BP~253

SMZ

Class:	Candidate Name:	Candidate Index Number:

SHUQUN SECONDARY SCHOOL
2017 End-Of-Year Examination
Secondary Three Express

ADDITIONAL MATHEMATICS 4047

4 October 2017

Additional Materials: Answer Paper
Graph Paper

2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and class index number in the spaces at the top of this page and all the work you hand in.
Write in blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of marks for this paper is 100.

This question paper consists of **7** printed pages.

[Turn over

Mathematical formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae of ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin c$$

Answer **all** the questions.

1 (a) Solve the inequality $(3x-1)(2x+3)+10x>0$. [3]

(b) Write $\frac{5\sqrt{2}+\sqrt{14}}{3\sqrt{2}-\sqrt{14}}$ in the form $a+b\sqrt{7}$, where a and b are integers. [3]

2 The function f is given by $f(x)=2\sin 2x-1$ for $0° \le x \le 180°$.

(i) State the amplitude of f. [1]

(ii) State the period of f. [1]

(iii) Find the x-coordinates of the points where the curve meets the x-axis. [2]

(iv) Sketch the graph of $y=2\sin 2x-1$, for $0° \le x \le 180°$. [2]

3 (a) The function $f(x)=x^3-6x^2+ax+b$, where a and b are constants, is exactly divisible by $x-3$ and leaves a remainder of -55 when divided by $x+2$. Find the value of a and of b. [4]

(b) Solve the equation $2\log_2(x-2)-\log_2(x+5)+3=0$. [5]

4 (a) A man bought a new car. After x months, its value \$ V is given by $V=120000e^{-px}$, where p is a constant.

(i) Find the value of the car at the time when the man bought it. [1]

The value of the car after 5 years is \$ 48 000. Calculate

(ii) the expected value, to the nearest dollar, of the car after 8 years of purchase, [3]

(iii) the age of the car, to the nearest month, when the expected value of the car will be \$ 12 000. [2]

(b) Express $\frac{8x^2+3x+1}{x(2x+1)^2}$ in partial fractions. [4]

5 (a) Given that $\tan A = 8$, $\cos B = \frac{7}{\sqrt{65}}$, A is acute and that A and B lie in different quadrants, find without the use of calculator, the exact values of

(i) $\tan(90° - A)$, [1]

(ii) $\sin B$. [2]

(b) Sketch the graph of $y = |2x-1|$ for $-3 \le x \le 5$. [3]

(c) Solve $|3x-4| = 5x$. [3]

6 (a) (i) Expand and simplify the first 4 terms, in decending powers of x, in the expansion of $\left(2-\frac{1}{x}\right)^6$. [2]

(ii) Hence find the coefficient of x^{-1} in the expansion of $\left(1-2x+x^2\right)\left(2-\frac{1}{x}\right)^6$. [3]

(b) Find in terms of a, the term independent of x, in the expansion of $\left(\frac{a}{x}+x\right)^{20}$. [3]

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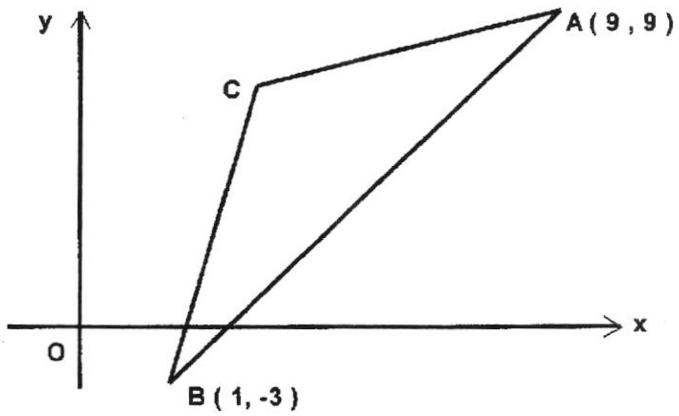
7 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a triangle ABC in which the point A is $(9,9)$ and the point B is $(1,-3)$. The point C lies on the perpendicular bisector of AB and the equation of the line BC is $y=8x-11$. Find

(i) the equation of the perpendicular bisector of AB, [4]

(ii) the coordinates of C. [2]

The point D is such that $ACBD$ is a rhombus.

(iii) Find the coordinates of D. [2]

(iv) Show that $AB=2CD$. [2]

8 (a) Solve the simultaneous equations
$3^x.\sqrt{3^y}=81,$
$\log_2(8)=3+\log_2(4x-y)$ [5]

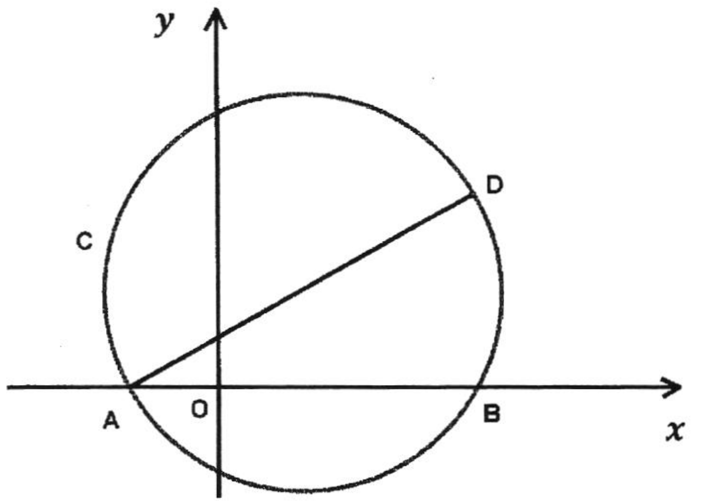
(b) Given that $\log_3(9a)-\log_9(9b)=2$, express b in terms of a. [4]

(c) Find the value(s) of m for which the straight line $mx+y+6=0$ is always a tangent to the curve $y=x^2+3x+2m$. [4]

9 (a) By using appropriate substitution, or otherwise, solve the equation $3^{2x+2}+1=2(3^{x+1})$. [5]

(b) Find the smallest integer value of k for which kx^2+8x+3 is always positive for all values of x. [3]

10 The equation of the circle, C, as shown in the diagram below is
$x^2 + y^2 - 6x - 10y - 135 = 0$.



(i) Find the coordinates of the centre and the radius of the circle, C. [2]

(ii) Given that the circle cuts the x-axis at the points A and B. Find the length of the line segment AB. [3]

Given that D is a point on the circle such that the line segment AD is the diameter of the circle. Find

(iii) the coordinates of D, [1]

(iv) the equation of the circle which is the reflection of C about the y-axis. [2]

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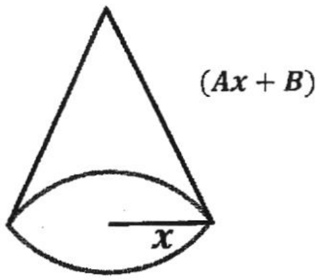
11 (a) **Answer the whole of this question on a piece of graph paper.**

The diagram, below, is the shape of a solid cone, with radius x cm and slant height $(Ax + B)$ cm, where A and B are constants. It has a total surface area of $y\,\text{cm}^2$.

(Formula : Curved surface area of cone = $\pi r l$)



Values of x and y are given below.

x	0.5	1.0	1.5	2.0	2.5	3.0
y	2.36	7.85	16.49	28.27	43.20	61.26

(i) Show that $\dfrac{y}{x} = \pi(A+1)x + \pi B$. [2]

(ii) On the graph paper, plot $\dfrac{y}{x}$ against x and draw a straight line graph. [2]

Use your graph to estimate,

(iii) the value of A and of B, [3]

(iv) the base radius for which $y = 10x$ [1]

(b) The roots of the quadratic equation $x^2 - 4x + 5 = 0$ are α and β. Find

(i) the sum of the roots, [1]

(ii) the product of the roots. [1]

(iii) Hence, find the quadratic equation whose roots are $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$. [3]

End of Paper

SA2

SHUQUN SECONDARY SCHOOL

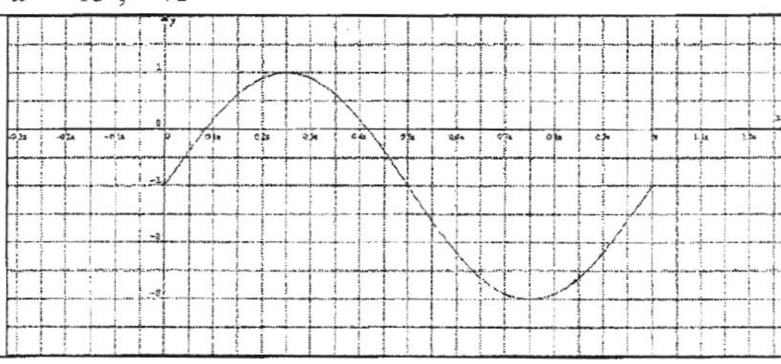
MATHEMATICS DEPARTMENT

SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS

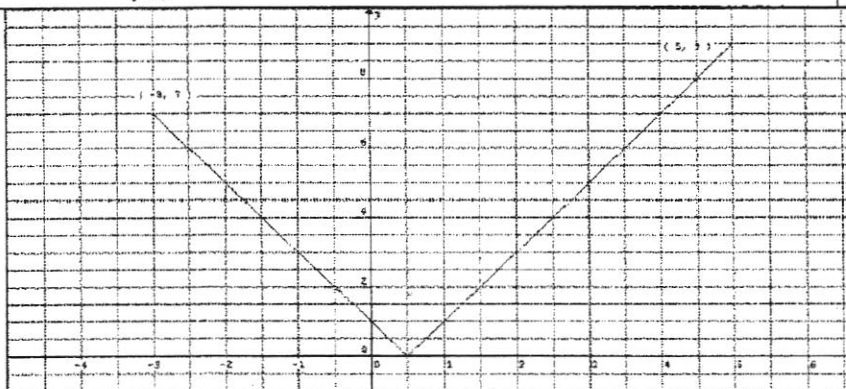
END-OF-YEAR Examination 2017

Marking scheme

1 (a)	$(3x-1)(2x+3)+10x>0$ $6x^2+7x-3+10x>0$ $6x^2+17x-3>0$ $(6x-1)(x+3)>0$ $x<-3$ or $x>\frac{1}{6}$	 M1 A2
1(b)	$\frac{5\sqrt{2}+\sqrt{14}}{3\sqrt{2}-\sqrt{14}}=\frac{5\sqrt{2}+\sqrt{14}}{3\sqrt{2}-\sqrt{14}}\times\frac{3\sqrt{2}+\sqrt{14}}{3\sqrt{2}+\sqrt{14}}$ $=\frac{15\times 2+3\sqrt{28}+5\sqrt{28}+14}{18-14}$ $=\frac{44+16\sqrt{7}}{4}$ $=11+4\sqrt{7}$	M1 M1 A1
		6 Marks
2(i)	Amplitude = 2	A1
(ii)	Period = 180°	A1
(iii)	$\sin 2x=\frac{1}{2}$ $2x=30°,150°$ $x=15°,\quad 75°$	A2
(iv)		Shape **1** Points on axes 1 Mark
		6 Marks

3(a)	$f(x)=x^3-6x^2+ax+b$ $f(3)=0$ $27-54+3a+b=0$ $3a+b=27$ $f(-2)=-55$ $-8-24-2a+b=-55$ $-2a+b=-23$ $a=10$ $b=-3$	 M1 M1 A1 A1
3(b)	$2\log_2(x-2)-\log_2(x+5)+3\log_2 2=\log_2 1$ $\log_2(x-2)^2-\log_2(x+5)+\log_2 2^3=\log_2 1$ $\log_2[\frac{(x-2)^2\times 8}{(x+5)}]=\log_2 1$ $\frac{8(x-2)^2}{(x+5)}=1$ $8(x^2-4x+4)=x+5$ $8x^2-33x+27=0$ $x=3$ or $\frac{9}{8}=1.125$ $x=1.125$ is rejected x = 3	M1 M1 B1 B1, only if rejected A1
		9 Marks

4(a) (i)	$V = \$\,120\,000.$	A1
(ii)	After 5 years, that is after 60 months $48000 = 120000e^{-60p}$ $e^{-60p} = \frac{48000}{120000} = \frac{2}{5}$ $-60p = \ln(\frac{2}{5})$ $p = 0.0152715$ Value after 8 years $V = 120000e^{-96p}$ $= 120000e_{-96\times0.0152715}$ $= 27699.87063$ $= \$27700$	M1 B1 A1
(iii)	$12000 = 120000e^{-px}$ $e^{-px} = \frac{1}{10}$ $x = \frac{-\ln 10}{-p} = \frac{\ln 10}{0.0152715}$ $= 150.7766$ 151 months	M1 A1
4(b)	$\frac{8x^2+3x+1}{x(2x+1)^2} = \frac{A}{x} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$ $8x^2+3x+1 = A(2x+1)^2 + Bx(2x+1) + Cx$ When $x = 0$, $A = 1$ When $x = -\frac{1}{2}$, $C = -3$ Comparing the coefficients of x^2 $8 = 4A + 2B$ $B = 2$ $\frac{8x^2+3x+1}{x(2x+1)^2} = \frac{1}{x} + \frac{2}{2x+1} - \frac{3}{(2x+1)^2}$	M1 M1 M1 M1
		10 Marks

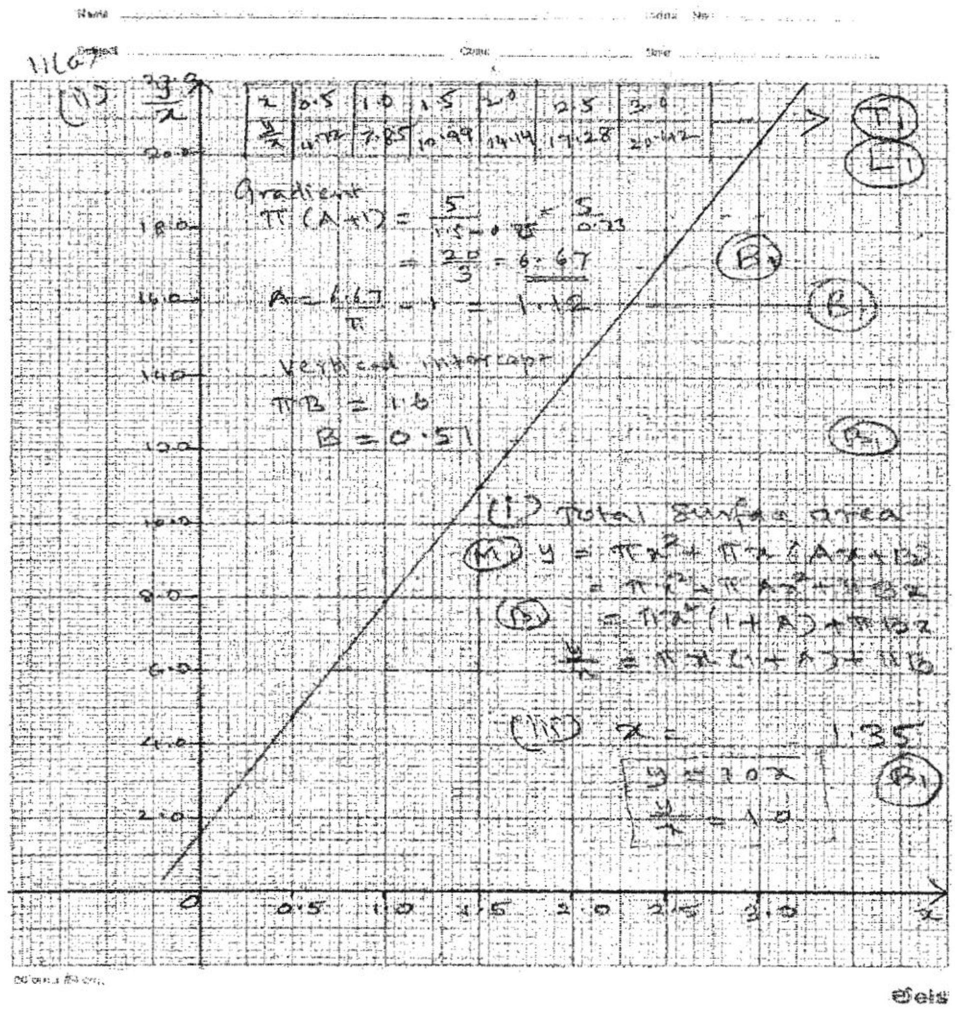
5 (a)(i)	$\tan(90^\circ - A) = \frac{1}{8}$	A1
(ii)	Angle B lies in the fourth quadrant Vertical or opposite side = -4 $\sin B = -\frac{4}{\sqrt{65}}$	 M1 A1
(b)		Shape 1 marks y-intercept 1 Mark Point on x-axis 1 Mark
(c)	$3x - 4 = \pm 5x$ $3x - 4 = -5x$ $x = \frac{1}{2}$ $3x - 4 = 5x$ $x = -2$ Re *jected*	M1 A1 A1 Only if it is rejected
		9 Marks
6(a) (i)	$\left(2 - \frac{1}{x}\right)^6 = 2^6 + \binom{6}{1}2^5\left[-\frac{1}{x}\right] + \binom{6}{2}2^4\left[-\frac{1}{x}\right]^2 + \binom{6}{3}2^3\left[-\frac{1}{x}\right]^3 + \ldots$ $64 - 6\times 32\times\frac{1}{x} + 15\times 16\times\frac{1}{x^2} - 20\times 8\times\frac{1}{x^3} + \ldots$ $= 64 - \frac{192}{x} + \frac{240}{x^2} - \frac{160}{x^3} + \ldots$	M1 B1
(ii)	$(1 - 2x + x^2)\left(2 - \frac{1}{x}\right)^6 = (1 - 2x + x^2)(64 - \frac{192}{x} + \frac{240}{x^2} - \frac{160}{x^3} + \ldots$ Coefficient of x^{-1} $= 1(-192) - 2(240) + 1(-160)$ $= -832$	M1 M1 A1

(b)	$\left(\frac{a}{x}+x\right)^{20}$ *Generalterm* $T_{r+1}=\binom{20}{r}\left(\frac{a}{x}\right)^{20-r}x^{r}$ $=\binom{20}{r}\frac{a^{20-r}}{x^{20-r}}\times x^{r}$ $=\binom{20}{r}a^{20-r}x^{2r-20}$ For the term independednt of x $2r-20=0$ $r=10$ Indoendent term $T_{11}=\binom{20}{10}a^{10}$ Or $184756a^{10}$	 M1 B1 B1
		8 Marks
7(i)	Mid point of $AB=\left(\frac{1+9}{2},\frac{-3+9}{2}\right)=(5,3)$ Gradient of $AB=\frac{9+3}{9-1}=\frac{3}{2}$ Gradient of the perpendicular $=-\frac{2}{3}$ $y=-\frac{2}{3}x+C$ It passes through (5 , 3) So $C=\frac{19}{3}$ Equation of the perpendicular bisector $y=-\frac{2}{3}x+\frac{19}{3}$ Or $3y+2x=19$	M1 M1 B1 B1
(ii)	To find the coordinates of C, solve simultaneously $y=8x-11$ *and* $y=-\frac{2}{3}x+\frac{19}{3}$ So $x=2$ *and* $y=5$ $C(2,5)$	M1 B1

(iii)	$ACBD$ is a rhombus Diagonals bisect each other If $D\ (a, b)$, then $\left(\frac{a+2}{2}, \frac{b+5}{2}\right) = (5, 3)$ $a = 8$ and $b = 1$ $D\ (8, 1)$	M1 B1
(iv)	$AB = \sqrt{(9-1)^2 + (9+3)^2} = \sqrt{208} = 2\sqrt{52}$ $CD = \sqrt{(8-2)^2 + (1-5)^2} = \sqrt{52}$ $AB = 2CD$	M1 B1
		10 Marks
8(a)	$3^x \times 3^{\frac{1}{2}y} = 3^4$ $x + \frac{1}{2}y = 4$ ------------(1) $\log_2 8 = 3\log_2 2 + \log_2(4x - y)$ $8 = 8(4x - y)$ $4x - y = 1$,,,,,,,,,,,,,,,,,,,,,,,,,,,,(2) *Solving* $x = 1.5$ $y = 5$	M1 M1 M1 A2
(b)	$\log_3(9a) - \frac{\log_3(9b)}{\log_3 9} = 2\log_3 3$ $\log_3(9a) - \frac{1}{2}\log_3(9b) = \log_3 9$ $\log_3(\frac{9a}{\sqrt{9b}}) = \log_2 9$ $\frac{9a}{\sqrt{9b}} = 9$ $a = \sqrt{9b}$ $a^2 = 9b$ $b = \frac{1}{9}a^2$	M1 M1 M1 B1

(c)	$x^2+3x+2m=-mx-6$ $x^2+(3+m)x+(2m+6)=0$ $(3+m)^2-4(1)(2m+6)=0$ $m^2-2m-15=0$ $(m-5)(m+3)=0$ $m=5$ or -3	M1 B1 A2
		13 Marks
9(a)	$3^{2x}\times 3^2-2(3^x\times 3)+1=0$ $9(3^x)^2-6(3^x)+1=0$ $9t^2-6t+1=0$ $(3t-1)^2=0$ $t=\frac{1}{3}$ $3^3=3^{-1}$ $x=-1$	M1 M1 M1 B1 A1
(b)	For the function to be positive $8^2-4(k)(3)<0$ $64<12k$ $k>\frac{64}{12}=5\frac{1}{3}$ Smallest positiveinteger $=6$	B1 B1 B1
		8 Marks
10(i)	Centre (3, 5) Radius = 13 units	A1 A1
(ii)	$x^2-6x-135=0$ $(x-15)(x+9)=0$ $x=15$ or -9 $A(-9,0)$ $B(15.0)$ $AB=24$	 B1 B1 A1
(iii)	$\left(\frac{a-9}{2},\frac{b+0}{2}\right)=(3,5)$ $a=15$ $b=10$ $D(15,10)$	 A1

(iv)	Cemtre (-3 , 5) $(x+3)^2+(y-5)^2=169$	B1 B1
		8 Marks
11(a)		8 Marks



11(b) (i)	$x^2 - 4x + 5 = 0$ $\alpha + \beta = 4$	A1
(ii)	$\alpha\beta = 5$	A1
(iii)	Sum of the roots = $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{16-10}{25} = \frac{6}{25}$ Product of the roots = $\frac{1}{\alpha^2} \times \frac{1}{\beta^2} = \frac{1}{25}$ Equation $x^2 - x(\frac{6}{25}) + \frac{1}{25} = 0$ $25x^2 - 6x + 1 = 0$	M1 M1 B1
		13 Marks