1 Find the coordinates of the midpoint of the straight line joining the points of intersection of the curve $2x^2 + x = 2y - 3y^2 + 8$ and the line $y = x - 2$. [5]

2 The area of a rectangle is $\left(3\sqrt{15} - 11\right)$ cm^2.
Given that its length is $\left(2\sqrt{3} - \sqrt{5}\right)$ cm, find the perimeter of the rectangle in the form $a\sqrt{3}$ cm, where a is an integer. [4]

3 **(a)** The function $f(x) = ax^3 + bx^2 - 11x + c$, where a, b and c are constants, is exactly divisible by $x - 3$. Given that $f(x)$ leaves a remainder of -6 and 6 when divided by $(1 - x)$ and x respectively. Find the values of a, b and c. [4]

(b) **(i)** Solve the cubic equation $3x^3 - x^2 = 8x + 4$. [4]

(ii) Hence solve the equation $3(y+1)^3 - (y+1)^2 = 8y + 12$. [3]

4 **(i)** Find the coordinates of all the points at which the graph of $y = |2x - 5| - 3$ meets the coordinates axes. [4]

(ii) Sketch the graph of $y = |2x - 5| - 3$. [2]

(iii) Calculate the coordinates of the point of intersection of the graph of $y = |2x - 5| - 3$ and the line $y = 2x + 2$. [2]

(iv) State the number of solutions to the equation $\dfrac{1}{\sqrt[3]{x^2}} = |2x - 5| - 3$ where $x > 0$.

[1]

5 **(a)** Solve the inequality $2(x - 1) \leq 6x^2 - 30$. [3]

(b) A curve has the equation $y = 4x^2 - 2kx + k$, where k is a constant.
Find the range of values of k for which the curve lies completely above the x-axis. [4]

6 Express $\frac{x-1}{(x-2)(x^2+1)}$ in partial fractions. [5]

7 The function f is defined by $f(x)=1+3\cos 2x$.

(i) State the amplitude of f. [1]

(ii) State the period of f. [1]

The equation of a curve is $y=1+3\cos 2x$ for $0° \le x \le 360°$.

(iii) Find the coordinates of the minimum point of the curve. [1]

(iv) Find all the values of x such that $y=0$. [2]

(v) Sketch the graph of $y=1+3\cos 2x$. [2]

(vi) On the diagram drawn in part (v), sketch the graph of $y=-2\sin x$ for $0° \le x \le 360°$. [1]

(vii) State the number of solutions, for $0° \le x \le 360°$, of the equation $3\cos 2x+2\sin x=-1$. [1]

Start Question 8 on a fresh sheet of Answer Paper.

Hand in Question 8 to Question 13 separately from Question 1 to Question 7.

8 The roots of the quadratic equation $10x^2 + 6x - 4 = 0$ are α and β.

(i) Find the values of $\alpha + \beta$ and of $\alpha\beta$. [2]

(ii) Find the quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [4]

9 Express $3 + 3^x = 3^{3+x} + 3^{2x+2}$ as a quadratic equation in 3^x and hence find the value of x which satisfies the equation $3 + 3^x = 3^{3+x} + 3^{2x+2}$. [4]

10 (a) Prove that $\frac{\cos 2x}{\sin 2x + 1} = \frac{\cos x - \sin x}{\cos x + \sin x}$. [4]

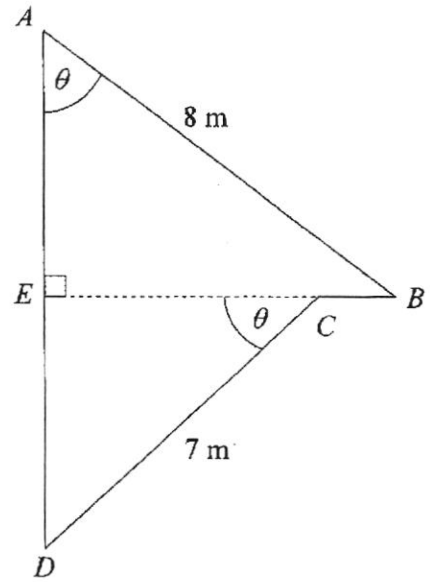
(b) Given that $\cos A = -\frac{3}{5}$ and $\tan B = \frac{5}{12}$, where A and B are in the same quadrant, find, **without using a calculator**, the exact value of $\sin(A + B)$. [4]

(c) Using $\tan x = \operatorname{cosec} 2x - \cot 2x$, show, **without using a calculator**, that $\tan 22.5° = -1 + \sqrt{2}$. [3]

11 (a) Find all the values of x between 0° and 360° for which $2\cos^2 x = 3\cos x \sin x$. [4]

(b) Solve the equation $\cos 2\theta = 1 - \sin\theta$ for $0 \le \theta \le 2\pi$, giving your answers, in radians, correct to 2 decimal places. [4]

12 The diagram shows a quadrilateral $ABCD$ in which $AB = 8$ m and $CD = 7$ m. $\angle AEB = 90°$ and $\angle DAB = \angle ECD = \theta$, where $0° \le \theta \le 90°$.



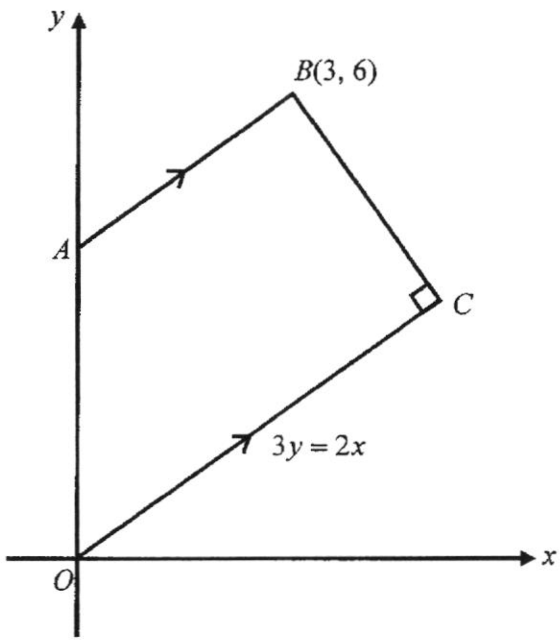
(i) Show that $AD = 8\cos\theta + 7\sin\theta$. [2]

(ii) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) State the maximum value of AD and find the corresponding value of θ. [3]

(iv) Find the values of θ for which $AD = 9.5$. [2]

13



The diagram shows a trapezium $OABC$ in which AB is parallel to OC and angle $BCO = 90°$. The side OC has equation $3y = 2x$. The point B is (3, 6). Given that the point A lies on the y-axis, find

(i) the equations of AB and of BC, [4]

(ii) the coordinates of A and of C, [3]

(iii) the area of the trapezium $OABC$. [3]

~ End of Paper ~

SA2 ANS

2017 Secondary Three Express Additional Mathematics SA2

1	Sub $y=x-2$ into $2x^2+x=2y-3y^2+8$ $2x^2+x=2(x-2)-3(x-2)^2+8$ ---M1 for substitution $5x^2-13x+8=0$ $(5x-8)(x-1)=0$ ----M1 for correct factorisation $x=\frac{8}{5}$ or $x=1$ $y=-\frac{2}{5}$ or $y=1$ $\left(\frac{8}{5},-\frac{2}{5}\right)$ ----B1 and $(1,1)$ ----B1 The coordinate of midpoint $\left(\frac{13}{10},-\frac{7}{10}\right)$ ----A1
2	Breadth $=\frac{3\sqrt{15}-11}{2\sqrt{3}-\sqrt{5}}\times\frac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$ ----M1 for rationalise with $2\sqrt{3}+\sqrt{5}$ $=\frac{18\sqrt{5}+15\sqrt{3}-22\sqrt{3}-11\sqrt{5}}{12-5}$ ---A1 numerator (2 or 4 terms) $=\frac{7\sqrt{5}-7\sqrt{3}}{7}$ $=\sqrt{5}-\sqrt{3}$ ---A1 denominator and completion soi Perimeter $=2(\sqrt{5}-\sqrt{3})+2(2\sqrt{3}-\sqrt{5})$ $=2\sqrt{3}$ cm ----A1
3 (a)	$f(x)=Ax^3+Bx^2-11x+C$ $f(1)=A+B-11+C=-6$ $A+B+C=5$ ---(1) $f(0)=6$ $C=6$ ---B1 $f(3)=27A+9B-33+6=0$ $3A+B-3=0$ ---(2) ---B1 From (1), $B=-1-A$ Sub (3) into (2), $3A+(-1-A)-3=0$ ---M1 $A=2$---A1 $B=-3$ ---A1

(b)(i)

$3x^3 - x^2 = 8x + 4$

Let $f(x) = 3x^3 - x^2 - 8x - 4$

$f(x) = (x+1)(3x^2 - 4x - 4)$ A1 correct quadratic factor soi

$f(-1) = 0$

$x+1$ is factor of $f(x)$ ---B1

$0 = (x+1)(3x+2)(x-2)$ ---M1 factorise quadratic factor and write as product of 3 linear factors

$x = -1$ or $x = -\frac{2}{3}$ or $x = 2$ ---A1

(ii)

Let $x = y+1$

Did not use part(i)

$3(y+1)^3 - (y+1)^2 - 8(y+1) - 4 = 0$

$y+1 = -1$ or $y+1 = -\frac{2}{3}$ or $y+1 = 2$ M1 for realising that $y+1$ is equivalent to x

$y = -2$ or $y = -1\frac{2}{3}$ or $y = 1$ ----A2, A1, A0

4(i)

Let $y = 0$

$2x - 5 = 3$ or $2x - 5 = -3$---M1

$x = 4$ or $x = 1$

The coordinates are $(4,0)$ and $(1,0)$ --- B1 B1

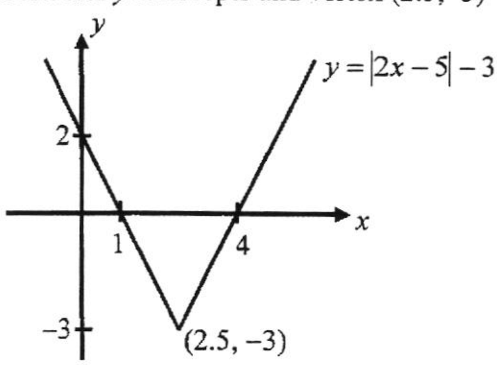
Let $x = 0$, $y = 2$

The coordinate is $(0,2)$----B1

(ii)

B1 : correct graph

B1: label x and y intercepts and vertex $(2.5,-3)$



Minimum point not labelled

(iii)

$2x + 2 = |2x - 5| - 3$---B1

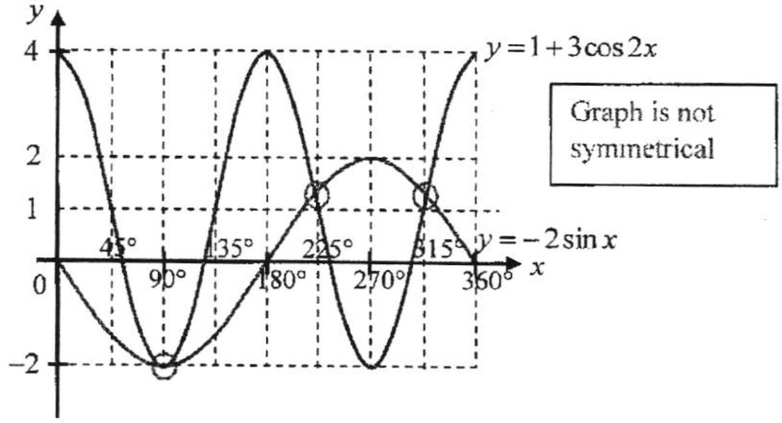
$2x - 5 = 2x + 5$ or $2x - 5 = -(2x+5)$ ---M1

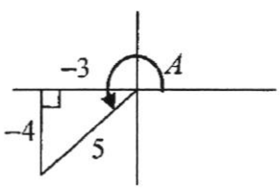
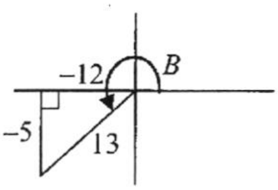
NA or $x = 0$ & $y = 2$ coordinate $(0,2)$ ---A1

(iv)

Number of solution = 1 ---B1

5(a)	$2(x-1) \le 6x^2-30$ $6x^2-2x-30+2\ge 0$ $(x+2)(3x-7)\ge 0$ ---M1 for correct factorisation Critical values, -2 and $\frac{7}{3}$ ----A1 soi $x\le -2$ or $x\ge \frac{7}{3}$ ----A1	Critical values not stated clearly
(b)	$y=4x^2-2kx+k$ $(-2k)^2-4(4)(k)<0$ ---M1 clear attempt at b^2-4ac $4k(k-4)<0$ ---M1 for factorisation Leads to critical values 0 and 4 ----M1 soi $0<k<4$ ---A1	
6	Let $\frac{x-1}{(x-2)(x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}$ ---B1 $x-1=A(x^2+1)+(Bx+C)(x-2)$ Let $x=2$, $A=\frac{1}{5}$ ---B1 Compare coefficient of x^2, $A+B=0$ $B=-\frac{1}{5}$ ---B1 Compare constant terms, $A-2C=1$ $C=\frac{3}{5}$ ---B1 $\frac{x-1}{(x-2)(x^2+1)}=\frac{1}{5(x-2)}+\frac{-x+3}{5(x^2+1)}$ ---B1	Students make mistake when substituting the values back to the fraction
7 (i)	3 ---B1	
(ii)	360° ---B1	
(iii)	$(90°,-2)$ and $(270°,-2)$ ---B1	Units not written; only 1 coordinate
(iv)	Let $y=0$ $(90°,-2)$ $\alpha=70.52878°$ $2x=180°-70.52878°,\ 180°+70.52878°,\ 360°+(180°-70.52878°),$ $360°+(180°+70.52878°)$ $x=54.7°, 125.3°, 234.7°, 305.2°$ (1d.p.) --- A1, A1 (A1 for any pair)	

(v)	B1: $y=\cos 2x$ all correct B1: a curve with highest value at $y=4$ and lowest value at $y=-2$ 
(vi)	B1: correct graph touching $y=1+3\cos 2x$ at critical points $(90°,-2)$ and ends at $(360°,0)$. Completely correct graph.
(vii)	3 ---B1
8(i)	$\alpha+\beta=-\frac{3}{5}$ ---B1 $\alpha\beta=-\frac{2}{5}$ ---B1 Well answered
(ii)	$\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ $=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$ $=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$ $=\frac{(-\frac{3}{5})^2-2(-\frac{2}{5})}{(-\frac{2}{5})^2}$ ---B1√for $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ substituted $=7\frac{1}{4}$ ---B1√for their $\alpha+\beta$ and $\alpha\beta$ $\frac{1}{(\alpha\beta)^2}=\frac{25}{4}$ ----B1√for their $\alpha\beta$ Equation is $x^2-7\frac{1}{4}x+\frac{25}{4}=0$ --- √ B1 f.t. OR $4x^2-29x+25=0$ Error in the omission of '=0'

9	$3+3^x=3^{3+x}+3^{2x+2}$ Wrong concept: $3^{1+x}=3^{3+x+2x+2}$ X $1+x=3+x+2x+2$ X Let $y=3^x$ $3+y=27y+9y^2$ --- M1 for attempting to form a quadratic $(y+3)(9y-1)=0$ --- DM1 for attempt to solve a 3 terms quadratic $y=-3$ or $y=\frac{1}{9}$ --- A1 for both (No solution) or $3^x=3^{-2}$ $x=-2$ --- A1 **No A1 for extra solutions seen**
10(a)	L.H.S.: $\frac{\cos 2x}{\sin 2x+1}$ $=\frac{\cos^2 x-\sin^2 x}{2\sin x\cos x+1}$ **M1** for $\cos 2x=\cos^2 x-\sin^2 x$; or $\sin 2x=2\sin x\cos x$ $=\frac{(\cos x-\sin x)(\cos x+\sin x)}{2\sin x\cos x+\sin^2 x+\cos^2 x}$ **M1** for $1=\sin^2 x+\cos^2 x$ $=\frac{(\cos x-\sin x)(\cos x+\sin x)}{(\sin x+\cos x)^2}$ **M1** for factorisation $=\frac{\cos x-\sin x}{\cos x+\sin x}$: R.H.S. (proven) **AG1** Difficult for majority of the students
(b)	 $\sin A=-\frac{4}{5}$ **B1** $\sin B=-\frac{5}{13}$ **B1** $\cos B=-\frac{12}{13}$ **B1** $\sin(A+B)=\sin A\cos B+\cos A\sin B$ $=\left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)+\left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right)$ $=\frac{63}{65}$ (c.a.o.) **B1** Majority of the students did not realise that $\sin A$, $\sin B$ and $\cos B$ are negative in the 3rd quadrant.

(c)

$\tan 22.5° = \text{cosec } 2(22.5°) - \cot 2(22.5°)$

$= \text{cosec } 45° - \cot 45°$ **B1**

$= \dfrac{1}{\sin 45°} - \dfrac{1}{\tan 45°}$

$= \dfrac{1}{\frac{1}{\sqrt{2}}} - \dfrac{1}{1}$ **B1**

$= \sqrt{2} - 1$

$= -1 + \sqrt{2}$ (shown) **AG1**

> The idea of using the given identity of $\tan x$ in the equation was NOT understood by almost all students.

11(a)

$2\cos^2 x = 3\cos x \sin x$

$2\cos^2 x - 3\cos x \sin x = 0$

$\cos x(2\cos x - 3\sin x) = 0$ --**M1**

$\cos x = 0$

$x = 90°$ or $270°$ ---**A1**

> Many students divided by $\cos x$ to obtain an equation in $\tan x$ and so lost some possible solutions.

OR

$2\cos x - 3\sin x = 0$

$\tan x = \dfrac{2}{3}$ (x lies in the 1^{st} / 3^{rd} quadrant) ----**M1**

$\alpha = 33.69°$

$x = \alpha$ OR $x = 180° + \alpha$

$x = 33.7°$ OR $x = 213.7°$ (1 d.p.) ---**A1**

11(b)

$\cos 2x = 1 - \sin x$

$1 - 2\sin^2 x = 1 - \sin x$ ---**M1** for $\cos 2x = 1 - 2\sin^2 x$

$\sin x\,(2\sin x - 1) = 0$

$\sin x = 0$

$x = 0, 3.14, 6.28$ ---**A1**

> The only significant problem was not to round to 2 decimal places

OR

$2\sin x - 1 = 0$

$\sin x = \dfrac{1}{2}$ (x lies in the 1^{st} / 2^{nd} quadrant) ---**M1**

$\alpha = 0.523599$

$x = \alpha$ OR $x = \pi - \alpha$

$x = 0.52$ OR $x = 2.62$ (2 d.p.) ---**A1**

12 (i)	$\cos\theta = \frac{AX}{8}$ ---B1 $\sin\theta = \frac{XD}{7}$ --- B1 $AD = (8\cos\theta + 7\sin\theta)$ (shown) AG
(ii)	$R = \sqrt{8^2 + 7^2}$ $= \sqrt{113}$ ---M1 A1 $\tan\alpha = \frac{7}{8}$ $\alpha = 41.186°$ (3 d.p.) $AD = \sqrt{113}\cos(\theta - 41.2°)$ ---M1 A1
(iii)	Max. $AD = \sqrt{113}$ $= 10.6$ cm (3 S.F.) --- B1 $\theta - 41.2° = 0$ $\theta = 41.2°$ (1 d.p.) ---M1 A1
(iv)	$9.5 = \sqrt{113}\cos(\theta - 41.186°)$ [Pre-mature rounding of 41.186°] $\cos(\theta - 41.186°) = \frac{9.5}{\sqrt{113}}$ $\alpha = 26.660°$ --- M1 $\theta - 41.186° = -26.660°, \quad 26.660°$ $\theta = 14.5°, \quad 67.8°$ (1 d.p.) ---A1
13 (i)	$3y = 2x$ $y = \frac{2}{3}x$ Gradient of $OC = \frac{2}{3}$ Gradient of $BC = -\frac{3}{2}$ ---M1 Equation of BC: $y - 6 = -\frac{3}{2}(x-3)$ $y = -\frac{3}{2}x + \frac{21}{2}$ or $2y = -3x + 21$ ----A1 Gradient of $AB = \frac{2}{3}$ ---M1 Equation of AB: $y - 6 = \frac{2}{3}(x-3)$ $y = \frac{2}{3}x + 4$ or $3y = 2x + 12$ ----A1

(ii)	Let $x = 0$, $y = 4$ $A(0, 4)$----B1 Solve equations of OC and BC simultaneously, $y = 3\frac{3}{13}$ and $x = 4\frac{11}{13}$ ---M1 $C\left(4\frac{11}{13}, 3\frac{3}{13}\right)$ ---A1 Or $-\frac{3}{2}x + \frac{21}{2} = \frac{2}{3}x$ $2\frac{1}{6}x = \frac{21}{2}$
(iii)	$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & \frac{63}{13} & 3 & 0 & 0 \\ 0 & \frac{42}{13} & 6 & 4 & 0 \end{vmatrix}$ ---√M1 *f.t. their coordinates of C* $= \frac{1}{2}\left\| \frac{378}{13} + 12 - \frac{126}{13} \right\|$ ---√M1 *f.t.* $= 15.7 \text{ units}^2$ ---A1 or $15\frac{9}{13}$