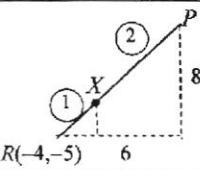
2017 Sec 3 End of Year Add Maths P2

No.	Solution	Remarks													
6(i)	$C(4, 8)$ $AB = \sqrt{(7-1)^2 + (12-4)^2}$ $= 10$ units Radius = 5 units Eqn of circle: $(x-4)^2 + (y-8)^2 = 25$ Or $x^2 + y^2 - 8x - 16y + 55 = 0$	B1 B1 Pythagoras' Theorem / Length formula with substitution B1 B1 \checkmark isw	4												
6(ii)	$m_{CP} = \frac{8-5}{4-8} = -\frac{3}{4}$ Equation of tangent: $y-5 = \frac{4}{3}(x-8)$ $y = \frac{4}{3}x - \frac{17}{3}$ or $3y = 4x - 17$	(must be from m_{CP}) M1 Perp gradient seen M1 Finding Equation A1 o.e.	3												
TOTAL			7												
7(a)(i)	$\tan \theta = \frac{-6-0}{-1-2}$ $\tan \theta = -2$ $\theta = 116.6^\circ$	M1 Relate $\tan \theta$ to grad Or $\tan \alpha = 2$ $\theta = 180^\circ - \alpha$ A1 to 1 dp	2												
7(a)(ii)	$xy = -2[(x+1) - 2]$ $xy = -2x+2$ $y = -2 + \frac{2}{x}$	M1 Find eqn of line & correct axes seen A1 o.e. [y must be subject]	2												
7(b)(i)	<table border="1"><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$\frac{y}{x}$</td><td>1.1</td><td>3.9</td><td>7.0</td><td>9.8</td><td>12.8</td></tr></table> $\frac{y}{x} = ax+b$ and a straight line drawn	x	2	3	4	5	6	$\frac{y}{x}$	1.1	3.9	7.0	9.8	12.8	T1 L1 with vert int seen	2
x	2	3	4	5	6										
$\frac{y}{x}$	1.1	3.9	7.0	9.8	12.8										
7(b)(ii)	$b = -4.7$ $a = \frac{9.8 - (-4.7)}{5 - 0} = 2.9$	B1 -4.8 to -4.6(fr graph) B1 Calculate grad B1 2.86 to 2.96(fr graph)	3												
7(b)(iii)	$y = ax^2 + bx$ $\frac{y}{x^2} = a + \frac{b}{x}$ Plot $\frac{y}{x^2}$ against $\frac{1}{x}$, from the straight line obtained, the gradient = b and the vertical intercept = a	B1 Eqn of str line B1 Explanation	2												
TOTAL			11												

2017 Sec 3 End of Year Add Maths P2

No.	Solution	Remarks	
8(i)	π	B1 Accept 180°	1
8(ii)	3	B1	1
8(iii)	$p = -1, q = 3$	B2	2
8(iv)		G1 Shape G1 Max pts, Min pt G1 The other 2 pts	3
TOTAL			7

2017 Sec 3 End of Year Add Maths P2

No.	Solution	Remarks	
9(i)	$X(-4+2, -5+\frac{1}{3}\times 8)$ $X(-2, -2\frac{1}{3})$ 	B1 for x B1 for y	2
9(ii)	Eqn of QR : $y+5 = -\frac{13}{6}(x+4)$ $6y = -13x - 82$ Eqn of PQ : $12y = -5x + 46$ (given) $2(-13x - 82) = -5x + 46$ $21x = 210$ $x = -10$ $y = (130-82)\div 6$ $y = 8$ $\therefore Q(-10, 8)$ (shown)	B1 Grad seen B1 Eqn seen B1 Manipulation seen to get ans	3
9(iii)	$\text{Area} = \frac{1}{2} \begin{vmatrix} 2 & -10 & -4 & 2 \\ 3 & 8 & -5 & 3 \end{vmatrix}$ $= \frac{1}{2} \{(16+50-12) - (-30-32-50)\}$ $= 63 \text{ units}^2$	B1 M1 Correct manipulation A1	3
9(iv)	$m_{PQ} = -\frac{5}{12}$ $m_{PR} = \frac{3-(-5)}{2-(-4)} = \frac{4}{3}$ Since $m_{PQ} \times m_{PR} = -\frac{5}{12} \times \frac{4}{3} \neq -1$ $\angle QPR \neq 90^\circ$, QR is not the diameter Not possible as it does not obey theorem- angle in semicircle	} B1 } B1 Correct conclusion with reason	2
TOTAL			10