SM

Name : | Class | Number

DUNEARN SECONDARY SCHOOL
End-of-Year Examination 2017 (4047)
Additional Mathematics

Secondary 3 Express

Monday **9th Oct 2017** **1030 – 1300** **2hr 30 mins**

INSTRUCTIONS TO CANDIDATES

Write your name, class and register number in the spaces at the top of this page.
Answer **all** questions.
Write your answers on the writing paper provided.
All working must be shown. Omission of essential working will result in loss of marks.
Do not use any highlighters, correction fluid or correction tape for the paper.

INFORMATION FOR CANDIDATES

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.
The use of an electronic calculator is expected where appropriate.
You are reminded of the need for clear presentation in your answers.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

PARENT'S SIGNATURE

FOR EXAMINER'S USE
100

Setter: **Ms Charmaine Goh**

This question paper consists of 6 printed pages, including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

1 Express $\dfrac{3x^2+x+2}{(2x+1)(x-1)^2}$ in partial fractions. [4]

2 Given that $\dfrac{5^x}{2^{3-y}} \times \dfrac{2^y}{(5^{x+1})^2} = 5(2)^6$, find the value of x and y. [4]

3 **(a)** Find the exact value of $\sin\dfrac{\pi}{6}\tan\dfrac{\pi}{4} - \cos\dfrac{\pi}{4}\tan\dfrac{\pi}{3}$, without using a calculator. [2]

(b) Solve $\tan\left(2x - \dfrac{\pi}{3}\right) = 0.4$ for which $0 \le x \le 2\pi$. [3]

4 The function $f(x) = x^3 + ax^2 + 5x + b$, where a and b are constants, has a factor $x - 2$. When $f(x)$ is divided by $x + 2$, it leaves a remainder that is four times the remainder when $f(x)$ is divided by $x + 1$.

(i) Find the value of a and of b. [4]

(ii) Hence solve the equation $f(x) = 0$, giving each root correct to 2 decimal places where appropriate. [2]

5 A curve has the equation $9x^2 + a(y-1)^2 = 36$, where a is a constant. This curve passes through the point (0, 4) and intersects the line $y = \dfrac{1}{2}bx - 2$, where b is a constant, at the points $M(-2, 1)$ and N.

(i) Show that $a + b = 1$. [3]

(ii) Find the coordinates of N. [3]

<u>Start this question on a brand new page.</u>

6 **(i)** Sketch the graph of $y = 1 + 3\sin 2x$ for the domain $0 \le x \le 2\pi$. [3]

(ii) Hence, by sketching another graph on the same diagram, show that the equation $3\sin 2x - x - 3 = 0$ has no real roots for $0 \le x \le 2\pi$. [3]

7 **(a)** Solve the equation $\log_2 y = 1 + \log_4(y+3)$. [4]

(b) Given that $\log_3 a = m$, without using a calculator, express 3^{m-2} in terms of a. [3]

8 The equation of a curve is given by $x^2 - xy + y^2 = 1$.

(i) Find the range of values of p for which the line $2x - y = p$ will meet the curve. [4]

(ii) Hence, state the positive integer value of p if the line becomes a tangent to the curve. [1]

(iii) Determine the number of point(s) of intersection of the two graphs if $p = 2\sqrt{3}$. [2]

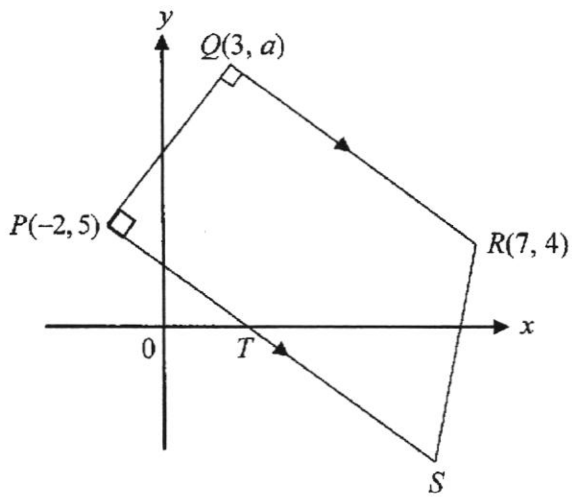
9 (i) Sketch the graph of $y = |2x^2 - 11x + 5|$ for $0 \le x \le 6$, stating clearly the x-y intercepts and turning point. [4]

(ii) Using your graph, find the range of value(s) of k for each of the number of solutions for the equation $|2x^2 - 11x + 5| = k$ for $0 \le x \le 6$.

(a) 3 solutions. [1]

(b) 2 solutions. [2]

10 **Solutions to this question by accurate drawing will not be accepted.**



The diagram (not drawn to scale) shows a trapezium *PQRS* in which *PS* is parallel to *QR* and *PQ* is perpendicular to *QR* and *PS*. The coordinates of *P*, *Q* and *R* are $(-2, 5)$, $(3, a)$ and (7, 4) respectively where $a \neq 0$. *PS* cuts the x-axis at *T*.

(i) Show that $a = 9$. [2]

(ii) Find the coordinates of *T*. [2]

Given further that *PT* : *TS* is 2 : 3, find

(iii) the coordinates of *S*, [2]

(iv) the area of the trapezium *PQRS*. [2]

11 The roots of the equation $3x^2 + 5x + 1 = 0$ are α and β while the roots of the equation $hx^2 - 4x + k = 0$ are $\alpha + 3$ and $\beta + 3$.

(i) Calculate the numerical values of h and k. [5]

(ii) Form an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

12 (a) Without using a calculator, solve the equation $x\sqrt{20} = (x-1)\sqrt{3} + \sqrt{5}$, leaving your answer in the form $\frac{a - \sqrt{b}}{17}$, where a and b are positive integers. [4]

(b) Solve the following simultaneous equations.

$$2^{x+2} + 5^{y} = 881$$
$$2^{x-1} - 5^{y-3} = 27$$ [5]

13 (a) Given that $\sin A = -\frac{3}{5}$, $\tan B = \frac{1}{2}$ and that A and B lie in the same quadrant, find the exact value of

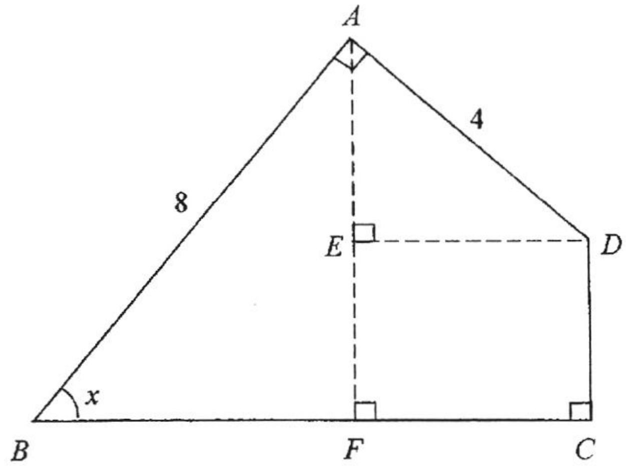
(i) $\cos(A - B)$, [2]

(ii) $\sin\left(\frac{A}{2}\right)$. [2]

(b) (i) Prove the identity $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$. [3]

(ii) Hence, solve $\frac{1+\sin(x+50°)}{\cos(x+50°)} + \frac{\cos(x+50°)}{1+\sin(x+50°)} = 7$ for $0° \le x \le 360°$. [4]

14 The diagram shows a quadrilateral $ABCD$ in which $\angle BAD$, $\angle AED$, $\angle AFC$ and $\angle BCD$ are right angles, $\angle ABC = x$, $AD = 4$ cm and $AB = 8$ cm.



(i) Show that $EF = 8\sin x - 4\cos x$. [2]

(ii) Show that the perimeter, P cm, of the quadrilateral $ABCD$ is given by $P = 4\cos x + 12\sin x + 12$. [2]

(iii) Express P in the form $R\sin(x+\alpha)+12$, where $R>0$ and $0° < \alpha < 90°$. [3]

Hence,

(iv) state the maximum value of P and the corresponding value of x, [2]

(v) find the value of x when $P = 19$. [2]

~ End of Paper ~

SA2

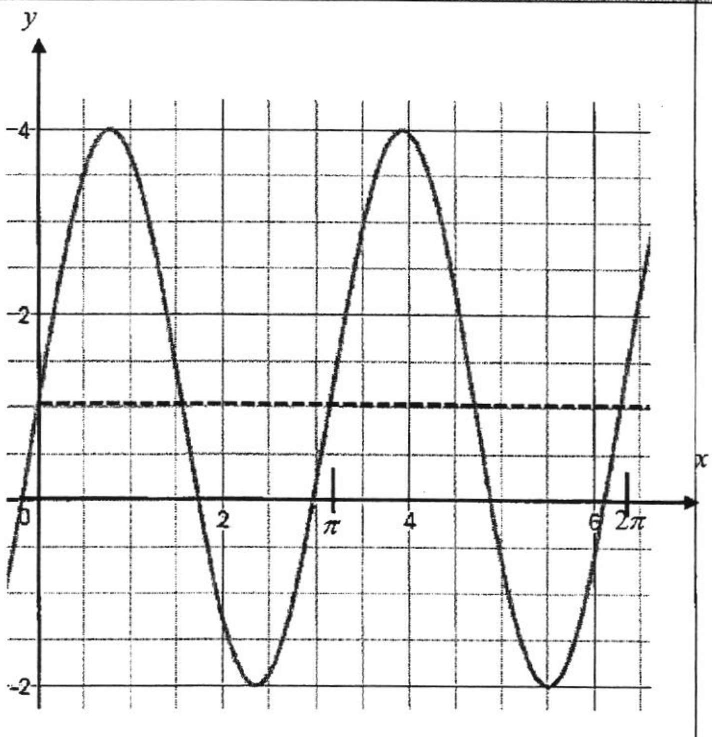
Dunearn Secondary School
Additional Mathematics for Secondary 3 Express
End-of-Year Examination 2017 Marking Scheme

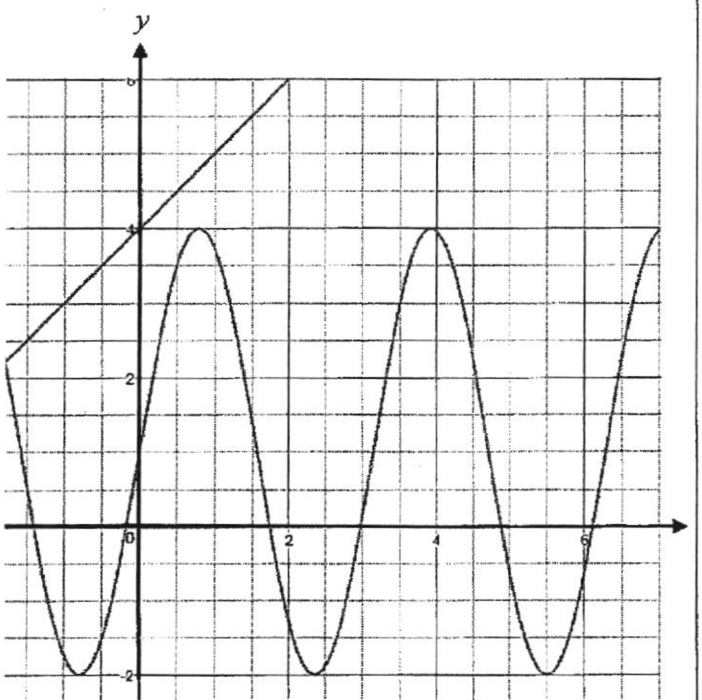
Maximum 2 marks will be deducted from overall paper for 3 s.f. error or missing/wrong units

Qns		Marking Scheme	Marks	Remarks
1		$\frac{3x^2+x+2}{(2x+1)(x-1)^2}=\frac{A}{2x+1}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$ $3x^2+x+2=A(x-1)^2+B(2x+1)(x-1)+C(2x+1)$	M1	The ability to form the correct partial fraction form.
		$Let\ x=1,$ $6=3C$ $C=2$ $Let\ x=-\frac{1}{2},$ $2.25=2.25A$ $A=1$ $Let\ x=0,$ $2=A-B+C$ $2=1-B+2$ $B=1$	M2	By substituting appropriate numbers to solve for unknowns (any mistake -1m)
		$\frac{3x^2+x+2}{(2x+1)(x-1)^2}=\frac{1}{2x+1}+\frac{1}{(x-1)}+\frac{2}{(x-1)^2}$	A1	
Qn1		Subtotal	4	

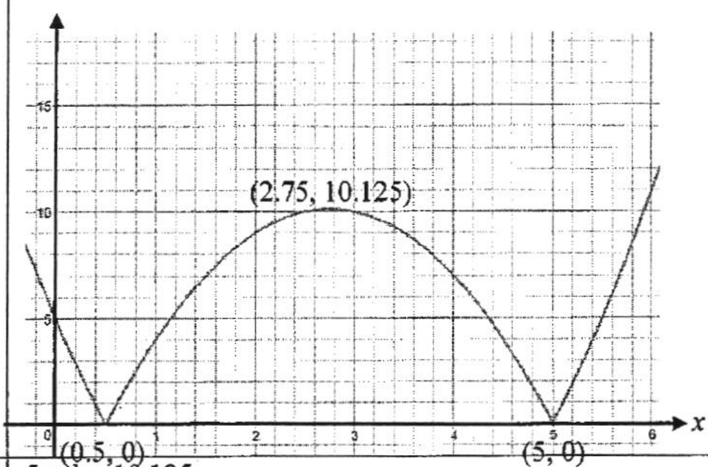
2		$\frac{5^x}{2^{3-y}} \times \frac{2^y}{(5^{x+1})^2} = 5(2)^6$ $(5^{x-(2x+2)})(2^{y-(3-y)}) = 5(2)^6$ $(5^{-x-2})(2^{2y-3}) = 5(2)^6$ *By comparison,* $-x-2=1$ $x=-3$ $2y-3=6$ $y=\frac{9}{2}$	 M1 M1 A1 A1	
Qn2		Subtotal	4	
3	(a)	$\sin\frac{\pi}{6}\tan\frac{\pi}{4} - \cos\frac{\pi}{4}\tan\frac{\pi}{3}$ $=\left(\frac{1}{2}\right)(1)-\left(\frac{\sqrt{2}}{2}\right)(\sqrt{3})$ $=\frac{1}{2}-\frac{\sqrt{6}}{2}$ $=\frac{1-\sqrt{6}}{2}$	 M1 A1	
	(b)	$0 \le x \le 2\pi$ $-\frac{\pi}{3} \le 2x-\frac{\pi}{3} \le \frac{11\pi}{3}$ $\tan(2x-\frac{\pi}{3})=0.4$ $\alpha = 0.38050$ $2x-\frac{\pi}{3} = 0.38050, 3.52209, 6.66368, 9.80527$ $x = 0.714rad, 2.28rad, 3.86rad, 5.43rad$	 M1 M1 A1	 For finding basic angle For finding all angles for $2x-\frac{\pi}{3}$
Qn3		Subtotal	5	

4	(i)	$f(x) = x^3 + ax^2 + 5x + b$ $f(2) = (2)^3 + a(2)^2 + 5(2) + b = 0$ $4a + b + 18 = 0 \quad ----(1)$ $f(-2) = 4f(-1)$ $(-2)^3 + a(-2)^2 + 5(-2) + b = 4[(-1)^3 + a(-1)^2 + 5(-1) + b]$ $4a + b - 18 = 4(a + b - 6)$ $4a + b - 18 = 4a + 4b - 24$ $3b = 6$ $b = 2$ *sub into* (1): $4a = -20$ $a = -5$	M1 M1 A1 A1	
	(ii)	$f(x) = x^3 - 5x^2 + 5x + 2$ $= (x-2)(x^2 - 3x - 1)$ $(x-2)(x^2 - 3x - 1) = 0$ $x = 2,\ 3.30,\ -0.30$	M1 (ecf) B1	The ability to factorise cubic equation. No M1 rewarded if students do not show workings for factorization. B1 for all answers
Qn4		Subtotal	6	
5	(i)	$9x^2 + a(y-1)^2 = 36$ *when* $x = 0$ *and* $y = 4$, $9a = 36$ $a = 4$ *sub* $(-2,1)$ *into* $y = \frac{1}{2}bx - 2$: $1 = \frac{1}{2}b(-2) - 2$ $3 = -b$ $b = -3$ $a + b = 4 + (-3) = 1$ (*shown*)	M1 M1 A1	Correct substitution Correct substitution

	(ii)	Sub $y=-\frac{3}{2}x-2$ into $9x^2+4(y-1)^2=36$ $9x^2+4(-\frac{3}{2}x-2-1)^2=36$ $9x^2+4(-\frac{3}{2}x-3)^2=36$ $9x^2+4(\frac{9}{4}x^2+9x+9)=36$ $9x^2+9x^2+36x=0$ $18x^2+36x=0$ $18x(x+2)=0$ $x=0\ or\ -2$ $y=-2$ $N(0,-2)$	M1 (ecf) M1 A1	Correct substitution Correct factorisation
Qn5		**Subtotal**	**6**	
6	(i)		D1 D1 D1	Correct amplitude Correct period Correct waveform
	(ii)	$3\sin 2x-x-3=0$ $3\sin 2x+1=x+4$ $y=x+4$	M1	ecf for below

			D1 (ecf)	$y = x + 4$ drawn ecf allowed if students managed to draw the line shown above.
		The line does not intersect the curve thus $3\sin 2x - x - 3 = 0$ has no real roots.	B1	
Qn6		**Subtotal**	**6**	
7	(a)	$\log_2 y = 1 + \log_4(y+3)$		
		$\log_2 y = \log_2 2 + \dfrac{\log_2(y+3)}{2}$	M1	Change of base
		$\log_2 y^2 = \log_2 2^2 + \log_2(y+3)$	M1	multiply by 2
		$y^2 = 4(y+3)$		
		$y^2 - 4y - 12 = 0$		
		$(y-6)(y+2) = 0$	M1	Factorisation
		$y = 6 \quad or \quad -2\,(rejected)$	A1	For both answers
	(b)	$\log_3 a = m$		
		$3^m = a$	M1	
		$3^{m-2} = \dfrac{3^m}{3^2}$	M1	
		$= \dfrac{a}{9}$	A1	

Qn7		**Subtotal**	**7**	
8	(i)	$2x - y = p$ $y = 2x - p \quad -----(1)$ $x^2 - x(2x-p) + (2x-p)^2 = 1$ $x^2 - 2x^2 + px + 4x^2 - 4px + p^2 - 1 = 0$ $3x^2 - 3px + p^2 - 1 = 0$ $Discriminant \geq 0$ $(-3p)^2 - 4(3)(p^2-1) \geq 0$ $9p^2 - 12p^2 + 12 \geq 0$ $-3p^2 + 12 \geq 0$ $p^2 - 4 \leq 0$ $(p+2)(p-2) \leq 0$ $-2 \leq p \leq 2$	M1 M1 M1 A1	For substituting line into curve Discriminant Solve inequality (independent of sign of discriminant)
	(ii)	For tangent, discriminant =0 $p^2 - 4 = 0$ $(p+2)(p-2) = 0$ $p = 2 \text{ or } -2(rejected)$	B1	
	(iii)	$Discriminant = -3(p^2 - 4)$ $= -3[(2\sqrt{3})^2 - 4]$ $= -24 < 0$ There is no intersection. Alternative answer Since $p = 2\sqrt{3} > 2$, using (a) answer, there is no intersection.	M1 (ecf) A1 M1 A1	For showing $D<0$ Conclusion for stating $p > 2$
Qn8		**Subtotal**	**7**	

9	(i)		D1 D1 D1 D1	Modulus graph ('W' shape) Correct x intercepts Correct y intercept Correct maximum point
	(iia)	$5 < k < 10.125$	B1	
	(iib)	$k = 0$ *or* $k = 10.125$	B2	B1 for each answer
Qn9		**Subtotal**	**7**	
10	(i)	Since PQ and QR is perpendicular to each other, $\frac{a-4}{3-7} \times \frac{a-5}{3+2} = -1$ $\frac{a^2-9a+20}{-20} = -1$ $a^2 - 9a = 0$ $a(a-9) = 0$ $a = 0$ (NA) or $a = 9$ (shown)	M1 A1	for equating the product of both gradient = – 1
	(ii)	Gradient of $PS = QR = -\frac{5}{4}$ $\frac{5-0}{-2-x} = -\frac{5}{4}$ $-5(-2-x) = 20$ $10 + 5x = 20$ $x = 2$ Coordinate of $T = (2, 0)$	M1 A1	equating gradient (accept if students find equation of PS)
	(iii)	Horizontal distance of PS $= \frac{3}{2} \times (2-(-2)) = 6$ x-coordinates of S is $2 + 6 = 8$ Vertical distance of PS $= \frac{3}{2} \times 5 = 7.5$	B1 (ecf)	

		y-coordinates of S is $0-7.5=-7.5$ $\therefore$ The coordinates of S are $(8,-7.5)$.	B1 (ecf)	
	(iv)	Area of trapezium $PQRS$ $=\frac{1}{2}\begin{vmatrix}3 & 7 & 8 & -2 & 3\\ 9 & 4 & -7.5 & 5 & 9\end{vmatrix}$ $=\frac{1}{2}\lvert(12-52.5+40-18)-(63+32+15+15)\rvert$ $=71\frac{3}{4}$ units^2	M1 A1	Shoelace method / trapezium formula method e.c.f. based on their coordinates of S.
Qn10		Subtotal	8	
11	(i)	$3x^2+5x+1=0$ $\alpha+\beta=-\frac{5}{3}$ ----(1) $\alpha\beta=\frac{1}{3}$ ----(2)	M1	Correct sum and product of roots for both
		$hx^2-4x+k=0$ $(\alpha+3)+(\beta+3)=\frac{4}{h}$ $\alpha+\beta=\frac{4}{h}-6$ ----(3) $(\alpha+3)(\beta+3)=\frac{k}{h}$	M1	Correct sum and product of roots for both
		$\alpha\beta+3\alpha+3\beta+9=\frac{k}{h}$ $\frac{1}{3}+3(-\frac{5}{3})+9=\frac{k}{h}$ $\frac{13}{3}=\frac{k}{h}$ ----(4)	M1	For substituting values from (1) and (2)
		Sub (1) into (3): $\frac{4}{h}-6=-\frac{5}{3}$ $h=\frac{12}{13}$	A1	
		From (4):		

		$\frac{13}{3}=\frac{k}{\frac{12}{13}}$ $k=4$	A1	
	(ii)	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$		
		$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	M1	for expressing $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
		$=\frac{(-\frac{5}{3})^2-2(\frac{1}{3})}{\frac{1}{3}}$		
		$=\frac{19}{3}$	M1	Correct value for sum of roots
		$\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=\frac{\alpha\beta}{\alpha\beta}=1$	M1	Correct value for product of roots
		$x^2-\frac{19}{3}x+1=0$	A1	
Qn11		**Subtotal**	**9**	
12	(a)	$x\sqrt{20}=(x-1)\sqrt{3}+\sqrt{5}$ $x\sqrt{20}=x\sqrt{3}-\sqrt{3}+\sqrt{5}$		
		$2x\sqrt{5}-x\sqrt{3}=-\sqrt{3}+\sqrt{5}$	M1	grouping of subject
		$x(2\sqrt{5}-\sqrt{3})=-\sqrt{3}+\sqrt{5}$		
		$x=\frac{-\sqrt{3}+\sqrt{5}}{2\sqrt{5}-\sqrt{3}}\times\frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$	M1	rationalising
		$=\frac{-2\sqrt{15}-3+10+\sqrt{15}}{17}$	M1	
		$=\frac{7-\sqrt{15}}{17}$	A1	
	(b)	$2^{x+2}+5^y=881 \quad ---(1)$ $2^{x-1}-5^{y-3}=27 \quad ---(2)$		
		Let $a=2^x$ and $b=5^y$, from (1): $4(2^x)+5^y=881$ $4a+b=881 \quad ---(3)$	M1	Correct substitution

		from (2): $\frac{1}{2}(2^x)-\frac{1}{125}(5^y)=27$ $125a-2b=6750\quad ---(4)$	M1	correct (3) and (4)
		$(4)+2(3)$: $125a-2b+8a+2b=8512$ $133a=8512$ $a=64$ Sub (3): $4(64)+b=881$ $b=625$	M1	correct simultaneous method
		$2^x=64$ $x=6$	A1	
		$5^y=625$ $y=4$	A1	
Qn12		Subtotal	9	
13	(ai)	$\cos(A-B)=\cos A\cos B+\sin A\sin B$ $=\left(-\frac{4}{5}\right)\left(-\frac{2}{\sqrt{5}}\right)+\left(-\frac{3}{5}\right)\left(-\frac{1}{\sqrt{5}}\right)$	M1	
		$=\frac{8}{5\sqrt{5}}+\frac{3}{5\sqrt{5}}$ $=\frac{11}{5\sqrt{5}}$	A1	
	(aii)	$\cos A=1-2\sin^2\left(\frac{A}{2}\right)$	M1	Using double angle formula
		$\sin\frac{A}{2}=\sqrt{\frac{1-\cos A}{2}}$ $=\sqrt{\frac{1-(-\frac{4}{5})}{2}}$ $=\frac{3}{\sqrt{10}}$	A1	Since $\frac{A}{2}$ is in second quadrant

	(bi)	$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$ $LHS: \frac{(1+\sin x)^2+\cos^2 x}{\cos x(1+\sin x)}=\frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$ $=\frac{2(1+\sin x)}{\cos(1+\sin x)}$ $=\frac{2}{\cos x}$ $=2\sec x$ *(proven)*	M1 M1 A1	combining fractions and expansion Using $\sin^2 x+\cos^2 x=1$
	(bii)	$\frac{1+\sin(x+50°)}{\cos(x+50°)}+\frac{\cos(x+50°)}{1+\sin(x+50°)}=7$ $2\sec(x+50°)=7$ $\cos(x+50°)=\frac{2}{7}$ $\alpha=73.398$ $(x+50)=73.398,\ 286.601$ $x=23.4°,\ 236.6°$	M1 M1 M1 A1	Using identity in (i) change to cosine For finding basic angle Both answers
Qn13		**Subtotal**	**11**	
14	(i)	$\angle EAD=x$ $EF=AF-AE$ $=8\sin x-4\cos x$ *(shown)*	M1 A1	Working for finding *AF* and *AE* must be seen for M1 to be awarded.
	(ii)	$BF=8\cos x$ $FC=4\sin x$ $P=12+8\sin x-4\cos x+8\cos x+4\sin x$ $=4\cos x+12\sin x+12$ *(shown)*	M1 A1	for both BF and FC
	(iii)	$R=\sqrt{12^2+4^2}=\sqrt{160}=4\sqrt{10}=12.6$ $\alpha=\tan^{-1}\frac{4}{12}=18.434$ $P=4\sqrt{10}\sin(\theta+18.4°)+12$	M1 M1 A1	accept $\sqrt{160}$ and 12.6
	(iv)	Max $P=4\sqrt{10}+12=24.6$ $(\theta+18.4°)=90$ $\theta=71.6°$	M1 A1	

	(v)	$4\sqrt{10}\sin(\theta+18.434°)+12=19$ $\sin(\theta+18.434)=0.55339$ $\alpha=33.600$ $\theta=15.2°$	M1 A1	
Qn14		Subtotal	11	