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GAN ENG SENG SCHOOL
End-Of-Year Examination 2017

CANDIDATE NAME			
CLASS		**INDEX NUMBER**	

ADDITIONAL MATHEMATICS
Paper 1

4047/01

6 Oct 2017
2 hours 30 minutes

Sec 3 Express

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

	For Examiner's Use
Total	100

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer ALL Questions

1 The graph of the curve $y = p(4^{2x}) - q(4^x)$ passes through the points $(0, 2)$ and $(0.5, 14)$. Find the value of p and of q. [2]

2 **(a)** The value, V dollars, of a car aged t years is given by $V = 12000e^{-0.2t}$.

(i) Write down the value of the car when it was new. [1]

(ii) Find the time it takes for the value to decrease to $\frac{2}{3}$ of the value when it was new. [2]

(b) **(i)** Solve the equation $4e^{-x} = 3e^x + 4$. [3]

(ii) Explain why the equation is satisfied by only one value of x. [1]

3 **(a)** The line $y = kx - 5$, where k is a positive constant, is a tangent to the curve $y = x^2 + 4x$ at the point A. Find the exact value of k. [3]

(b) Show that the roots of $px^2 + (p - q)x - q = 0$ are real for all real values of p and q. [4]

4 The acute angle x radians is such that $\sin x = k$, where k is a positive constant. Express, in terms of k,

(i) $\cos(\pi - x)$, [2]

(ii) $\sin\left(\frac{1}{2}\pi - x\right)$, [1]

(iii) $\tan x$. [1]

5 Express $\frac{4x^2 + 5x + 3}{2x^2 + 5x + 2}$ in partial fractions. [5]

6 **(i)** On the same axes, sketch the curves $y = x^{\frac{2}{3}} - 1$ and $y = x^{\frac{1}{3}} + 1$ for $x \geq 0$. [2]

(ii) Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

7 **Do not use a calculator in this question.**

(a) Show that $\sqrt{24}\times\sqrt{27}+\dfrac{9\sqrt{30}}{\sqrt{15}}$ can be written in the form $a\sqrt{2}$, where a is an integer. [3]

(b) Solve the equation $\sqrt{3}(1+x)=2(x-3)$, giving your answer in the form $b+c\sqrt{3}$, where b and c are integers. [3]

8 **(a)** **(i)** Expand $(a+b)^4$, giving each term in its simplest form. [2]

(ii) Hence, find the term independent of x in the expansion of $\left(2x+\dfrac{1}{5x}\right)^4$. [2]

(b) The coefficient of x^3 in the expansion of $\left(1+\dfrac{x}{2}\right)^n$ equals $\dfrac{5n}{12}$. Find the value of the positive integer n. [3]

9 **(a)** Solve the equation $3^{x+4}=5^{2x}$. [3]

(b) It is given that $y=\log_a ax+2\log_a(4x-3)-1$, where a is a positive integer.

(i) Explain why x must be greater than 0.75. [1]

(ii) Show that y can be written as $\log_a(16x^3-24x^2+9x)$. [2]

(iii) Find the value of x for which $y=\log_a(9x)$. [2]

10 It is given that $\mathrm{P}(x)=2x^3+ax^2+4x+b$, where a and b are constants. It is given also that $2x+1$ is a factor of $\mathrm{P}(x)$ and that when $\mathrm{P}(x)$ is divided by $x-1$ there is a remainder of -12 .

(i) Find the value of a and of b. [4]

(ii) Using your values of a and b, write $\mathrm{P}(x)$ in the form $(2x+1)\mathrm{Q}(x)$, where $\mathrm{Q}(x)$ is a quadratic expression. [2]

(iii) Hence find the exact solutions of the equation $\mathrm{P}(x)=0$. [2]

11 The quadratic equation $3x^2-6x+2=0$ has roots α and β.

(i) Write down the numerical values of $\alpha+\beta$ and $\alpha\beta$. [2]

(ii) Find the value of $\alpha^3+\beta^3$. [3]

(iii) Find a quadratic equation with roots α^3 and β^3. [2]

12 The table below shows values of the variables x and y which are related by the equation $y=\dfrac{a}{x+b}$, where a and b are constants.

x	0.1	0.4	1.0	2.0	3.0
y	8.0	6.0	4.0	2.6	1.9

(i) Using graph paper, plot xy against y and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

An alternative method for obtaining a straight line graph for the equation $y=\dfrac{a}{x+b}$ is to plot x against $\dfrac{1}{y}$.

(iii) Without drawing a second graph, use your values of a and b to estimate the gradient and the intercept on the vertical axis of the graph of x plotted against $\dfrac{1}{y}$. [3]

13 (a) A circle with centre C has equation $x^2+y^2+6x-4y=12$.

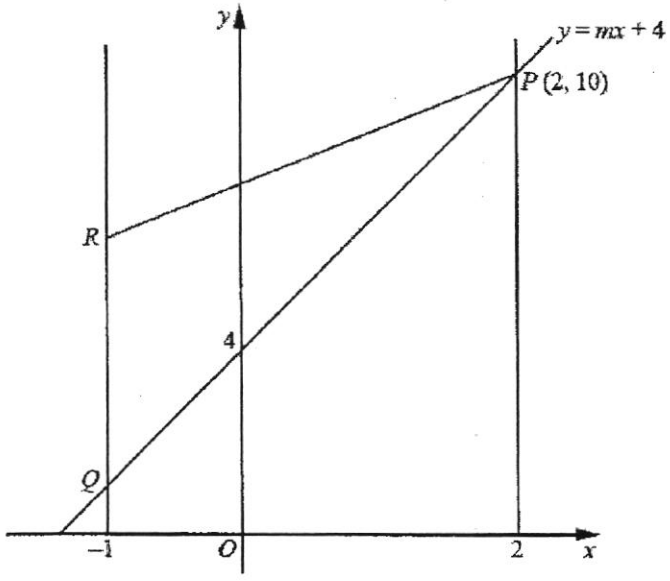
(i) Find the coordinates of C. [2]

(ii) Find the radius of the circle. [1]

(iii) Show that the point $N(0,-2)$ lies on the circle. [1]

(b) Another circle touches the line $y=x$ at the point $A(3,3)$ and passes through the point $B(5,9)$. Find the equation of the circle. [8]

14 Solutions by accurate drawing will not be accepted.



The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and Q respectively. The point R is such that QR is parallel to the y-axis and the gradient of RP is 1. The point P has coordinates (2, 10).

(i) Find the value of m. [1]

(ii) Find the y-coordinate of Q. [1]

(iii) Find the coordinates of R. [2]

(iv) Find the equation of the line through P, perpendicular to PQ, giving your answer in the form $ax + by = c$, where a, b and c are integers. [3]

(v) Find the coordinates of the midpoint, M, of the line PQ. [2]

(vi) Find the area of triangle QRM. [2]

~ End of Paper ~