SA1

Name:	Class:	Index Number:

ST. HILDA'S SECONDARY SCHOOL

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MID-YEAR EXAMINATION 2017

Additional Mathematics **4047**

Level: Secondary 3 Express Duration: 2 hours

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Writing Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

For Examiner's Use	
Total	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

1 Express $\dfrac{6x^3-19x^2+31x-14}{3x-2}$ in the form ax^2+bx+c, stating the values of the integers a, b and c. [3]

2 Given that the curve $x-1=9y-2y^2$ intersects the line $x=10-2y$ at points P and Q, find the coordinates of P and Q. [4]

3 A curve has the equation $y=5x^2+13x+k$, where k is a constant.

(i) In the case where $k=-6$, find the range of values of x for which $y \geq 0$. [2]

(ii) Find the value of k for which the line $y-4=3x$ is a tangent to the curve. [3]

4 Express $\dfrac{x-5}{(2x-3)(x-2)^2}$ as partial fractions. [5]

5 Given that $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ are the roots of the equation $3x^2-6x+2=0$,

(i) find the values of $\alpha^2+\beta^2$, [4]

(ii) find the values of $\alpha^3+\beta^3$, [2]

(iii) find the equation whose roots are $\dfrac{\beta^2}{\alpha}$ and $\dfrac{\alpha^2}{\beta}$. [3]

6 Write down in ascending powers of x, the first four terms in the expansion of $(1-5x)^9$. [2]

(i) Find the coefficient of the 7th term. [2]

(ii) Find the coefficient of x^2 in the expansion of $\left(2-\dfrac{1}{x}\right)(1-5x)^9$. [2]

(iii) Using the expansion of $(1-5x)^9$, find the value of $(0.995)^9$, to the nearest 4 decimal places. [2]

7 (a) Find the value of $16^{n+1} \div 64^{\frac{2n}{3}}$. [3]

(b) A rectangle has sides $\left(2+\sqrt{18}\right)$ metres and $\left(5-\dfrac{4}{\sqrt{2}}\right)$ metres. The area of the rectangle is A m^2. **Without using a calculator**, show that A can be expressed as $a+b\sqrt{2}$, where a and b are integers. [3]

(c) Express $\dfrac{1+\sqrt{5}}{7-3\sqrt{5}}$ in the form $\dfrac{a+b\sqrt{5}}{2}$, where a and b are integers. [3]

8 (a) Find the smallest integer p for which $(p+1)x^2+4x=1-p$ has real roots. [5]

(b) The curve $y=x^2+2kx-6x+25$ is always positive for all real values of x. Find the greatest value of k such that k is a composite number. [4]

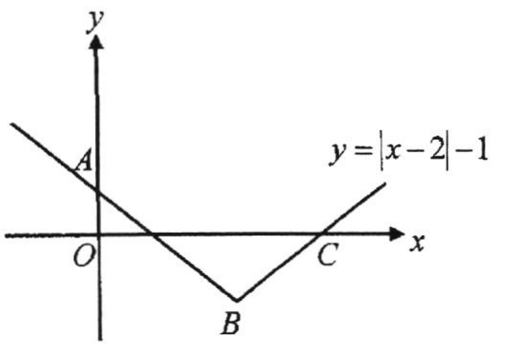
9 The polynomial f(x) is given by $f(x)=ax^3-x^2+bx-12$, where a and b are constants. It is given that f(4) = 0 and the remainder when f(x) is divided by $x+1$ is 10.

(i) Show that $a=2$ and $b=-25$. [4]

(ii) Show that $2x+1$ is a factor of f(x). [2]

(iii) Showing your working clearly, solve the equation f(x) = 0. [3]

10



The diagram shows part of the graph of $y=|x-2|-1$.

(i) Find the coordinates of the points A, B and C. [4]

(ii) Find the range of values of x for which $y<1$. [3]

(iii) A straight line $y=2x+3$ is drawn on the same axes. Find the x coordinate(s) of the intersection(s) with the graph $y=|x-2|-1$. [3]

11 A curve has the equation $y=(3x-1)^2-4$.

(i) Explain why the lowest point on the curve has coordinates $\left(\frac{1}{3}, -4\right)$. [1]

(ii) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(iii) Sketch the graph of $y=\left|(3x-1)^2-4\right|$. [3]

(iv) Using your graph, state the number of solutions to each of the following equations.

(a) $\left|(3x-1)^2-4\right|=4$

(b) $\left|(3x-1)^2-4\right|=2$

(c) $\left|(3x-1)^2-4\right|+1=0$ [3]

End of Paper

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Suggested Solutions

No	Solution
1	$\dfrac{6x^3-19x^2+31x-14}{3x-2}=2x^2+bx+7$ $(3x-2)(2x^2+bx+7)=6x^3-19x^2+31x-14$ Compare coefficient of x^2: $3b-4=-19$ $b=-5$ $\dfrac{6x^3-19x^2+31x-14}{3x-2}=2x^2-5x+7$ $a=2$, $b=-5$ and $c=7$
2	$x-1=9y-2y^2 \quad -(1)$ $x=10-2y \quad -(2)$ Sub (2) into (1), $10-2y-1=9y-2y^2$ $2y^2-11y+9=0$ $(2y-9)(y-1)=0$ $y=4.5$ or 1 Sub $y=\frac{9}{2}$ into (2), $x=10-2(4.5)$ $=1$ Sub $y=1$ into (2), $x=10-2(1)$ $=8$ The coordinates of P and Q are $(1,4.5)$ and $(8,1)$.
3(i)	$5x^2+13x-6\geq 0$ $(5x-2)(x+3)\geq 0$ $x\leq -3$ or $x\geq \frac{2}{5}$
3(ii)	$3x+4=5x^2+13x+k$ $5x^2+10x+k-4=0$ discriminant = 0 or $b^2-4ac=0$ where $a=5,\ b=10,\ c=k-4$ $(10)^2-4(5)(k-4)=0$ $k=9$

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4	$\frac{x-5}{(2x-3)(x-2)^2}=\frac{A}{(2x-3)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}$ $=\frac{A(x-2)^2+B(2x-3)(x-2)+C(2x-3)}{(2x-3)(x-2)^2}$ $x-5=A(x-2)^2+B(2x-3)(x-2)+C(2x-3)$ When $x=2$, $-3=C$ $C=-3$ When $x=\frac{3}{2}$, $-\frac{7}{2}=\frac{1}{4}A$ $A=-14$ When $x=0,\ A=-14,\ C=-3$, $-5=4(-14)+6B-3(-3)$ $-5=6B-47$ $B=7$ $\frac{x-5}{(2x-3)(x-2)^2}=-\frac{14}{(2x-3)}+\frac{7}{(x-2)}-\frac{3}{(x-2)^2}$
5(i)	From $3x^2-6x+2=0$, $a=3,\ b=-6,\ c=2$ Sum of roots: $\frac{1}{\alpha}+\frac{1}{\beta}=-\left(-\frac{6}{3}\right)$ $\frac{\alpha+\beta}{\alpha\beta}=2$ Product of roots: $\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{2}{3}$ $\frac{1}{\alpha\beta}=\frac{2}{3}$ $\therefore \alpha\beta=\frac{3}{2}$ $\alpha+\beta=2\times\frac{3}{2}$ $=3$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=(3)^2-2(\frac{3}{2})$ $=6$

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5(ii)	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(3)\left(6-\frac{3}{2}\right)$ $=13\frac{1}{2}$
5(iii)	$\frac{\beta^2}{\alpha}+\frac{\alpha^2}{\beta}=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $=13\frac{1}{2}\div\frac{3}{2}$ $=9$ $\frac{\beta^2}{\alpha}\times\frac{\alpha^2}{\beta}=\frac{3}{2}$ $x^2-9x+1\frac{1}{2}=0$ Alternative: $2x^2-18x+3=0$
6	$(1-5x)^9=1+\binom{9}{1}(-5x)+\binom{9}{2}(-5x)^2+\binom{9}{3}(-5x)^3+...$ $=1-45x+900x^2-10500x^3+...$
6(i)	Coefficient of 7th term $=\binom{9}{6}(1)(-5)^6$ $=1312500$
6(ii)	$\left(2-\frac{1}{x}\right)(1-5x)^9=\left(2-\frac{1}{x}\right)(1-45x+900x^2-10500x^3+...)$ Coefficient of x^2= $2(900)-1(-10500)$ $=12300$
6(iii)	Let $x=0.001$ $0.995^9=[1-5(0.001)]^9$ $=1-45(0.001)+900(0.001)^2-10500(0.001)^3+...$ $=0.9559$ (4 decimal places)
7(a)	$16^{n+1}\div64^{\frac{2n}{3}}=(4^2)^{n+1}\div(4^3)^{\frac{2n}{3}}$ $=(4^{2n+2})\div(4^{2n})$ $=(4^2)$ $=16$ **Alternative** $16^{n+1}\div64^{\frac{2n}{3}}=(2^4)^{n+1}\div(2^6)^{\frac{2n}{3}}$ $=(2^{4n+4})\div(2^{4n})$ $=(2^4)$ $=16$

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7(b)	$A=\left(2+\sqrt{18}\right)\left(5-\frac{4}{\sqrt{2}}\right)$ $=\left(2+3\sqrt{2}\right)\left(5-2\sqrt{2}\right)$ $=10-4\sqrt{2}+15\sqrt{2}-12$ $=-2+11\sqrt{2}$
7(c)	$\frac{1+\sqrt{5}}{7-3\sqrt{5}}=\frac{1+\sqrt{5}}{7-3\sqrt{5}}\times\frac{7+3\sqrt{5}}{7+3\sqrt{5}}$ $=\frac{22+10\sqrt{5}}{7^2-\left(3\sqrt{5}\right)^2}$ $=\frac{22+10\sqrt{5}}{49-45}$ $=\frac{11+5\sqrt{5}}{2}$
8(a)	$(p+1)x^2+4x=1-p$ $(p+1)x^2+4x+p-1=0$ $a=p+1,\ b=4,\ c=p-1$ Discriminant ≥ 0 $b^2-4ac\geq 0$ $(4)^2-4(p+1)(p-1)\geq 0$ $16-4(p^2-1)\geq 0$ $4-p^2+1\geq 0$ $5-p^2\geq 0$ $p^2-5\leq 0$ $(p+\sqrt{5})(p-\sqrt{5})\leq 0$ $-\sqrt{5}\leq p\leq\sqrt{5}$ $p=-2$
8(b)	$x^2+2kx-6x+25=0$ $a=1,\ b=2k-6,\ c=25$ $b^2-4ac<0$ $(2k-6)^2-4(1)(25)<0$ $4k^2-24k-64<0$ $k^2-6k-16<0$ $(k-8)(k+2)<0$ $-2<k<8$ $k=6$

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9(i)	$f(x)=ax^3-x^2+bx-12$ $f(4)=0$ $f(4)=a(4)^3-(4)^2+b(4)-12$ $0=64a-16+4b-12$ $0=16a+b-7$ $16a+b=7 \quad -(1)$ $f(-1)=10$ $f(-1)=a(-1)^3-(-1)^2+b(-1)-12$ $10=-a-1-b-12$ $10=-a-b-12$ $a+b=-13-10$ $a+b=-23 \quad -(2)$ $(1)-(2), \quad 15a=30$ $a=2$ (Shown) Sub. $a=2$ into (2), $2+b=-23$ $b=-25$ (Shown)
9(ii)	$a=2, b=-25$ Let $f(x)=2x^3-x^2-25x-12$ Let $2x+1=0$, $x=-\frac{1}{2}$ $f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3-\left(-\frac{1}{2}\right)^2-25\left(-\frac{1}{2}\right)-12$ $=-\frac{2}{8}-\frac{1}{4}+\frac{25}{2}-12$ $=0$ Hence, $2x+1$ is a factor of $f(x)$. (Shown)
9(iii)	$f(x)=2x^3-x^2-25x-12$ $f(x)=(x-4)(2x+1)(ax+b)$ By comparing coefficients of, $x^3: \quad 2=2a$ $a=1$ $x^0: -12=-4b$ $b=3$ $f(x)=2x^3-x^2-25x-12$ $=(x-4)(2x+1)(x+3)$ $f(x)=0 \quad \therefore \quad (x-4)(2x+1)(x+3)=0$ $x=4, -\frac{1}{2}, -3$

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Suggested Solutions

10(i)	$y=\|x-2\|-1$ When $x=0,\ y=1\ \therefore A(0,1)$ When $y=0,\ x-2=1\ \ or\ \ -1$ $x=3\ \ or\ \ 1$ (rejected) $\therefore C(3,0)$ Vertex, $B(2,-1)$
10(ii)	$\|x-2\|-1=1$ $\|x-2\|=2$ $x-2=2$ or $x-2=-2$ $x=4$ or $x=0$ are the 2 intersections. For $y<1,\ 0<x<4$ **Alternative Solutions** $\|x-2\|-1<1$ $\|x-2\|<2$ $x-2>-2$ or $x-2<2$ $x>0$ or $x<4$ $0<x<4$
10(iii)	$\|x-2\|-1=2x+3$ $\|x-2\|=2x+4$ $x-2=2x+4$ or $x-2=-(2x+4)$ $x=-6$ or $x=-\frac{2}{3}$ (Rejected)
11(i)	$3x-1=0$ $x=\frac{1}{3}$ (line of symmetry) the line of symmetry for the curve cuts the curve at the lowest point. When $x=\frac{1}{3},\ y=-4$. Therefore, the lowest point on the curve has coordinates $\left(\frac{1}{3},-4\right)$. **Alternative Solutions** $(3x-1)^2\geq 0$ $(3x-1)^2-4\geq -4$ $y\geq -4$ $\therefore$ minimum value of y is -4.

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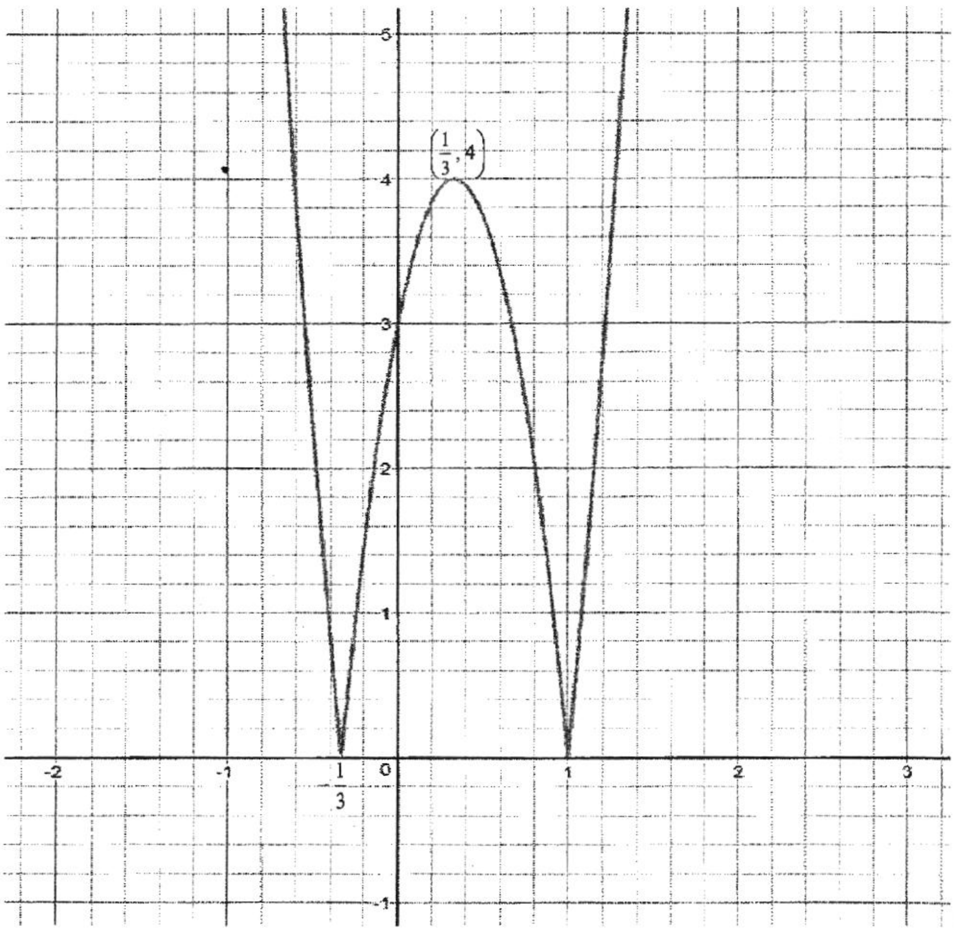
Suggested Solutions

11(ii)	$(3x-1)^2-4=0$ $(3x-1)^2=4$ **Alternative:** $3x^2-2x-1=0$ $3x-1=\pm 2$ $(x-1)(3x+1)=0$ $x=1$ *or* $-\frac{1}{3}$ Coordinates: $(1,0)$ *or* $\left(-\frac{1}{3},0\right)$
11(iii)	
(iv)(a)	3
(iv)(b)	4
(iv)(c)	0