SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 1

4048/01

28 AUGUST 2018 (Tuesday) **2 hours**

CANDIDATE NAME

CLASS

INDEX NUMBER

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.

Write your name, class and index number in the spaces above.

Write in dark blue or black pen in the space provided for each question.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES

Answer **all** the questions.

Write your answers in the space provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use		
Q1	1	
Q2	1	
Q3	2	
Q4	2	
Q5	2	
Q6	2	
Q7	2	
Q8	4	
Q9	2	
Q10	3	
Q11	3	
Q12	3	
Q13	3	
Q14	3	
Q15	3	
Q16	2	
Q17	4	
Q18	5	
Q19	6	
Q20	5	
Q21	3	
Q22	7	
Q23	9	
Q24	3	
Total	/ 80	

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^{n}$$

Mensuration

$$\textit{Curved surface area of a cone} = \pi r l$$
$$\textit{Surface area of a sphere} = 4\pi r^2$$
$$\textit{Volume of a cone} = \frac{1}{3}\pi r^2 h$$
$$\textit{Volume of a sphere} = \frac{4}{3}\pi r^3$$
$$\textit{Area of triangle ABC} = \frac{1}{2}ab\sin C$$
$$\textit{Arc length} = r\theta, \textit{ where } \theta \textit{ is in radians}$$
$$\textit{Sector area} = \frac{1}{2}r^2\theta, \textit{ where } \theta \textit{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\textit{Mean} = \frac{\sum fx}{\sum f}$$

$$\textit{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Factorise $5(3x-y)^2-(3x-y)$ completely.

Answer .... [1]

2 The enrolment for a school in 2017 was 3450. This was 15% more than the enrolment in 2016. Calculate the enrolment in 2016.

Answer .... [1]

3 Ms Chew invested $\$P$ in a bank that pays compound interest at the rate of 4 % per annum compounded half yearly. If she received \$6341.21 from the bank after 6 years, find the value of P, giving your answer to the nearest whole number.

Answer $P =$ \$............................ [2]

4 **(a)** Express 315 as a product of its prime factors.

Answer .... [1]

(b) Find the **smallest whole number** by which 315 must be multiplied to obtain a perfect square.

Answer .... [1]

---

5 Given that $4^{\left(\frac{1}{2n}\right)} \div 64^{-2} = 2^5$, find the value of n.

Answer $n =$ .... [2]

6 Given that $OABC$ is a parallelogram such that $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and A (5, 1). Find $|\overrightarrow{OB}|$.

Answer ....units [2]

---

7 A map is drawn to a scale of 1 : 50000.

(a) Two towns are 24 km apart. Calculate, in centimetres, their distance apart on the map.

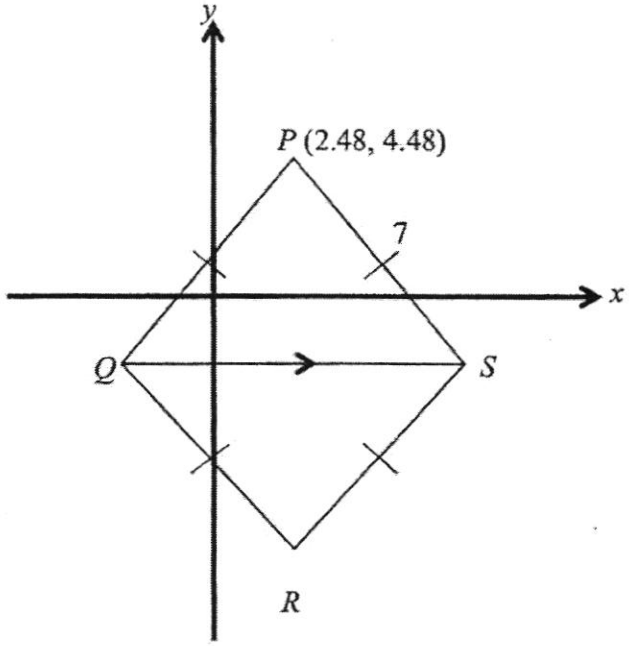
Answer ....cm [1]

(b) On the map, a farm has an area of 20 cm^2. Calculate, in square kilometres, the actual area of the farm.

Answer ....km^2 [1]

---

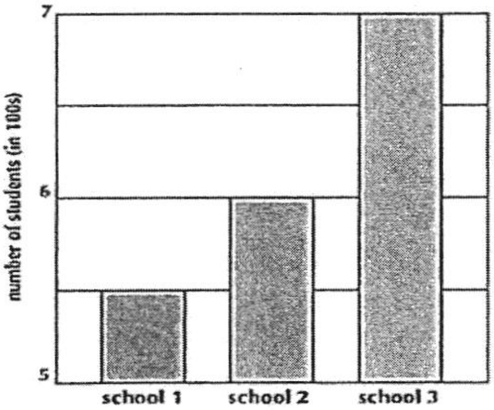
8 The diagram below shows 2 congruent equilateral triangles PQS and SQR with sides 7 cm. Point P has coordinates (2.48, 4.48).
The base SQ of the equilateral triangle PSQ is parallel to the x-axis. Find the coordinates of R and S, giving your answers correct to two decimal places.



Answer $R = (\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots)$

$S = (\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots)$ [4]

9 Jenny drew a bar chart to compare the enrolment (number of students) in school 1, 2 and 3.



State one aspect of the bar chart that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

....

....

.... [2]

---

10 Given that $3xy + x = \sqrt{3yz + x^2}$, express x in terms of y and z.

Answer $x =$ .... [3]

---

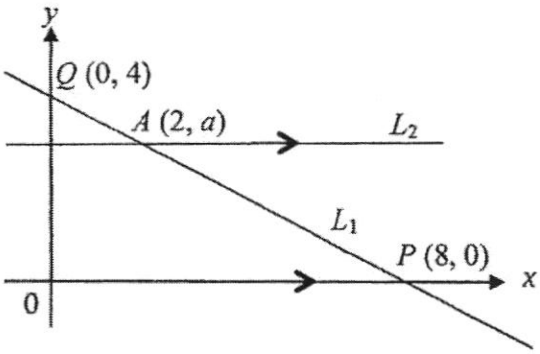
11 (a) Solve the inequalities $4 \leq 7 - \frac{x+3}{2} < \frac{13}{2}$.

Answer (a)........................ [2]

(b) Write down all the integers that satisfy $4 \leq 7 - \frac{x+3}{2} < \frac{13}{2}$.

Answer (b)............................ [1]

12 In the figure below, line L_1 cuts the x-axis at P (8, 0) and the y-axis at Q (0, 4).
On the same axes line L_2 meets line L_1 at A (2, a). Line L_2 is parallel to the x-axis.



(a) Write down the equation of line L_1.

Answer .... [1]

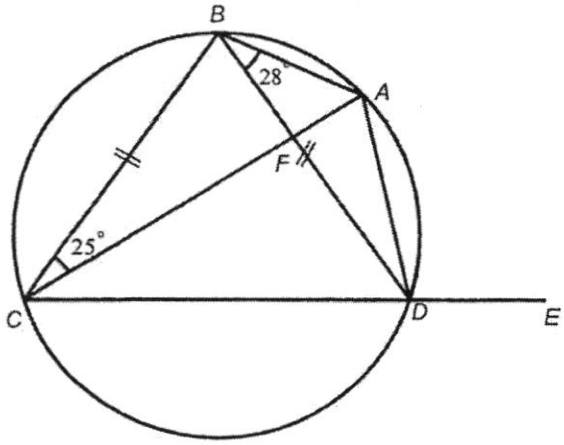
(b) Calculate the value of a.

Answer $a =$ .... [1]

(c) Write down the value of $\dfrac{QA}{AP}$.

Answer .... [1]

13 The diagram shows a circle $ABCD$ with $BC = BD$. CDE is a straight line. Given that angle $ABD = 28°$ and angle $ACB = 25°$,



(a) explain why is angle $ACD = 28°$.

Answer

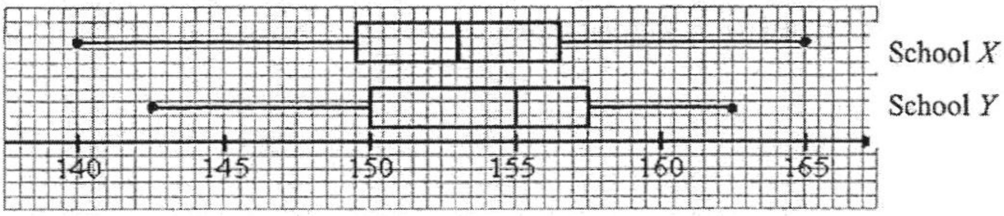
....

.... [1]

(b) Hence, find angle BAD, giving reasons for your answer.

Answer.............................° [2]

14 The box-and-whisker plots show the distribution of heights of girls in 2 schools.



(a) Find the median height for School X.

Answer ....cm [1]

(b) Find the interquartile range for School Y.

Answer ....cm [1]

(c) Janet said the girls in School X are generally taller than the girls in School Y.
Do you agree? Give a reason for your answer.

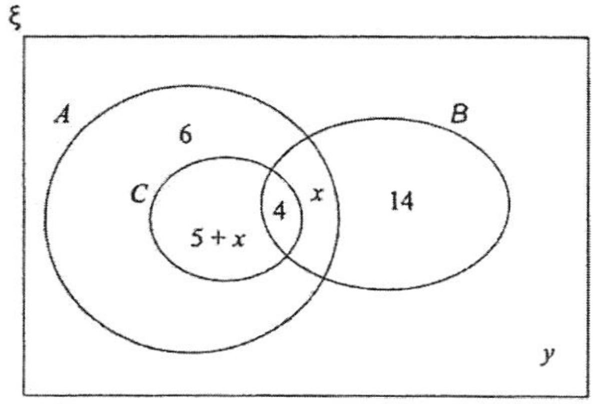
Answer

....

.... [1]

15 The Venn diagram shows the number of elements of sets A, B and C. Given that $n(A) = 27$ and $n(A \cup B)' = 4$

(a) find the value of x and y,

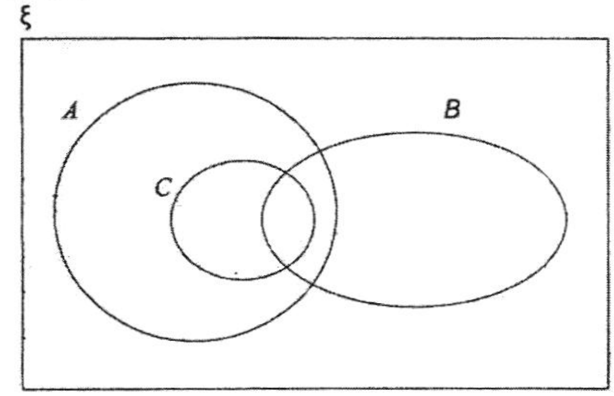


Answer $x =$ .... [1]

$y =$ .... [1]

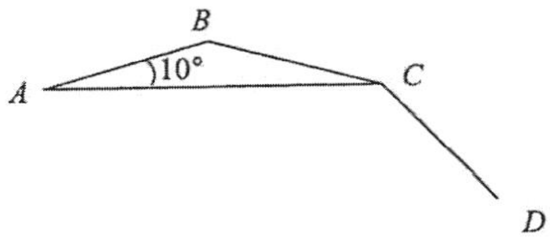
(b) shade the region $A \cap B'$.

Answer



[1]

16 AB, BC and CD are adjacent sides of a regular polygon. Given that $\angle CAB = 10°$,



calculate

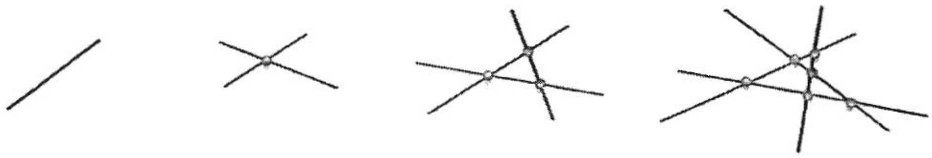
(a) the exterior angle of the polygon,

Answer ....° [1]

(b) the number of sides of the polygon.

Answer .... [1]

17



The above diagrams show the maximum number of intersections obtained from 1, 2, 3 and 4 lines respectively.

Number of lines, n	Maximum number of line segments, E	Maximum number of intersections, P	Maximum number of regions, R
1	1	0	$2 = 1+1$
2	$2^2 = 4$	$1 = \dfrac{2(1)}{2}$	$4 = 1+3$
3	$3^2 = 9$	$3 = \dfrac{3(2)}{2}$	$7 = 1+6$
4		$6 = \dfrac{4(3)}{2}$	$11 = 1+10$
5		$10 = \dfrac{5(4)}{2}$	
6	$6^2 = 36$	$15 = \dfrac{6(5)}{2}$	$22 = 1+21$

(a) Complete the above table. [1]

(b) What is the maximum number of intersections P obtained from n straight lines in terms of n?

Answer $P =$............................ [1]

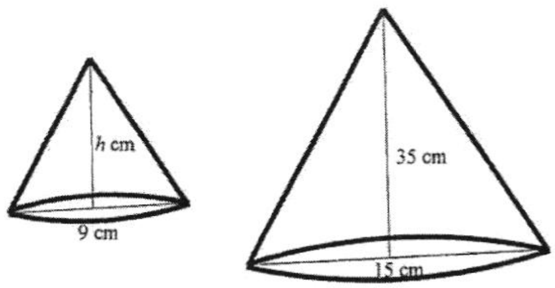
(c) What is the maximum number of regions R obtained by using n straight lines in terms of n?

Answer $R =$............................ [1]

(d) Hence, write down an expression connecting R, E and P.

Answer .... [1]

18 Two solid cones are geometrically similar. The diameters of the base of the smaller cone and the base of the larger cone are 9 cm and 15 cm respectively. The heights of the smaller cone and the larger cone are h cm and ~~15~~ cm respectively.
35



(a) Find the value of h.

Answer $h =$ ....cm [1]

(b) If it costs \$9 to paint the smaller cone with 1 coat of paint, how much does it cost to paint the larger cone with 1 coat of the same kind of paint?

Answer \$............................ [2]

(c) Given that the mass of the larger cone is 25 g, find the mass of the smaller cone, assuming that both cones are made of the same kind of material.

Answer ....g [2]

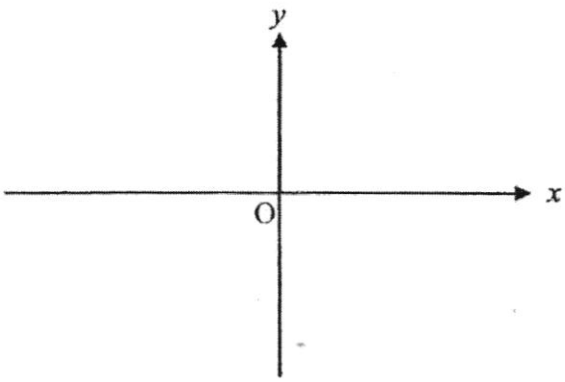
19 **(a)** Express $-x^2-6x-7$ in the form $-(x+a)^2+b$, [1]

Answer

(b) hence, solve $-x^2-6x-7=0$, showing your working clearly.
Give your answers correct to two decimal places.

Answer $x =$ .... *or* .... [2]

(c) Sketch the graph of $y = -x^2-6x-7$,



[2]

(d) With reference to graph drawn above, explain why there is no solution for the equation $-x^2-6x-7=3$.

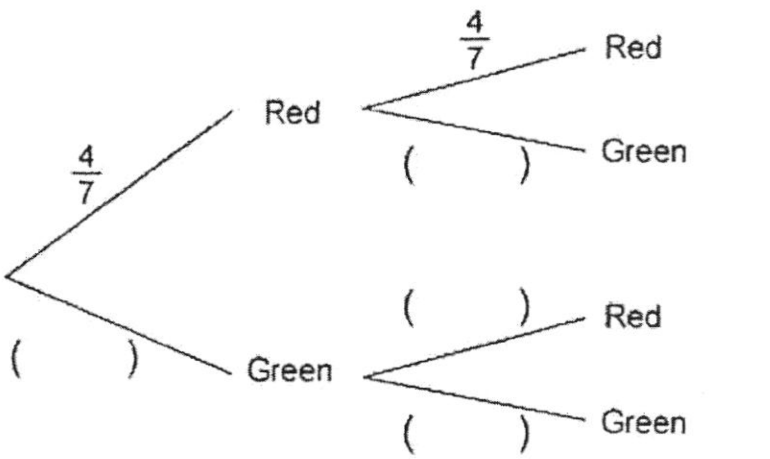
....

....

.... [1]

20 A box contains 4 red balls and 3 green balls. One ball is drawn at random. If a green ball is drawn, it will **not** be replaced, a second ball is then drawn. If a red ball is drawn, it will be replaced, a second ball is then drawn from the box.
Complete the probability tree diagram to show the probabilities of possible outcomes.

Answer [1]



Find the probability that

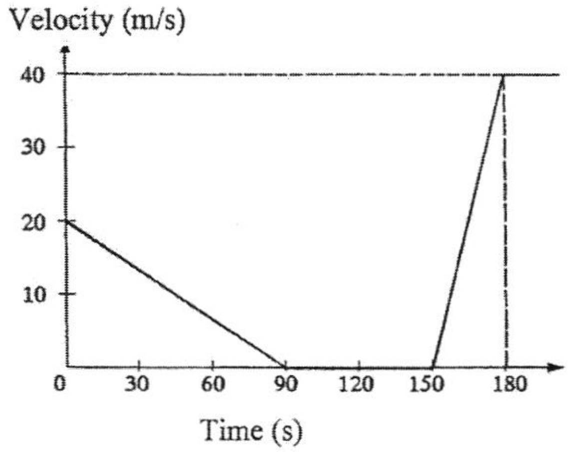
(i) the two balls are of different colours,

Answer .... [2]

(ii) at least 2 green balls are left in the box after the second draw.

Answer .... [2]

21 A train slows down to a stop on entering a station P as shown in the velocity-time graph. After a brief stop of 60 s, it starts to move off with an acceleration of $1\frac{1}{3}$ m/s^2 for 30 s before it gets out of station P. It then continues its journey with this velocity until it reaches another station Q.



(a) Find the deceleration of the train when it enters the station P.

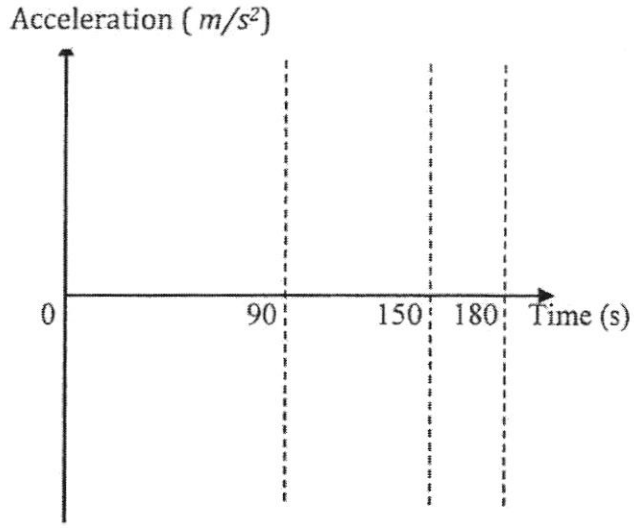
Answer ....m/s^2 [1]

(b) Calculate the total distance travelled by the train in its first 3 minutes journey.

Answer ....m [1]

(c) On the axes below, sketch an acceleration-time graph of the train for the whole 3 minutes of its motion.

[1]



22

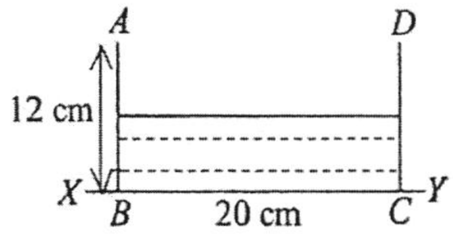


Figure 1

Figure 1 shows a vertical cross-section of a rectangular tank that stands on a horizontal table represented by *XY*. The tank is 12 cm high and has a square base of side 20 cm and contains 3000 cm^3 of water. Calculate

(a) (i) the volume of the tank,

Answer ....cm^3 [1]

(ii) the depth of the water.

Answer ....cm [2]

The tank is now tilted about a base edge through C, so that some of the water spills out until the position shown in Figure 2 below. Calculate

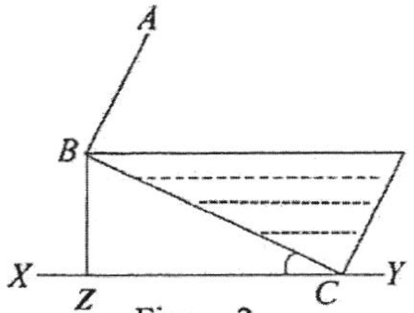


Figure 2

(b) Calculate

(i) the volume of water remaining in the tank,

Answer ....cm^3 [1]

(ii) angle BCZ,

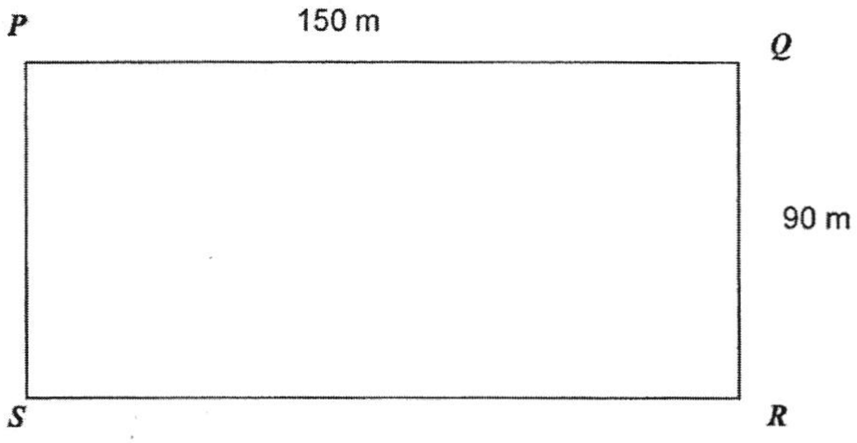
Answer ....° [2]

(iii) BZ, the vertical height of B above the table where Z is the foot of the perpendicular of B to YZ.

Answer ....cm [1]

[Turn over

23



The diagram shows a rectangular field where $PQ = 150$ m and $QR = 90$ m.
Jason starts from P and walks towards Q at a constant speed of 1.5 m/s.
At the same time, John starts from Q and walks towards R at a constant speed of 0.5 m/s.

(a) Write down in terms of t (where $t < 100$s)

(i) the distance of Jason from Q after t seconds,

Answer ....m [1]

(ii) the distance of John from Q after t seconds.

Answer ....m [1]

(b) Given that after t seconds, the two men are h m apart, show that

$$h^2 = 2.5t^2 - 450t + 22500.$$

Answer [2]

(c) Find the distance between the two men one minute after the start.

Answer ....m [2]

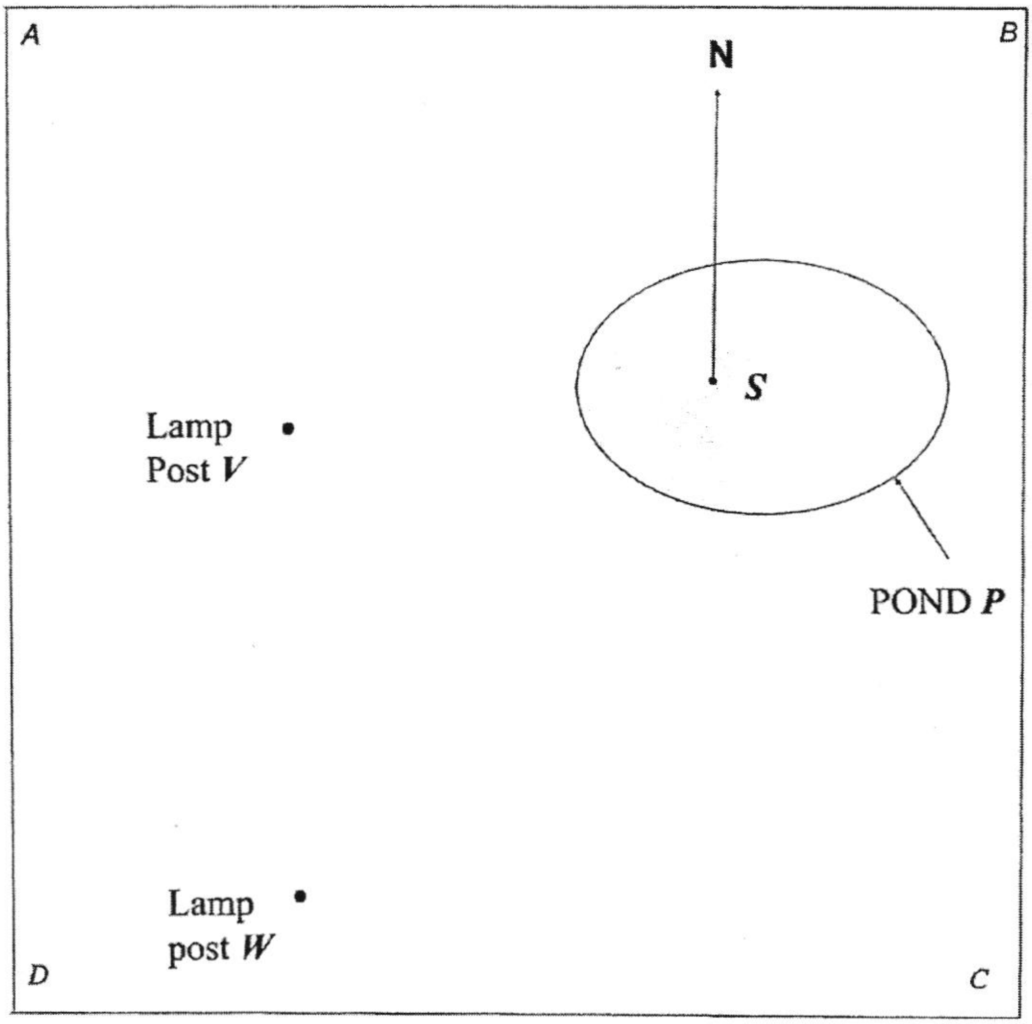
(d) Find the value of t when the two men are 100 m apart.

Answer ....s [2]

(e) Hence, find the distance that Jason is from Q when the two men are 100 m apart.

Answer ....m [1]

24 **(a)** The diagram shows a plot of land *ABCD* with a pond *P* at a corner and two lamp-posts *V* and *W*. *S* is the fixed position of a stick placed in the pond.
A gardener wants to plant a tree *T* equidistant from the two lamp posts *V* and *W* and also equidistant from the lines *VS* and *WS*. By appropriate constructions, mark the point *T* on the diagram below. [2]



(b) Measure the bearing of *W* from *S*.

Answer ....° [1]

End of Paper

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 2

4048/02

11 SEPTEMBER 2018 (Tuesday)

2 hours 30 minutes

CANDIDATE NAME

CLASS

INDEX NUMBER

For Examiner's Use		
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Q8	11	
Q9	13	
Q10	16	
Total	/ 100	

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1. **(a)** The following table shows some information on the population of a country in 2016 and 2017. The total population of the country consists of residents and non-residents.

Year	Total population	Residents	Non-residents
2016	55.9 million	x	y
2017	56.1 million	54.45 million	1.65 million

[1 million people = 1×10^6 people]

From 2016 to 2017, the number of non-residents decreased by 1.6%.

(i) Find the value of y, expressing your answer as k million, where k is a constant correct to 3 decimal places. **[1]**

(ii) Hence, find the value of x, giving your answer in standard form correct to 2 decimal places. **[2]**

(b) P is proportional to Q^n, where n is an integer.

State the value of n when

(i) P units is the force between two particles which is inversely proportional to the square of the distance Q mm between them, **[1]**

(ii) P m^3 is the volume of a sphere with radius Q m, **[1]**

(iii) P cm^3 is the volume of a cone with radius Q cm and a fixed height. **[1]**

(c) Express as a single fraction in its simplest form

$$\frac{2}{2c-b} - \frac{3c-11b}{5b^2-20c^2}.$$ **[3]**

2. Jenny bought some jars of cookies for \$900. She paid \$$n$ for each jar of cookies.

(a) Write down an expression, in terms of n, for the number of jars of cookies she bought. **[1]**

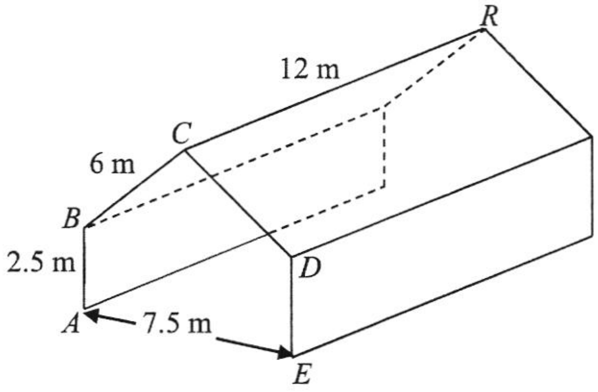
(b) Jenny found that 2 jars of cookies were spoilt and could not be sold. Jenny sold each remaining jar of cookies for \$3 more than she paid for it. Write down an expression, in terms of n, for the total sum of money she received from the sale of the jars of cookies. **[1]**

(c) Given that she made a profit of \$92 from the sale of the jars of cookies, form an equation in n and show that it reduces to $n^2 + 49n - 1350 = 0$. **[2]**

(d) Solve $n^2 + 49n - 1350 = 0$, giving your solutions correct to 3 decimal places. **[2]**

(e) Hence, find the selling price of each jar of cookies sold by Jenny, giving your solution correct to the nearest cent. **[2]**

3. The diagram shows a tent. The cross-section of the tent forms a pentagon *ABCDE* with two vertical sides of height 2.5 m and two slant sides of equal length 6 m. It is also given that the length of the tent *CR* is 12 m and the width of the tent *AE* is 7.5 m.



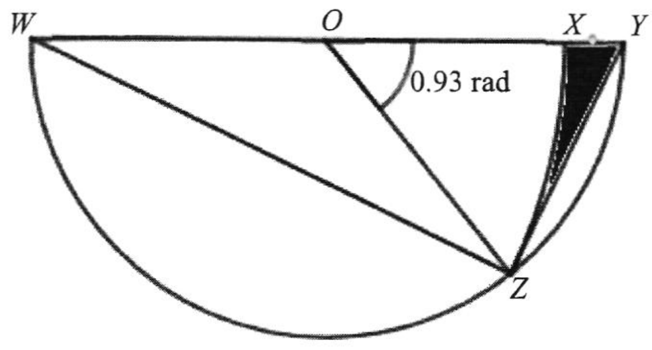
Find

(a) the area of the cross section *ABCDE*, **[2]**

(b) the angle of elevation of *R* from *A*. **[4]**

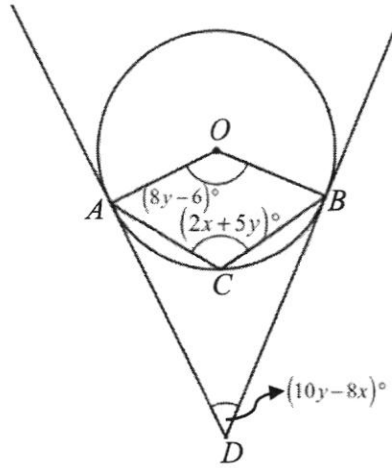
4. **(a)** Explain, with mathematical calculations, why it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm. **[3]**

(b) In the diagram below, *WZY* is a semicircle with centre *O*, radius 7 cm and angle $ZOX = 0.93$ radians. *WZX* is a sector of another circle with centre *W* and radius 12.5 cm.



Find the perimeter of the shaded area. **[4]**

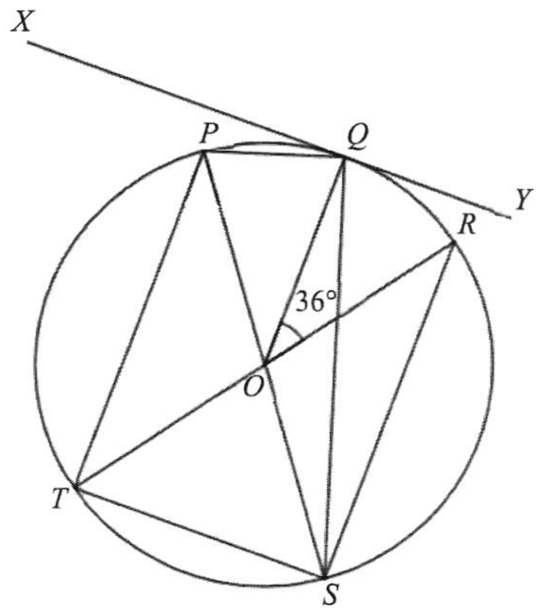
5. **(a)** In the diagram, A, B and C lie on a circle with centre O.
The tangents at A and B meet at D.
It is given that angle $AOB = (8y-6)°$, angle $ACB = (2x+5y)°$ and angle $ADB = (10y-8x)°$.



(i) Stating your reasons clearly, show that
$(8y-6)°+(10y-8x)°=180°$ **[3]**

(ii) Hence, by solving a pair of simultaneous equations, find the value of x and of y. **[3]**

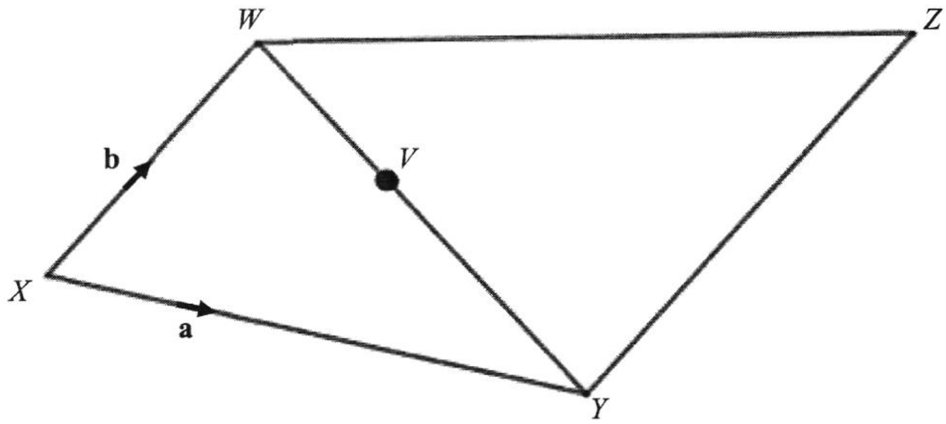
(b) The diagram shows a circle $PQRST$, with centre O. XY is a tangent to the circle at Q. It is given that angle QOR= 36°. TOR and POS are straight lines.



(i) Find, giving reasons for your answer, angle RSQ. **[1]**

(ii) Prove that triangle PTS and triangle RST are congruent. **[3]**

6. In the diagram, $WXYZ$ is a quadrilateral such that $\overrightarrow{XY} = \mathbf{a}$, $\overrightarrow{XW} = \mathbf{b}$ and $\overrightarrow{XW} = \frac{2}{3}\overrightarrow{YZ}$. V is a point on WY such that $5\overrightarrow{VY} = 3\overrightarrow{WY}$.



(a) What is the special name given to the quadrilateral $WXYZ$? **[1]**

(b) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{WY}$, **[1]**

(ii) $\overrightarrow{XZ}$, **[1]**

(iii) $\overrightarrow{XV}$. **[2]**

(c) Explain why X, V and Z lie on a straight line. **[2]**

(d) Prove that triangle XWV and triangle ZYV are similar. **[2]**

(e) Find

(i) $\dfrac{\text{Area of triangle } XWV}{\text{Area of triangle } ZYV}$, **[1]**

(ii) $\dfrac{\text{Area of triangle } ZWV}{\text{Area of triangle } ZYV}$. **[1]**

7. **(a)** The cash price of a waffle maker is \$149.

Rose wants to start a waffle shop business and buys 5 waffle makers on hire purchase. She pays a deposit of 15% of the cash price followed by 24 equal monthly instalments with interest charged at a flat rate of 1.5% per annum.

Calculate the amount of the monthly instalment, correct to the nearest cent. **[2]**

(b) Rose offers three types of waffle fillings at her shop: chocolate, cheese and blueberry.

The price of each type of waffle is shown in the table below.

Chocolate	Cheese	Blueberry
\$1.80	\$2.50	\$1.50

The table below shows the sale of waffles at Rose's shop for the months of June and July.

Month/ Fillings	Chocolate	Cheese	Blueberry
June	52	8	27
July	48	13	21

(i) Represent the prices of each type of waffle in a column matrix **P**. **[1]**

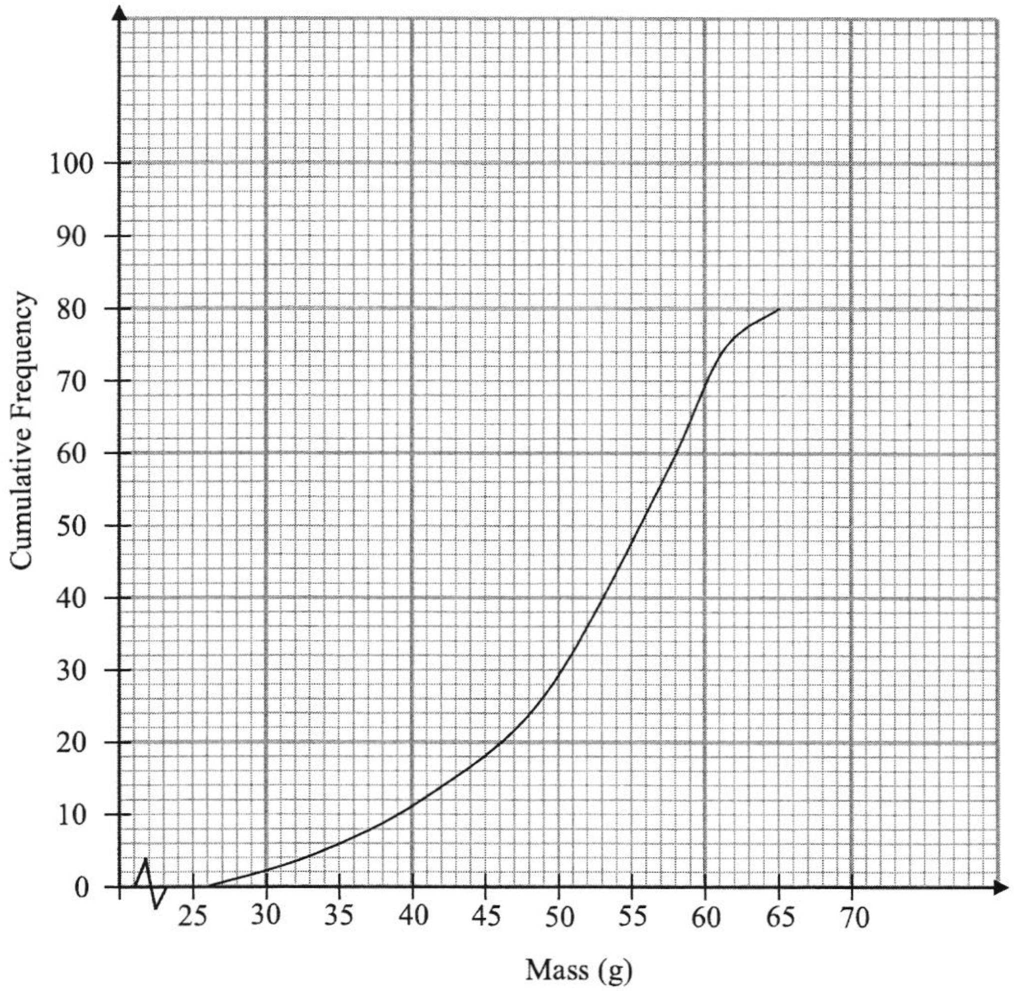
(ii) Represent the sale of waffles at Rose's shop for the months of June and July in a 2×3 matrix **W**. **[1]**

(iii) Evaluate the matrix $\mathbf{R} = \mathbf{WP}$. **[2]**

(iv) State what the elements of **R** represent. **[1]**

(v) By multiplying matrix **W** with a row matrix, find the matrix that represents the total number of each type of waffles Rose sold in June and July. **[2]**

8. The masses of 80 eggs collected at Farm A are recorded.
The cumulative frequency curve below shows the distribution of their masses.



(a) Use the curve to estimate

(i) the median mass, **[1]**

(ii) the 30^{th} percentile, **[1]**

(iii) the interquartile range. **[2]**

(b) The distribution of the masses of the eggs can be represented by the grouped frequency table below.

Mass (x g)	$25 < x \leq 35$	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$
Frequency	6	m	30	32

(i) Show that the value of m is 12. **[1]**

(ii) A worker in Farm A select two eggs at random, one after another, without replacement.
Find, as a fraction in its simplest form, the probability that both eggs are more than 55g. **[2]**

8 **(c)** The masses of eggs at Farm B are also measured and recorded. Information relating to the masses of eggs at Farm B are given below.

Mean = 53g
Standard Deviation = 9g

(i) A worker at Farm B says to a worker at Farm A:

"The masses of the eggs at my farm are more consistent than the masses of the eggs at your farm."

Do you agree with the worker at Farm B? Explain with mathematical calculations. **[2]**

(ii) The worker at Farm B realises that the weighing machine is spoilt. Hence the mass of each egg should be 1g more than the measured mass.
State the correct mean and standard deviation of the masses of eggs at Farm B. **[2]**

9. **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{x}{4}\left(8 - 6x + x^2\right).$$

Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	-1	0	0.5	1	1.5	2	2.5	3	5
y	-3.75	0	k	0.75	0.47	0	-0.47	-0.75	3.75

(a) Calculate the value of k. **[1]**

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-1 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-4 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. **[3]**

(c) The equation $x^3 - 6x^2 + 8x = -12$ has only one solution.
Explain how this can be seen from your graph. **[2]**

(d) By drawing a tangent, find the gradient of the curve at the point (1, 0.75). **[2]**

(e) **(i)** Line L has gradient -0.5 and passes through the point (3, 1).
Draw line L on the same axes for $-1 \le x \le 5$. **[2]**

(ii) Write down the x-coordinate of the point where the two graphs intersect. **[1]**

(iii) This value of x is a solution of a cubic equation. Write down the cubic equation in the form $x^3 + Px^2 + Qx + R = 0$, where P, Q and R are integers. **[2]**

10. A soda can may be modelled as a cylinder with a closed top and a hollow hemisphere hollowed in at the base of the can as shown in the diagram below.

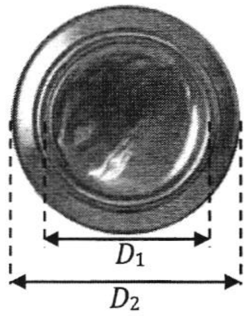
Information about the model of the soda can is given below.

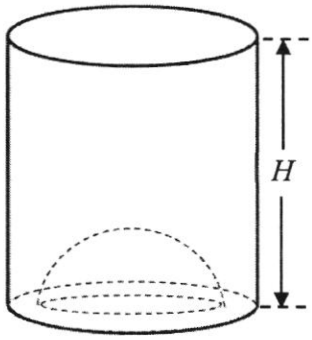
Height (H) = 12.4 cm

Inner Diameter (D_1) of base = 6.7 cm

Outer diameter (D_2) of base = 7.9 cm

Mass of empty can = 15g





(a) Using the model of the soda can in the diagram above, calculate

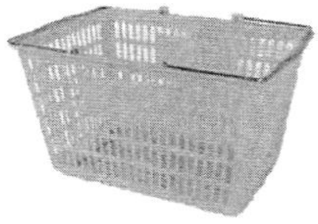
(i) the total surface area, in square centimetres, of the soda can. **[5]**

(ii) the volume, in cubic centimetres, of the soda can. **[3]**

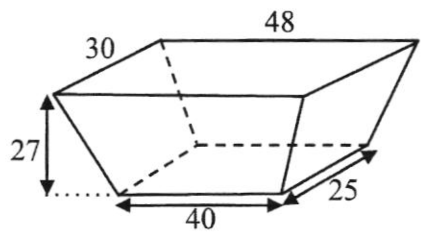
10 **(b)** Harry uses a shopping basket to transport the soda cans filled with carbonated drink.

The soda cans will be placed with the base of the can lying on the base of the basket then stacked up vertically within the basket.

For safety reasons, all the soda cans must be contained inside the shopping basket. The maximum load that the shopping basket can carry is 55 pounds.



The shopping basket can be modelled by a frustrum of a inverted pyramid as shown in the diagram below.



The frustrum above is obtained by removing the top portion of an inverted right rectangular pyramid. The flat rectangular base of the frustum has length 40 cm and width 25 cm. The remaining vertical height is 27 cm. The flat rectangular top of the frustum is 48 cm by 30 cm.

Other Useful Information

- Density of carbonated drink = 1.3 g/cm^3
- 1 pound is equivalent to 0.45 kg
- Mass = Volume $\times$ Density
- Safety information: Soda can is filled with carbonated drink up to a maximum of 90% of its total volume.

Assuming that each soda can is filled with carbonated drink to the maximum safe volume, find the maximum number of soda cans Harry can transport with the shopping basket at any one time.
Justify your answer with mathematical calculations. **[8]**

END OF PAPER 2

School of Science and Technology
Sec 4 Prelim Exam 2018

ANSWER KEY

Qn	Answer/Solution
1.	$(3x-y)[5(3x-y)-1]$ $= \underline{\mathbf{(3x-y)[15x-5y-1]}}$
2.	115% represents 3450. 100% represents $\frac{100}{115}\times 3450 = \underline{\mathbf{3000}}$
3.	$6341.21 = P\left[1+\frac{4/2}{100}\right]^{6\times 2}$ $6341.21 = P\,(1.02)^{12}$ P = $\underline{\mathbf{\$5000}}$
4.	**a)** $315 = \underline{\mathbf{3^2 \times 5 \times 7}}$ b) $3^2 \times 5 \times 7 = \underline{\mathbf{35}}$
5.	$\left(2^2\right)^{\frac{1}{2n}} \div \left(2^6\right)^{-2} = 2^5$ $2^{\frac{1}{n}} \div 2^{-12} = 2^5$ $2^{\frac{1}{n}-(-12)} = 2^5$ $\frac{1}{n}+12=5$ $\frac{1}{n}=-7$ $n=-\frac{1}{7}$
6.	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$ $=\binom{5}{1}+\binom{3}{5}=\binom{8}{6}$ $\left\|\overrightarrow{OB}\right\|=\sqrt{8^2+6^2}=10$
7.	a) 1 cm represent 0.5 km 0.5 km is represented by 1cm $\therefore$ 24km is represented by $\frac{24}{0.5} = \underline{\mathbf{48\text{ cm}}}$ b) 1 cm ^{2} represents $(0.5\text{ km})^2$ 1cm^2 represents 0.25 km^2 $\therefore$ 20 cm^2 represents $0.25\times 20 = \underline{\mathbf{5\text{ km}^2}}$

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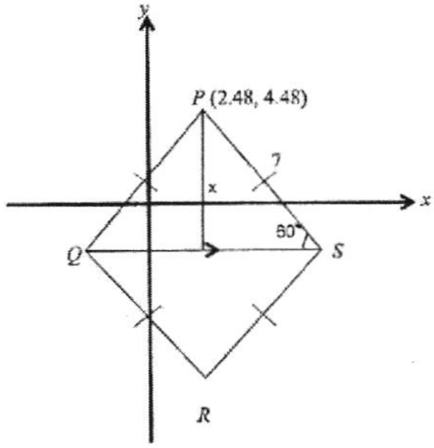
ANSWER KEY

<table>
<tr><td>8.</td><td>

Let perpendicular line from P to QS be x
$\sin 60° = \frac{x}{7} \Rightarrow x = 7sin60° = 6.0622$
$PR = 6.0622 \times 2 = 12.1244$
$\therefore y - cord.of\ R = 4.48 - 12.1244$
$\therefore R(2.48, -7.64)$
$\frac{y}{7} = \cos 60° \Rightarrow y = 3.5$
$x - cord.of\ S = 5.98$
$y - cord.of\ S = 4.48 - 6.0622 = -1.58$
$\therefore S(5.98, -1.58)$</td></tr>
<tr><td>9.</td><td>The vertical axis did not start from 0.
This would exaggerate the difference in the enrolment of the schools.</td></tr>
<tr><td>10.</td><td>$(3xy + x)^2 = 3yz + x^2$
$9x^2y^2 + 6x^2y + x^2 = 3yz + x^2$
$9x^2y^2 + 6x^2y = 3yz$
$9x^2y(3y + 2) = 3yz$
$x^2 = \frac{3yz}{3y(3y+2)}$
$x = \pm\sqrt{\frac{z}{3y+2}}$</td></tr>
<tr><td>11.</td><td>a)</td></tr>
</table>

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ANSWER KEY

	$4 \le 7-\left(\dfrac{x+3}{2}\right)$ $\left(\dfrac{x+3}{2}\right) \le 7-4$ $\left(\dfrac{x+3}{2}\right) \le 3$ $x+3 \le 6$ $x \le 3$ $7-\left(\dfrac{x+3}{2}\right) < \dfrac{13}{2}$ $14-(x+3) < 13$ $-(x+3) < -1$ $x+3 > 1$ $x > -2$ **$-2 < x \le 3$** **b) -1, 0, 1, 2, 3**
12.	a) Gradient of L_1 = Equation of L_1 is y. $\boldsymbol{y = -\frac{1}{2}x + 4}$ b) Sub. $x = 2$, $y =$ a into $y = -\frac{1}{2}x + 4$ $a = -\dfrac{1}{2}(2)+4 \Rightarrow a = 3$ c) $\dfrac{QA}{AP} = \dfrac{2}{6} = \dfrac{1}{3}$
13.	a) $\angle ACD = \angle DBA$ (L in the same segment) b) $\angle BAD = 180° - 25° - 28° = 127°$
14.	a) Median Height for school X = 153cm b) Interquartile range for school Y = 157.5 – 150 = **7.5cm** c) No 1 disagreement median of school Y = 155cm is larger than the median of school X (153cm)

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ANSWER KEY

15.

a) $6 + 5 + x + 4 + x = 27$

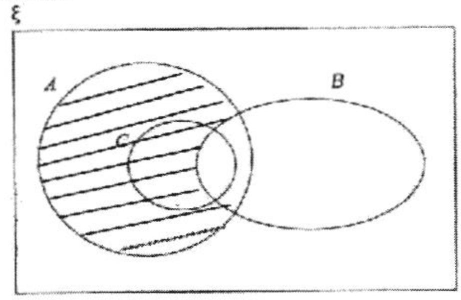
$2x = 27 - 15 = 12$

$\underline{\mathbf{x = 6}}$

$\underline{\mathbf{y = 4}}$

b)

Answer



16.

a)

$\angle BCA = 10° (AB = BC)$

$Exterior\ \angle = 10° + 10° = 20°$

b) Number of sides = 360/ 20 = 18

17.

a)

Number of lines, n	Maximum number of line segments, E	Maximum number of intersections, P	Maximum number of regions, R
1	1	0	2 = 1+1
2	$2^2 = 4$	$1 = \frac{2(1)}{2}$	4 = 1+3
3	$3^2 = 9$	$3 = \frac{3(2)}{2}$	7 = 1+6
4	$4^2 = 16$	$6 = \frac{4(3)}{2}$	11 = 1+10
5	$5^2 = 25$	$10 = \frac{5(4)}{2}$	16 = 1 + 15
6	$6^2 = 36$	$15 = \frac{6(5)}{2}$	22 = 1+21

b) $P = \frac{n(n-1)}{2}\ or\ \frac{n^2 - n}{2}\ or\ \frac{1}{2}(n^2 - n)$

c) $R = 1 + \frac{n(n+1)}{2}\ or\ \frac{2 + n + n^2}{2}$

d) R + P – E = 1

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ANSWER KEY

18.

a) $\frac{h}{35}=\frac{9}{15}\Rightarrow h=\frac{9}{15}\times 35=21$

b)

$$\frac{A_1}{A_2}=\left(\frac{L_1}{L_2}\right)^2$$

$$\frac{9}{A}=\left(\frac{3}{5}\right)^2=\frac{9}{25}$$

$$A=25$$

c)

$$\frac{M_1}{M_2}=\left(\frac{L_1}{L_2}\right)^3$$

$$\frac{M_1}{25}=\left(\frac{3}{5}\right)^2=\frac{27}{125}$$

$$M_1=\frac{27}{125}\times 25=5.4$$

19.

a)

$$=\frac{1}{2}\times 90\times 20+\frac{1}{2}\times 30\times 40$$

$$=900+600$$

$$=1500m$$

b)

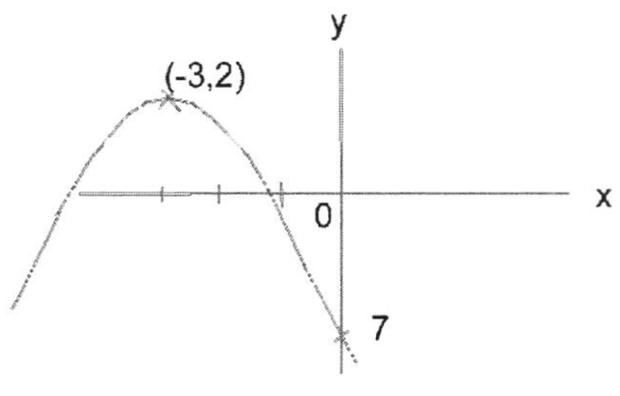
$$-(x+3)^2+2=0$$

$$(x+3)^2=2$$

$$(x+3)=\pm\sqrt{2}$$

$$x=-1.59\ or-4.41$$

c)



d) Inserting the graph y = 3 will not intercept as max y = 2 the graph y

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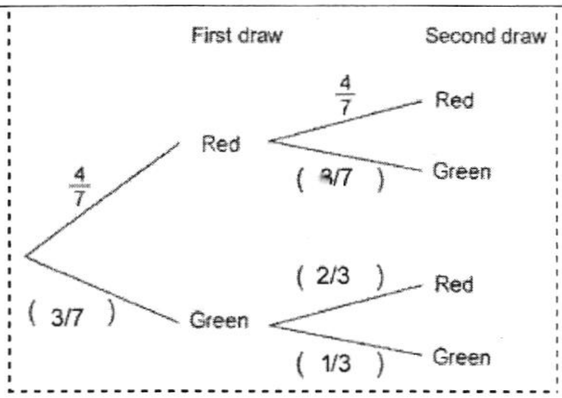
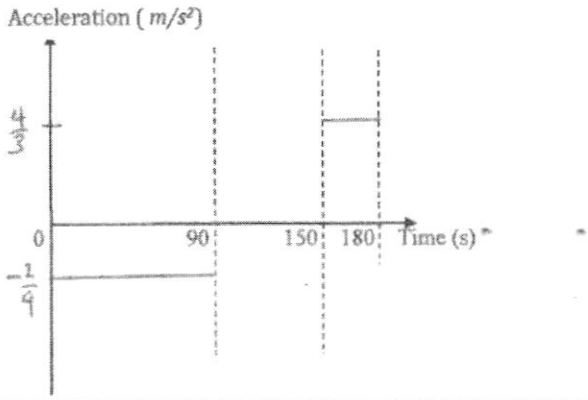
ANSWER KEY

	$=-x^2-6x-7$. Therefore, there is no solution.
20.	 a) $P(R,G)+P(G,R)=\frac{4}{7}\times\frac{3}{7}+\frac{3}{7}\times\frac{2}{3}=\frac{26}{49}$ b) $1-P(G,G)=1-\frac{3}{7}\times\frac{1}{3}=\frac{6}{7}$ *or* $P(R,R)+P(R,G)+P(G,R)=\frac{4}{7}\times\frac{4}{7}+\frac{4}{7}\times\frac{3}{7}+\frac{3}{7}\times\frac{1}{3}=\frac{6}{7}$
21.	a) $Deceleration=\frac{20}{90}=\frac{2}{9}m/s^2$ b) Total Distance Travelled $=\frac{1}{2}\times 90\times 20+\frac{1}{2}\times 30\times 40$ $=900+600$ $=1500m$ c) 
22.	a) i. $20\times 20\times 12=4800$ cm^3

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ANSWER KEY

	ii. $20 \times 20 \times d = 3000$ $d = \frac{3000}{20 \times 20}$ $= 7.5$ cm b) i. $\frac{1}{2} \times 12 \times 20 \times 20 = 2400cm^3$ ii. $\tan \angle BCZ = \frac{12}{20} = 30.9638$ $= 31°$ iii. $\sin 30.9638 = \frac{BZ}{20} = 10.3cm$
23.	a) i. $150 - 1.5t$ m ii. $0.5t$ m b) $h^2 = (150-1.5t)^2 + (0.5t)^2$ $= 22500 - 450t + 2.25t^2 + 0.25t^2$ $= 22500 - 450t + 2.5t^2$ $= 2.5t^2 - 450t + 22500(shown)$ c) $h^2 = 2.5(60)^2 - 450(60) + 22500$ $h^2 = 4500$ $h = 67.1m$ d) $100^2 = 2.5t^2 - 450t + 22500$ $2.5t^2 - 450t + 12500 = 0$ $t = \frac{450 \pm \sqrt{450^2 - 4(2.5) + 12500}}{2(2.5)}$ $= 145.68, 34.3224 \quad (145.68\ (rej))$ $Answer = 34.3s$ e) $150 - 1.5(34.3224) = 98.5m$
24.	a)

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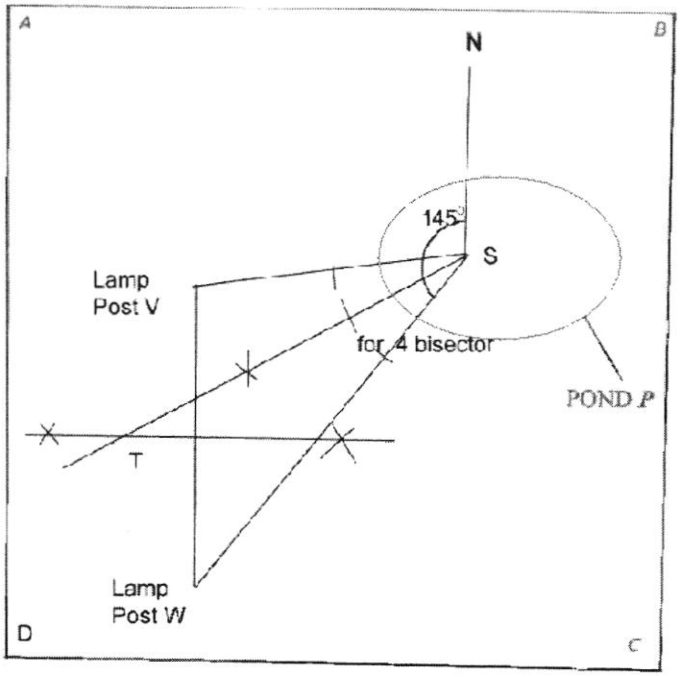
ANSWER KEY



b)

Bearing of W from s $= 360° - 145° = 218°$ $\quad$ (*Accept* $217° \pm 1°$)

Answer $217° \pm 1°$

BP~613

603

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION

MATHEMATICS
Paper 2

4048/02

11 SEPTEMBER 2018 (Tuesday) **2 hours 30 minutes**

CANDIDATE NAME	**Solutions (for students)**

CLASS

INDEX NUMBER

For Examiner's Use		
Q1	9	
Q2	8	
Q3	6	
Q4	7	
Q5	10	
Q6	11	
Q7	9	
Q8	11	
Q9	13	
Q10	16	
Total	/ 100	

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES

Answer **all** the questions.

Write your answers on the separate writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The use of a scientific calculator is expected, where appropriate.

If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1. [9] **(a)** The following table shows some information on the population of a country in 2016 and 2017. The total population of the country consists of residents and non-residents.

Year	Total population	Residents	Non-residents
2016	55.9 million	x	y
2017	56.1 million	54.45 million	1.65 million

[1 million people $= 1\times10^6$ people]

From 2016 to 2017, the number of non-residents decreased by 1.6%.

(i) Find the value of y, expressing your answer as k million, where k is a constant correct to 3 decimal places. [1]

(ii) Hence, find the value of x, giving your answer in standard form correct to 2 decimal places. [2]

(b) P is proportional to Q^n, where n is an integer.

State the value of n when

(i) P units is the force between two particles which is inversely proportional to the square of the distance Q mm between them, [1]

(ii) P m^3 is the volume of a sphere with radius Q m, [1]

(iii) P cm^3 is the volume of a cone with radius Q cm and a fixed height [1]

(c) Express as a single fraction in its simplest form

$$\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}.$$ [3]

Solution:

(a)(i)

$$y=\frac{1.65\text{million}}{100-1.6}\times100$$

$y=1.677\text{million (3dp)}$

(a)(ii)

$x=55.9\text{million}-1.677\text{million}$

$x=54.22317073\text{million}$

$x=54.22317073\times10^6$

$x=5.422317073\times10^7$

$x=5.42\times10^7\text{(2dp)}$

(b)(i)

P is proportional to Q^n,

P units is the force between two particles which is inversely proportional to the square of the distance Q mm between them

$n= -2$

(b)(ii) P is proportional to Q^n, P m^3 is the volume of a sphere with radius Q m $n = 3$	
(b)(iii) P is proportional to Q^n, P cm^3 is the volume of a cone with radius Q cm and a fixed height. $n = 2$	
(c) $\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}$ $=\frac{2}{2c-b}-\frac{3c-11b}{5(b^2-4c^2)}$ $=-\frac{2}{b-2c}-\frac{3c-11b}{5(b+2c)(b-2c)}$ $=\frac{-2(5)(b+2c)-(3c-11b)}{5(b+2c)(b-2c)}$ $=\frac{-10b-20c-3c+11b}{5(b+2c)(b-2c)}$ $=\frac{b-23c}{5(b+2c)(b-2c)}$	
OR	

$\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}$ $=\frac{2}{2c-b}-\frac{3c-11b}{5\left(b^2-4c^2\right)}$ $=\frac{2}{2c-b}-\frac{3c-11b}{5(b+2c)(b-2c)}$ $=\frac{2(5)(b+2c)(b-2c)-(3c-11b)(2c-b)}{5(b+2c)(b-2c)(2c-b)}$ $=\frac{-b^2+25bc-46c^2}{5(b+2c)(b-2c)(2c-b)}$ $=\frac{(2c-b)(b-23c)}{5(b+2c)(b-2c)(2c-b)}$ $=\frac{b-23c}{5(b+2c)(b-2c)}$	
OR	
$\frac{2}{2c-b}-\frac{3c-11b}{5b^2-20c^2}$ $=\frac{2\left(5b^2-20c^2\right)-(2c-b)(3c-11b)}{(2c-b)\left(5b^2-20c^2\right)}$ $=\frac{-b^2+25bc-46c^2}{(2c-b)\left(5b^2-20c^2\right)}$ $=\frac{(2c-b)(b-23c)}{(2c-b)(5)(b+2c)(b-2c)}$ $=\frac{b-23c}{5(b+2c)(b-2c)}$	

2. [8]	Jenny bought some jars of cookies for \$900. She paid \$$n$ for each jar of cookies.		
	(a)	Write down an expression, in terms of n, for the number of jars of cookies she bought.	**[1]**
	(b)	Jenny found that 2 jars of cookies were spoilt and could not be sold. Jenny sold each remaining jar of cookies for \$3 more than she paid for it. Write down an expression, in terms of n, for the total sum of money she received from the sale of the jars of cookies.	**[1]**
	(c)	Given that she made a profit of \$92 from the sale of the jars of cookies, form an equation in n and show that it reduces to $n^2+49n-1350=0$.	**[2]**
	(d)	Solve $n^2+49n-1350=0$, giving your solutions correct to 3 decimal places.	**[2]**
	(e)	Hence, find the selling price of each jar of cookies sold by Jenny, giving your solution correct to the nearest cent.	**[2]**

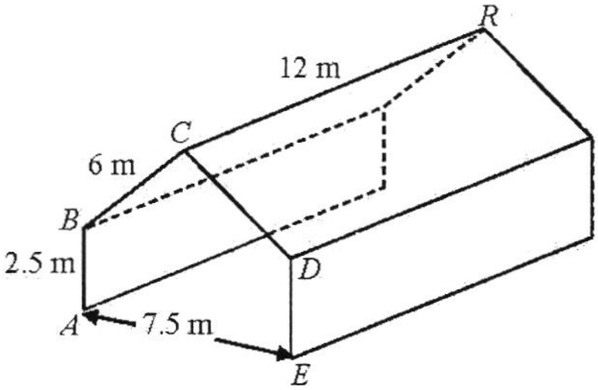
Remark: Deduct U for missing/ incorrect units in any part for this question where applicable

Solution: **(a)** $\left(\frac{900}{n}\right)$ jars of cookies	
(b) $\$\left(\frac{900}{n}-2\right)(n+3)$ **OR** $\$\left(900+\frac{2700}{n}-2n-6\right)$	**Remark:** Remember to write the correct units
(c) $\left(\frac{900}{n}-2\right)(n+3)-900=92$ $900+\frac{2700}{n}-2n-6-900=92$ $\frac{2700}{n}-2n-6=92$ $\frac{2700}{n}-2n-98=0$ $2700-2n^2-98n=0$ $n^2+49n-1350=0$ (Shown)	

(d) $n^2+49n-1350=0$ $n=\dfrac{-49\pm\sqrt{(49)^2-4(1)(-1350)}}{2(1)}$ $n=19.662$ (3dp) or $n=-68.662$ (3dp)	**Remark:** Must show working
(e) (Reject $n=-68.662$ as n>0) Selling price $=19.662+3$ =\$22.66 (nearest cent)	

3. [6]

The diagram shows a tent. The cross-section of the tent forms a pentagon *ABCDE* with two vertical sides of height 2.5 m and two slant sides of equal length 6 m. It is also given that the length of the tent *CR* is 12 m and the width of the tent *AE* is 7.5 m.



Find

(a)	the area of the cross section *ABCDE*,	**[2]**
(b)	the angle of elevation of *R* from *A*.	**[4]**

Solution:

(a)

height of triangle BCD = $\sqrt{6^2-\left(\frac{7.5}{2}\right)^2}=\sqrt{\frac{351}{16}}$

Area of the cross section *ABCDE*

$=\left(\frac{1}{2}\times 7.5\times\sqrt{\frac{351}{16}}\right)+(7.5\times 2.5)$

$=17.56405687+18.75$

$=36.31405687$

$=36.3\text{m}^2(3\text{sf})$

Remark: Remember to write the correct units

OR

$7.5^2=6^2+6^2-2(6)(6)\cos\angle BCD$

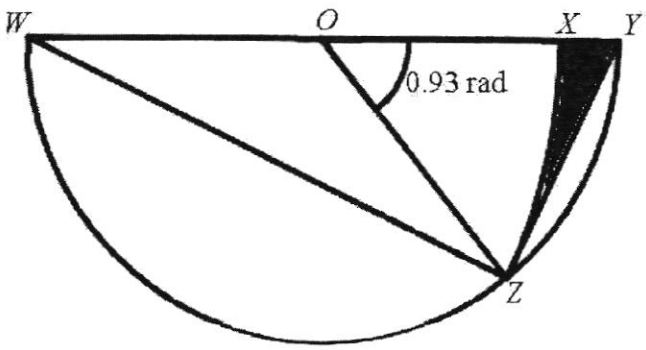
$\cos\angle BCD=\frac{6^2+6-7.5^2}{2(6)(6)}$

$\angle BCD=\cos^{-1}\frac{6^2+6-7.5^2}{2(6)(6)}$

$\angle BCD=77.36437491°$

OR

Let M is midpoint of BD. $\sin\angle BCM = \frac{(7.5 \div 2)}{6}$ $\angle BCM = \sin^{-1}\frac{(7.5 \div 2)}{6}$ $\angle BCD = 2\sin^{-1}\frac{(7.5 \div 2)}{6}$ $\angle BCD = 77.36437491°$ Area of the cross section *ABCDE* $= \left(\frac{1}{2}\times 6\times 6\times \sin 77.36437491°\right)+(7.5\times 2.5)$ $= 17.56405687 + 18.75$ $= 36.31405687$ $= 36.3\text{m}^2(3\text{sf})$	**Remark:** Remember to write the correct units
(b) Let N be the point vertically below R on the ground $RN = \sqrt{\frac{351}{16}} + 2.5$ $= 7.183748499m$ $AN = \sqrt{12^2 + \left(\frac{7.5}{2}\right)^2}$ $= 12.57229096m$ $\tan\angle RAN = \frac{RN}{AN}$ $\tan\angle RAN = \frac{7.183748499}{12.57229096}$ $\angle RAN = \tan^{-1}\frac{7.183748499}{12.57229096}$ $\angle RAN = 29.7°(1\text{dp})$ Angle of elevation = $29.7°(1\text{dp})$	**Remark:** -Remember to write statements to define clearly what you are finding -Remember to write the correct units

4. [7]	**(a)**	Explain, with mathematical calculations, why it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm.	**[3]**
	(b)	In the diagram below, *WZY* is a semicircle with centre *O*, radius 7 cm and angle $ZOX = 0.93$ radians. *WZX* is a sector of another circle with centre *W* and radius 12.5 cm.  Find the perimeter of the shaded area.	**[4]**

Solution:

(a)

$\pi(8)(\text{slant height}) = 115$

$\text{slant height} = \dfrac{115}{8\pi}$

$\text{height} = \sqrt{\left(\dfrac{115}{8\pi}\right)^2 - 8^2}$

$\left(\dfrac{115}{8\pi}\right)^2 - 8^2 = -43.06 < 0$

As height^2 is negative, height is undefined.
Hence it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm.

OR

Curved surface area of cone = 115

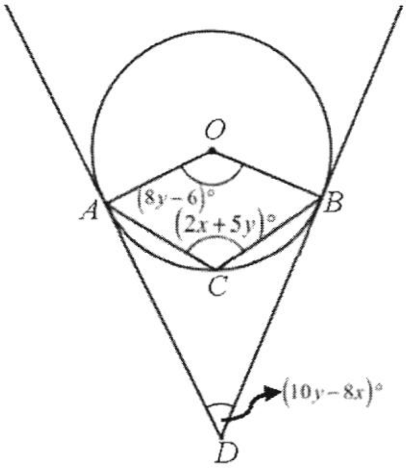
Base area of cone = $\pi(8)^2 = 201.06(2dp) > 115$

As base area of cone is greater than the curved surface area of cone, it is not possible to fold a sector of area 115 cm^2 into a cone of base radius 8 cm.

(b) $XY = 2(7) - 12.5 = 1.5$ Arc XZ $= 12.5\left(\dfrac{0.93}{2}\right)$ $\angle WZY = 90°$(angle in a semicircle is a right angle) $\sin\left(\dfrac{0.93}{2}\right) = \dfrac{YZ}{14}$ $YZ = 14\sin\left(\dfrac{0.93}{2}\right)$ Perimeter $= 14\sin\left(\dfrac{0.93}{2}\right) + 12.5\left(\dfrac{0.93}{2}\right) + 1.5$ $= 13.6\text{cm}(3\text{sf})$	

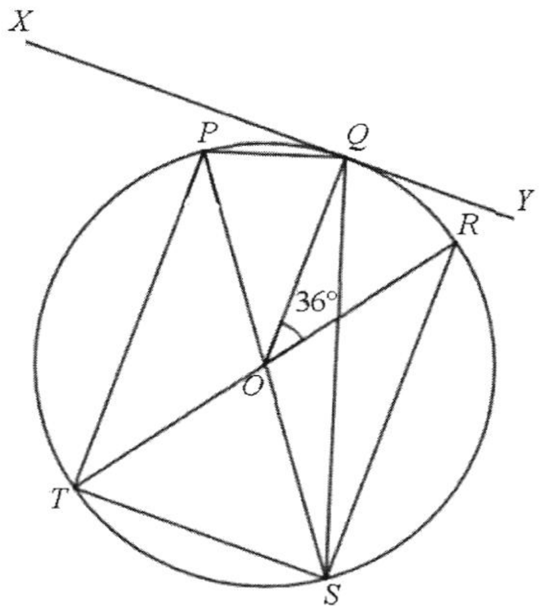
5. [10]

(a) In the diagram, A, B and C lie on a circle with centre O.
The tangents at A and B meet at D.
It is given that angle $AOB = (8y-6)^\circ$, angle $ACB = (2x+5y)^\circ$ and angle $ADB = (10y-8x)^\circ$.



(i) Stating your reasons clearly, show that $(8y-6)^\circ + (10y-8x)^\circ = 180^\circ$	[3]
(ii) Hence, by solving a pair of simultaneous equations, find the value of x and of y.	[3]

(b) The diagram shows a circle $PQRST$, with centre O. XY is a tangent to the circle at Q. It is given that angle $QOR = 36°$. TOR and POS are straight lines.



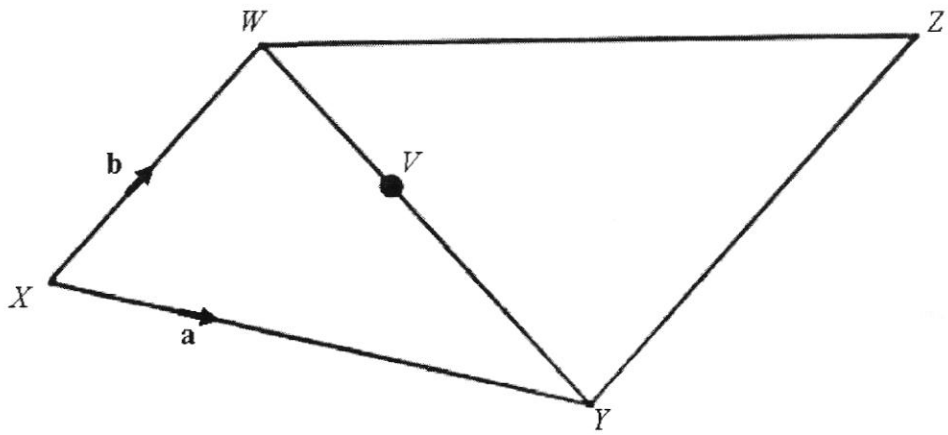
(i) Find, giving reasons for your answer, angle RSQ.	[1]
(ii) Prove that triangle PTS and triangle RST are congruent.	[3]

Solution: **(a)(i)** $\angle OAD = \angle OBD = 90°$ ($\angle$ between tangent and radius of a circle is a right angle) $90° + 90° + (8y-6)° + (10y-8x)° = 360°$ ($\angle$ sum of quadrilateral) $(8y-6)° + (10y-8x)° = 180°$ (Shown)	**Remark:** Remember to write the correct reasons in full, no short form
(a)(ii) $(8y-6)+(10y-8x)=180$ $-8x+18y=186$ $-4x+9y=93$ -----(1) $360-(8y-6)=2(2x+5y)$ $360-8y+6=4x+10y$ $4x+18y=366$ $2x+9y=183$ -----(2) Equations (1) + 2(2): $-4x+9y+2(2x+9y)=93+2(183)$ $27y=459$ $y=17$ Sub y=17 into (1): $-4x+9(17)=93$ $-4x=-60$ $x=15$ $\therefore x=15, y=17$	
(b) Remark: Remember to write the correct units	
(b)(i) $\angle RSQ = \frac{36°}{2} = 18°$ ($\angle$ at the centre is twice the $\angle$ at the circumference)	**Remark:** Remember to write the correct reasons in full, no short form

(b)(ii) $\angle PTS = \angle RST = 90°$ ($\angle$ in a semicircle is a right angle) $TS = ST$ (common length) $PS = RT$ (diameter of circle) Triangle PTS and Triangle RST are congruent (RHS)	**Remark:** -Remember to write the correct reasons in full, no short form -Remember to write the vertices in their corresponding order for all statements and conclusion -Remember to state the congruency test used
OR	
$\angle PTS = \angle RST = 90°$ ($\angle$ in a semicircle is a right angle) $\angle TPS = \angle SRT$ ($\angle$s in the same segment are equal) $\angle OTS = \angle OST$ (base angles of isosceles triangle) so $\angle PST = \angle RTS$ $TS = ST$ (common length) **OR** $PS = RT$ (diameter of circle) Triangle PTS and Triangle RST are congruent (AAS/ ASA)	**Remark:** Remember to write the correct reasons in full, no short form -Remember to write the vertices in their corresponding order for all statements and conclusion -Remember to state the congruency test used -2 angles and 1 side sufficient for AAS or ASA congruency test
OR	
$\angle OTS = \angle OST$ (base angles of isosceles triangle) so $\angle PST = \angle RTS$ $TS = ST$ (common length) $PS = RT$ (diameter of circle) Triangle PTS and Triangle RST are congruent (SAS)	**Remark:** -Remember to write the correct reasons in full, no short form -Remember to write the vertices in their corresponding order for all statements and conclusion -Remember to state the congruency test used -For SAS congruency test, the angle must be the included angle between the 2 sides

6. [11]

In the diagram, $WXYZ$ is a quadrilateral such that $\overrightarrow{XY}=\mathbf{a}, \overrightarrow{XW}=\mathbf{b}$ and $\overrightarrow{XW}=\frac{2}{3}\overrightarrow{YZ}$. V is a point on WY such that $5\overrightarrow{VY}=3\overrightarrow{WY}$.



(a)	What is the special name given to the quadrilateral $WXYZ$?		[1]
(b)	Express, as simply as possible, in terms of **a** and **b**.		
	(i)	$\overrightarrow{WY}$,	[1]
	(ii)	$\overrightarrow{XZ}$,	[1]
	(iii)	$\overrightarrow{XV}$.	[2]
(c)	Explain why X, V and Z lie on a straight line.		[2]
(d)	Prove that triangle XWV and triangle ZYV are similar.		[2]
(e)	Find		
	(i)	$\dfrac{\text{Area of triangle } XWV}{\text{Area of triangle } ZYV}$,	[1]
	(ii)	$\dfrac{\text{Area of triangle } ZWV}{\text{Area of triangle } ZYV}$.	[1]

Remark: Remember to write correct vector notation for vectors

Solution:

(a) Trapezium

(b)(i)

$\overrightarrow{WY}=\overrightarrow{WX}+\overrightarrow{XY}$

$\overrightarrow{WY}=-\mathbf{b}+\mathbf{a}$

OR

$\overrightarrow{WY}=\overrightarrow{XY}-\overrightarrow{XW}$

$\overrightarrow{WY}=\mathbf{a}-\mathbf{b}$

(b)(ii)

$\overrightarrow{XZ}=\overrightarrow{XY}+\overrightarrow{YZ}$

$\overrightarrow{XZ}=\mathbf{a}+\frac{3}{2}\mathbf{b}$

(b)(iii) $5\overrightarrow{VY} = 3\overrightarrow{WY}$ $\overrightarrow{VY} = \frac{3}{5}\overrightarrow{WY}$ $\overrightarrow{WV} = \frac{2}{5}\overrightarrow{WY}$ $\overrightarrow{XV} = \overrightarrow{XW} + \overrightarrow{WV}$ $\overrightarrow{XV} = \overrightarrow{XW} + \frac{2}{5}\overrightarrow{WY}$ $\overrightarrow{XV} = \mathbf{b} + \frac{2}{5}(\mathbf{a} - \mathbf{b})$ $\overrightarrow{XV} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ **OR** $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ **OR** $\frac{2}{5}(\mathbf{a} + \frac{3}{2}\mathbf{b})$	
OR	
$\overrightarrow{XV} = \overrightarrow{XY} + \overrightarrow{YV}$ $\overrightarrow{XV} = \overrightarrow{XY} + \frac{3}{5}(-\overrightarrow{WY})$ $\overrightarrow{XV} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$ $\overrightarrow{XV} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ **OR** $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ **OR** $\frac{2}{5}(\mathbf{a} + \frac{3}{2}\mathbf{b})$	
(c) $\overrightarrow{XZ} = \mathbf{a} + \frac{3}{2}\mathbf{b} = \frac{1}{2}(2\mathbf{a} + 3\mathbf{b})$ $\overrightarrow{XV} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ $\frac{XV}{XZ} = \frac{\left(1/5\right)}{\left(1/2\right)} = \frac{2}{5}$ $\overrightarrow{XV} = \frac{2}{5}\overrightarrow{XZ}$ $\overrightarrow{XV}$ and $\overrightarrow{XZ}$ are parallel. X is a common point. Hence X, V and Z lie on a straight line.	**Remarks:** -Can also use vectors XV and VZ or vectors VZ and XZ. -Division of vectors is undefined

<table>
<tr><td>(d)
$\angle XVW = \angle ZVY$ (vertically opposite $\angle$ s)
$\angle XWV = \angle ZYV$
(alternate $\angle$s, XW parallel to YZ)
$\angle WXV = \angle YZV$
(alternate $\angle$s, XW parallel to YZ)
Triangle XWV and Triangle ZYV are similar
(AA similarity test)</td><td rowspan="2">Remark:
-Remember to write the correct reasons in full, no short form
-Remember to write the vertices in their corresponding order for all statements and conclusion</td></tr>
<tr><td>OR
$\frac{XW}{ZY}=\frac{XV}{ZV}=\frac{WV}{YV}=\frac{2}{3}$
Triangle XWV and Triangle ZYV are similar
(SSS similarity test)</td></tr>
<tr><td colspan="2">OR</td></tr>
<tr><td colspan="2">$\frac{XV}{ZV}=\frac{WV}{YV}=\frac{2}{3}$
$\angle XVW = \angle ZVY$ (vertically opposite $\angle$ s)
Triangle XWV and Triangle ZYV are similar (SAS similarity test)
OR
$\frac{XW}{ZY}=\frac{WV}{YV}=\frac{2}{3}$
$\angle XWV = \angle ZYV$ (alternate $\angle$s, XW parallel to YZ)
OR
$\frac{XW}{ZY}=\frac{XV}{ZV}=\frac{2}{3}$
$\angle WXV = \angle YZV$ (alternate $\angle$s, XW parallel to YZ)
Triangle XWV and Triangle ZYV are similar (SAS similarity test)</td></tr>
<tr><td>(e)(i)
$\frac{\text{Area of triangle } XWV}{\text{Area of triangle } ZYV}=\left(\frac{2}{3}\right)^2=\frac{4}{9}$</td><td>Remarks: Division of vectors is undefined</td></tr>
<tr><td></td><td></td></tr>
<tr><td>(e)(ii)
$\frac{\text{Area of triangle } ZWV}{\text{Area of triangle } ZYV}=\frac{\left(\frac{1}{2}\times WV\times h\right)}{\left(\frac{1}{2}\times YV\times h\right)}=\frac{2}{3}$</td><td>Remarks: Division of vectors is undefined</td></tr>
</table>

7. [9] (a) The cash price of a waffle maker is \$149. Rose wants to start a waffle shop business and buys 5 waffle makers on hire purchase. She pays a deposit of 15% of the cash price followed by 24 equal monthly instalments with interest charged at a flat rate of 1.5% per annum. Calculate the amount of the monthly instalment, correct to the nearest cent. [2]

(b) Rose offers three types of waffle fillings at her shop: chocolate, cheese and blueberry.

The price of each type of waffle is shown in the table below.

Chocolate	Cheese	Blueberry
\$1.80	\$2.50	\$1.50

The table below shows the sale of waffles at Rose's shop for the months of June and July.

Month/ Fillings	Chocolate	Cheese	Blueberry
June	52	8	27
July	48	13	21

(i) Represent the prices of each type of waffle in a column matrix **P**. [1]

(ii) Represent the sale of waffles at Rose's shop for the months of June and July in a 2×3 matrix **W**. [1]

(iii) Evaluate the matrix **R** = **WP**. [2]

(iv) State what the elements of **R** represent. [1]

(v) By multiplying matrix **W** with a row matrix, find the matrix that represents the total number of each type of waffles Rose sold in June and July. [2]

Solution:

(a)

Balance

$$= \frac{100-15}{100}\times(149\times5) = \$633.25$$

Interest

$$= 633.25\times\frac{1.5}{100}\times\frac{24}{12}$$
$$= \$18.9975$$

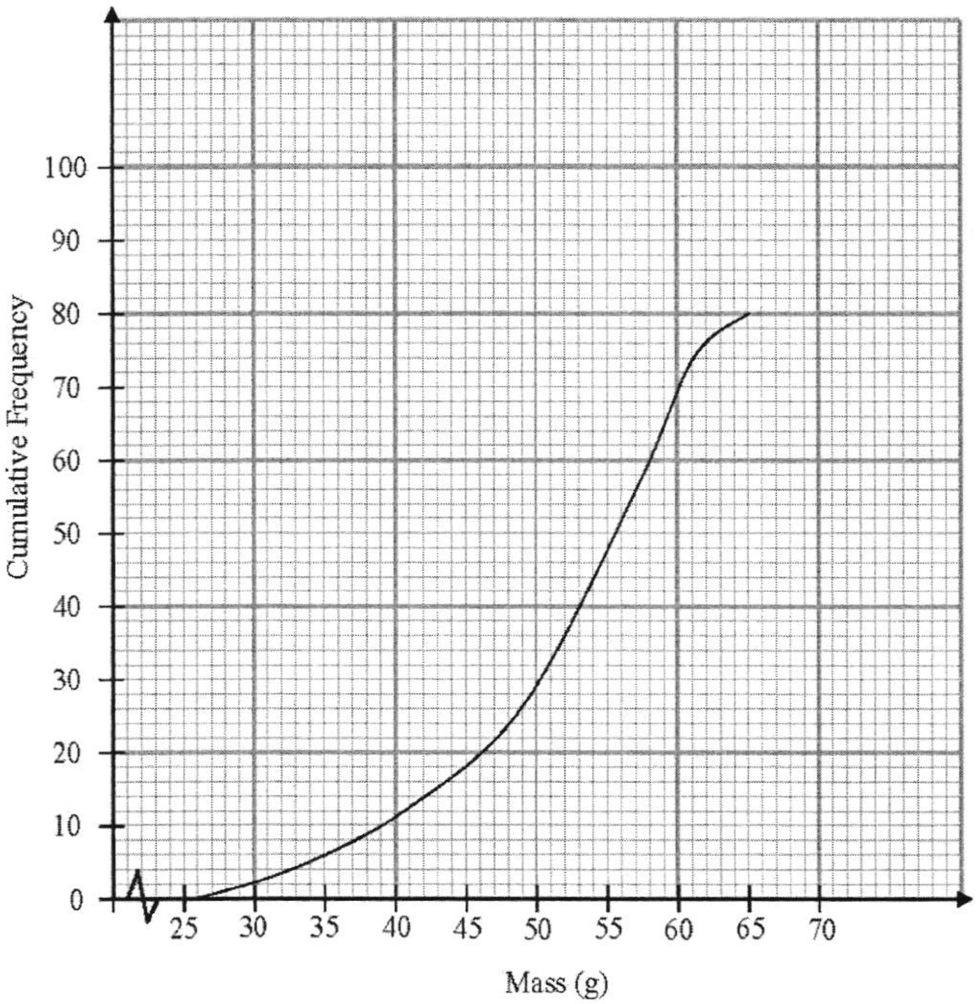
Amount of monthly instalment

$$= \frac{633.25+18.9975}{24}$$
$$= \$27.17697917\ (8\text{dp})$$
$$= \$27.18\ (2\text{dp})$$

Remark: Remember to write the correct units

(b)(i) $\mathbf{P}=\begin{pmatrix}1.8\\2.5\\1.5\end{pmatrix}$	**OR** $\begin{pmatrix}1.80\\2.50\\1.50\end{pmatrix}$
(b)(ii) $\mathbf{W}=\begin{pmatrix}52 & 8 & 27\\48 & 13 & 21\end{pmatrix}$	
(b)(iii) $\mathbf{R}=\mathbf{WP}$ $=\begin{pmatrix}52 & 8 & 27\\48 & 13 & 21\end{pmatrix}\begin{pmatrix}1.8\\2.5\\1.5\end{pmatrix}$ $=\begin{pmatrix}154.1\\150.4\end{pmatrix}$	
(b)(iv) The elements of matrix **R** represent the amount collected from the sales of waffles for the months of June and July **respectively**. **OR** The elements of matrix **R** represent the amount collected from the sales of waffles for **each** month. **OR** 154.1 represents the amount collected from the sales of waffles for the month of June. 150.4 represents the amount collected from the sales of waffles for the month of July.	**Remark:** Profit means selling price – cost price, different from amount of money collected
(b)(v) $\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}52 & 8 & 27\\48 & 13 & 21\end{pmatrix}$ $=\begin{pmatrix}100 & 21 & 48\end{pmatrix}$	

8. [11] The masses of 80 eggs collected at Farm A are recorded.
The cumulative frequency curve below shows the distribution of their masses.



(a) Use the curve to estimate

(i) the median mass, **[1]**

(ii) the 30^{th} percentile, **[1]**

(iii) the interquartile range. **[2]**

(b) The distribution of the masses of the eggs can be represented by the grouped frequency table below.

Mass (x g)	$25 < x \leq 35$	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$
Frequency	6	m	30	32

(i) Show that the value of m is 12. **[1]**

(ii) A worker in Farm A select two eggs at random, one after another, without replacement.
Find, as a fraction in its lowest form, the probability that both eggs are more than 55g. **[2]**

8	(c)	The masses of eggs at Farm B are also measured and recorded. Information relating to the masses of eggs at Farm B are given below. Mean = 53g Standard Deviation = 9g		
		(i)	A worker at Farm B says to a worker at Farm A: "The masses of the eggs at my farm are more consistent than the masses of the eggs at your farm." Do you agree with the worker at Farm B? Explain with mathematical calculations.	[2]
		(ii)	The worker at Farm B realises that the weighing machine is spoilt. Hence the mass of each egg should be 1g more than the measured mass. State the correct mean and standard deviation of the masses of eggs at Farm B.	[2]

Remark: Deduct U for missing/ incorrect units in any part for this question where applicable

Solution:	
(a)(i) $\frac{50}{100}\times 80 = 40\text{eggs}$ Median = 53g	**Remark:** Remember to write the correct units
(a)(ii) $\frac{30}{100}\times 80 = 24\text{eggs}$ 30^{th} percentile = 48g	**Remark:** Remember to write the correct units
(a)(iii) $\frac{25}{100}\times 80 = 20\text{eggs}$ Lower quartile = 46g $\frac{75}{100}\times 80 = 60\text{eggs}$ Upper quartile = 58g Interquartile range = 58 – 46 = 12g	**Remark:** -Remember to show working -Remember to write the correct units
(b)(i) Frequency for mass, $x \leq 45 = 18$ Frequency for mass, $x \leq 35 = 6$ $m = 18 - 6 = 12$ (Shown) **OR** $6 + m + 30 + 32 = 80$ $m = 80 - 6 - 30 - 32 = 12$ (Shown)	

(b)(ii)
P(both eggs more than 55g)
$=\frac{32}{80}\times\frac{31}{79}$
$=\frac{62}{395}$

(c)(i)

Mid value (x)	30	40	50	60
Mass (x g)	$25<x\leq 35$	$35<x\leq 45$	$45<x\leq 55$	$55<x\leq 65$
Frequency	6	12	30	32

Mean of masses of eggs at Farm A = 51g
Standard deviation of masses of eggs at Farm A
= 9.16515139=9.17g (3sf)

The standard deviation of the masses of eggs at Farm B, 9g., is **smaller than that of Farm A.** Hence the masses of eggs at Farm B is more consistent than that of Farm A. So **I agree** with the worker at Farm B.

(c)(ii)
Correct Mean = 54g
Correct Standard Deviation = 9g

9. [13] **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{x}{4}\left(8 - 6x + x^2\right).$$

Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	-1	0	0.5	1	1.5	2	2.5	3	5
y	-3.75	0	k	0.75	0.47	0	-0.47	-0.75	3.75

(a) Calculate the value of k. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-1 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-4 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) The equation $x^3 - 6x^2 + 8x = -12$ has only one solution.
Explain how this can be seen from your graph. [2]

(d) By drawing a tangent, find the gradient of the curve at the point (1, 0.75). [2]

(e) **(i)** Line L has gradient -0.5 and passes through the point (3, 1).
Draw line L on the same axes for $-1 \le x \le 5$. [2]

(ii) Write down the x-coordinate of the point where the two graphs intersect. [1]

(iii) This value of x is a solution of a cubic equation. Write down the cubic equation in the form $x^3 + Px^2 + Qx + R = 0$, where P, Q and R are integers. [2]

Solution: [Graph attached]

(a) $k = 0.66$ (2dp)

Remark: Remember to write k=, not y=

(b)

(c) $x^3-6x^2+8x=-12$ $x\left(x^2-6x+8\right)=-12$ $\frac{x}{4}\left(x^2-6x+8\right)=-3$ $\frac{x}{4}\left(8-6x+x^2\right)=-3$ $y=-3$ (Draw line $y=-3$. See graph in (b)) The line $y=-3$ cuts the curve $y=\frac{x}{4}\left(8-6x+x^2\right)$ at only one point. Hence $x^3-6x^2+8x=-12$ has only one solution.	
(d) Gradient = −0.25	
(e)(i) $y=-\frac{1}{2}x+c$ $1=-\frac{1}{2}(3)+c$ $c=\frac{5}{2}$ $y=-\frac{1}{2}x+\frac{5}{2}$ See graph in (b)	**Remark:** line $y=-\frac{1}{2}x+\frac{5}{2}$ must be drawn for $-1\le x\le 5$ as defined in question: Passes through (−1, 3), (3, 1) and (5, 0)
(e)(ii) $x=4.18$	(Accept $4.1\le x\le 4.3$)
(e)(iii) $\frac{x}{4}\left(8-6x+x^2\right)=-\frac{1}{2}x+\frac{5}{2}$ $x\left(8-6x+x^2\right)=-2x+10$ $8x-6x^2+x^3=-2x+10$ $x^3-6x^2+10x-10=0$	

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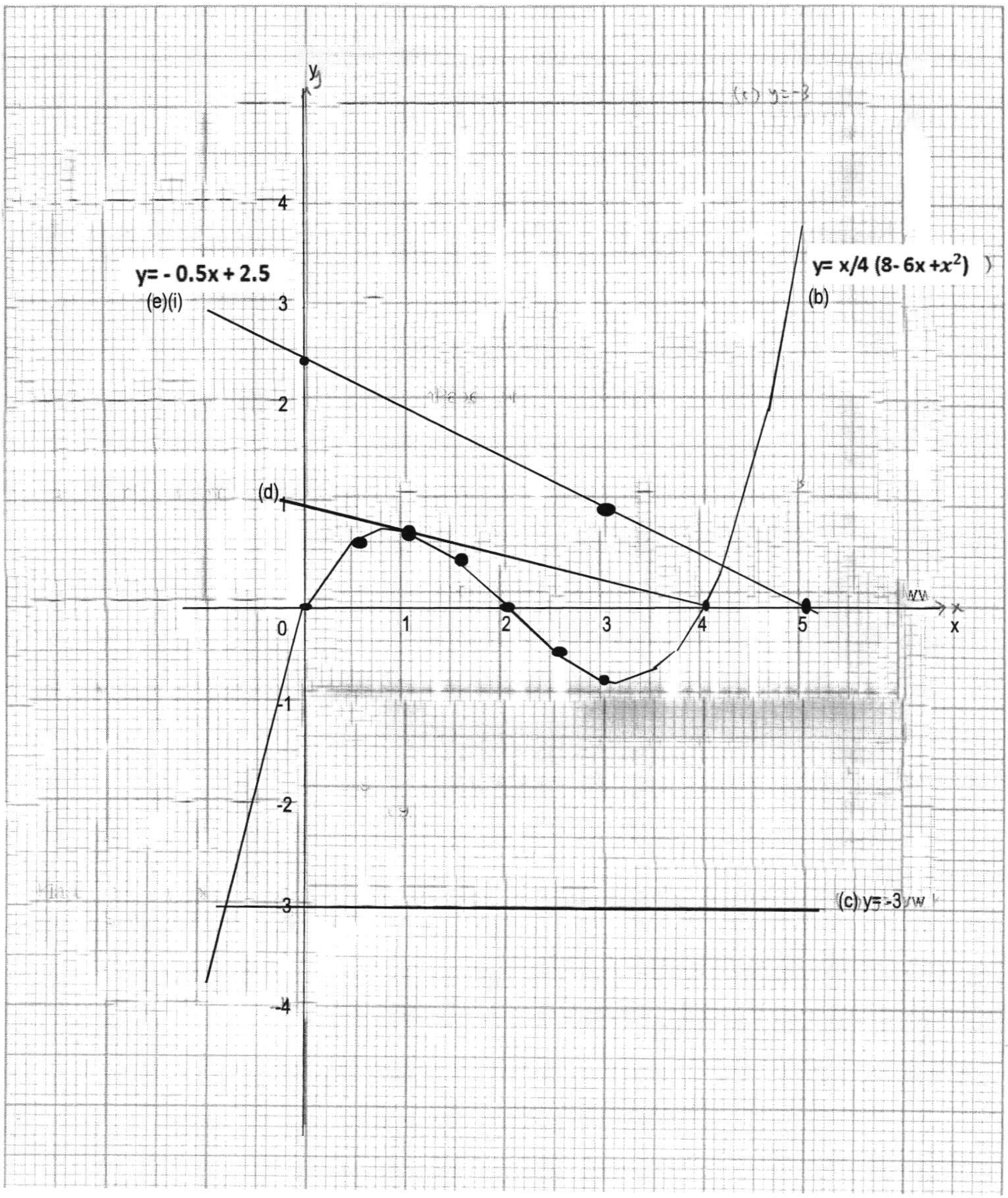


(c) x^3-6x^2+ 8x= -12

x (x^2 – 6x +8) = - 12

x/4 (8- 6x +x^2) = -3

Draw line y =-3

y= -3 intersects graph y= x/4 (8- 6x +x^2) only at 1 point

Hence x^3-6x^2+ 8x= -12 only has one solution

(d) Gradient at (1, 0.75) = 0 - 1/ 4 - 0

= - 0.25

(e) (i) y= - 0.5 x + b

1= - 0.5 (3) + b

b= 2.5

y= - 0.5x + 2.5

x	-1	0	5
y	3	2.5	0

(ii) x= 4.2 (Accept 4.1 ≤ x ≤ 4.3)

(iii) x/4 (8- 6x +x^2) = - 0.5 x + 2.5

8x - 6x^2 + x^3 = -2x + 10

x^3-6x^2+ 10x -10 = 0

10. [16]

A soda can may be modelled as a cylinder with a closed top and a hollow hemisphere hollowed in at the base of the can as shown in the diagram below

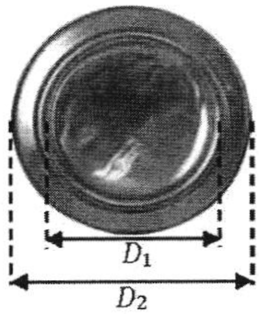
Information about the model of the soda can is given below.

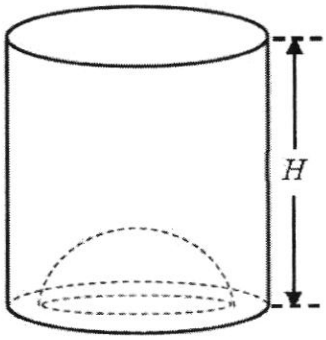
Height $(H) = 12.4$ cm

Inner Diameter (D_1) of base $= 6.7$ cm

Outer diameter (D_2) of base $= 7.9$ cm

Mass of empty can $= 15$g





(a) Using the model of the soda can in the diagram above, calculate

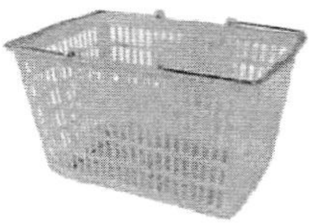
(i) the total surface area, in square centimetres, of the soda can. [5]

(ii) the volume, in cubic centimetres, of the soda can. [3]

(b) Harry uses a shopping basket to transport the soda cans filled with carbonated drink.

The soda cans will be placed with the base of the can lying on the base of the basket then stacked up vertically within the basket.

For safety reasons, all the soda cans must be contained inside the shopping basket. The maximum load that the shopping basket can carry is 55 pounds.



The shopping basket can be modelled by a frustrum of a inverted pyramid as shown in the diagram below.

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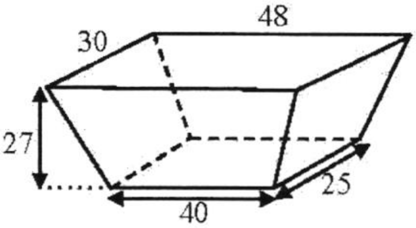

The frustrum above is obtained by removing the top portion of an inverted right rectangular pyramid. The flat rectangular base of the frustum has length 40 cm and width 25 cm. The remaining vertical height is 27 cm. The flat rectangular top of the frustum is 48 cm by 30 cm.

Other Useful Information

- Density of carbonated drink = 1.3 g/cm^3
- 1 pound is equivalent to 0.45 kg
- Mass = Volume × Density
- Safety information: Soda can is filled with carbonated drink up to a maximum of 90% of its total volume.

Assuming that each soda can is filled with carbonated drink to the maximum safe volume, find the maximum number of soda cans Harry can transport with the shopping basket at any one time.
Justify your answer with mathematical calculations. **[8]**

Solution: **(a)(i)** Total surface area of soda can $=\pi\left(\frac{7.9}{2}\right)^2+2\pi\left(\frac{7.9}{2}\right)(12.4)$ $+\left\{\pi\left(\frac{7.9}{2}\right)^2-\pi\left(\frac{6.7}{2}\right)^2\right\}+2\pi\left(\frac{6.7}{2}\right)^2$ $=441.0403387$ $=441\text{cm}^2$ (3sf)	**Remark:** Remember to write the correct units
(a)(ii) Volume of soda can $=\pi\left(\frac{7.9}{2}\right)^2(12.4)-\frac{2}{3}\pi\left(\frac{6.7}{2}\right)^3$ $=529.067503$ $=529\text{cm}^3$ (3sf)	**Remark:** Remember to write the correct units

(b) Volume of carbonated drink in each soda can $=\frac{90}{100}\times 529.067503$ $=476.1607527\text{cm}^3$ Mass of each soda can and carbonated drink $=(476.1607527\times 1.3)+15$ $=634.0089785\text{g}$ Maximum number of cans based on mass $=\frac{55\times 0.45\times 1000}{634.0089785}$ $=39.03730205$ $=39\text{cans}$ (nearest whole number rounded down) Number of layers of cans based on height $=\frac{27}{12.4}$ $=2.177419355$ $=2$ layers of cans (nearest whole number rounded down) Number of cans based on length of basket $=\frac{40}{7.9}$ $=5.063291139$ $=5$ cans (nearest whole number rounded down) Number of cans based on width of basket $=\frac{25}{7.9}$ $=3.164556962$ $=3$ cans (nearest whole number rounded down) Maximum number of cans based on dimensions $=5\times 3\times 2$ $=30$ cans Since the maximum number of cans by dimensions, 30 cans, is less than the maximum number of cans by mass, 39 cans, the maximum number of cans Harry can transport by basket is 30 cans.	**Remarks:** Remember to write clear statements and the correct units at each solution step