COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018

MATHEMATICS
PAPER 1

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL
SECONDARY FOUR NORMAL (O)

Monday 20 August 2018
08 00 – 10 00
2 h

4048/1

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Mrs Tan HP

For Examiner's Use	
Presentation	
Accuracy	
Total	80

Parent's Signature: _____

This paper consists of **17** printed pages including the cover page.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 Given that $\frac{1}{343} = 7^k$, find k.

Answer $k =$ ..... [1]

2(a) Simplify $\frac{3a}{8} - \frac{a+2}{4}$.

Answer ..... [2]

(b) Hence solve $\frac{3a}{8} - \frac{a+2}{4} = 0$.

Answer $a =$ ..... [1]

3(a) Solve the inequalities $-6 < 3 - 2x \leq 9$.

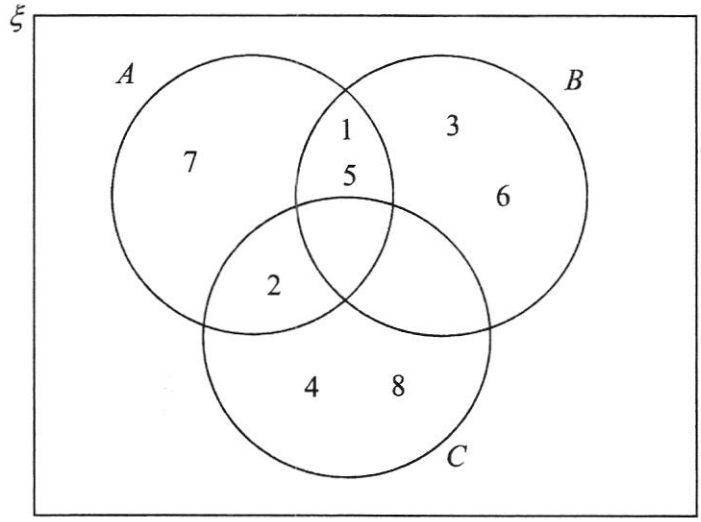
Answer ..... [2]

(b) Write down all the prime numbers that satisfy $-6 < 3 - 2x \leq 9$.

Answer $x =$ ..... [1]

4 $\xi = \{\text{ integers } x : 1 \leq x \leq 8\}$

The Venn diagram shows the elements of ξ and three sets, A, B and C.



Use one of the symbols below to complete each statement.

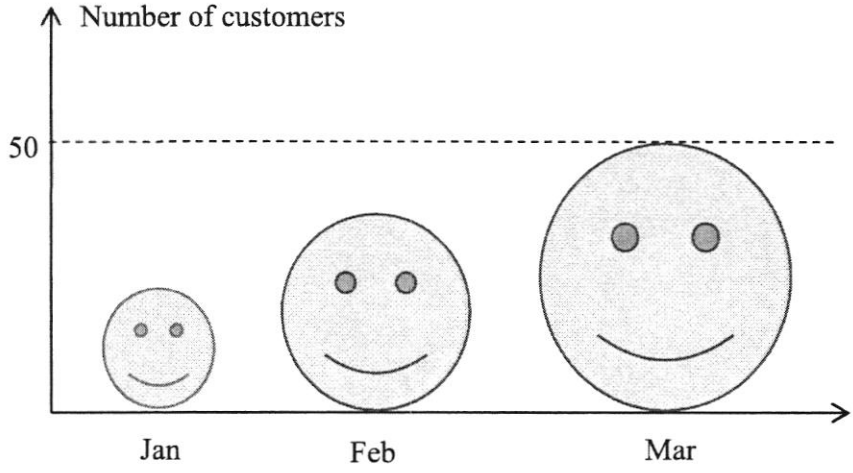
$\varnothing \quad \subset \quad \not\subset \quad \in \quad \notin \quad \xi$

(a) 3 ..... C [1]

(b) $\{1\}$ ..... $(A \cap B)$ [1]

(c) $(A \cup B \cup C)' =$ ..... [1]

5 Anil draws this graph to show the number of customers who visited his store for each of the first three months since it opened.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

..... [2]

6 A man normally takes 1 hour 45 minutes to cycle at a constant speed from Town A to Town B. One day, he increases his speed by 5 km/h and finds that the journey takes 25 minutes less than the normal time. Find his normal speed.

Answer ..... km/h [2]

7 A shopkeeper bought a pair of shoes for $56. He still made a percentage profit of 15% even though he offered a 20% discount to his customer. Calculate the normal selling price of this pair of shoes.

Answer $ ..... [2]

8 Two similar cans of beans, A and B, have total surface areas of 40 cm^2 and 90 cm^2 respectively. The smaller can contains 400 g of beans.
Jane worked out the amount of beans in the bigger can in the following way.

Amount of beans in smaller can = $40 \times 10 = 400$g
So amount of beans in bigger can = $90 \times 10 = 900$g

Is Jane's working correct? Support your answer with clear workings.

Answer .....

..... [2]

9 Alec has written down six numbers. The mean of these numbers is 4.5, the median is 5.5 and the mode is 6. The smallest number appears twice. Find the six numbers.

Answer ....., ..., ..., ..., ..., ... [3]

10(a) Factorise $x^2 - 169$.

Answer ..... [1]

(b) Use your answer to part (a) to find two factors of 731 other than 1 and 731.

Answer ..... , ... [2]

11 One day, Tom used this formula to estimate the air temperature at different heights above sea level.

$$T = 21 - \frac{h}{120},$$

where T is the temperature in degrees Celsius, and h is the height, in metres, above sea level.

(a) An aircraft records the air temperature as $-5°C$. Use the formula to estimate the height of the aircraft above sea level.

Answer ..... m [1]

(b) Two aircrafts are flying at different heights. The difference in temperatures at their two heights is $10°C$. Estimate the difference between the heights of the two aircraft.

Answer ..... m [2]

12 A packet of sweets is divided among Ahmad, Benny, Carl and Dexter. The number of sweets that Ahmad, Benny and Carl have are in the ratio 5 : 9 : 7. If Carl gets 12 sweets more than Ahmad and Benny gets twice as many sweets as Dexter, find

(a) the total number of sweets,

Answer ..... sweets [2]

(b) the number of sweets that Dexter gets.

Answer ..... sweets [1]

13 Jane can varnish 3 jars in 5 hours. Jim can varnish 2 jars in 3 hours.

Jane and Jim work together to varnish a total of 20 jars.

If they continue to varnish at the same rate, how long will it take them to varnish the 20 jars? Give your answer in hours and minutes, to the nearest minute.

Answer ..... hr ..... mins [3]

14 A club has between 50 to 60 members. During a donation drive, each member contributes \$12. The total proceeds is shared equally amongst 14 children in an orphanage, with each child getting \$$x$, where x is an integer.

(a) How many members are there in the club?

Answer ..... members [2]

(b) Find the value of x.

Answer ..... [1]

15 A driving theory test is set every month. John takes the test each month until he passes. Each time he takes the test, the probability he passes is 0.9.
Find the probability that John

(a) passes on his second attempt,

Answer ..... [1]

(b) takes at least two attempts to pass the test,

Answer ..... [1]

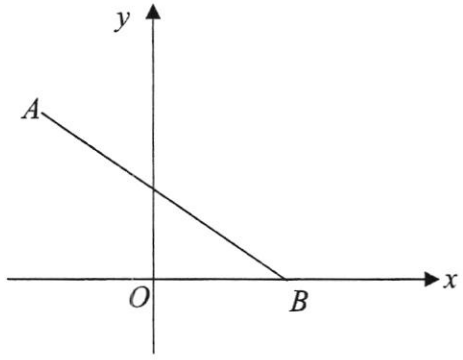
(c) fails the first n test,

Answer ..... [1]

(d) passes the test in one of the first n months.

Answer ..... [1]

16 In the diagram, the point $A=(-4,6)$ and B lies on the x-axis. The midpoint of AB lies on the y-axis.



(a) Find the coordinates of the midpoint of AB.

Answer (..... , ...) [2]

(b) The point $C(6,k)$ lies on a line parallel to AB and passing through the origin. Find the value of k.

Answer $k=$ ..... [2]

17(a) The ratio of an interior angle to an exterior angle of a regular polygon is 5 : 1. Find
(i) the size of each exterior angle,

Answer ..... ° [1]

(ii) the number of sides of the regular polygon.

Answer ..... sides [1]

(b) A decagon has 6 interior angles of 125°. The remaining interior angles are all equal. Find the size of the each of the remaining interior angles.

Answer ..... ° [2]

18(a) Convert 482 cm^3 per second to litres per hour.

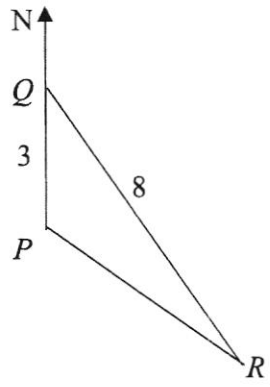
Answer ..... litres / hour [2]

(b) The average volume of water flowing over a waterfall is 7.79×10^3 litres per second. After a rainstorm the volume of water increased to 2.38×10^4 litres per second.

Calculate the percentage increase in the volume of water flowing over the waterfall.

Answer ..... % [2]

19 The diagram shows the positions of three points P, Q and R on level ground.
$PQ = 3$ m, $QR = 8$ m and $\sin Q\hat{P}R = \frac{7}{8}$. Q is due north of P.



(a) Calculate the bearing of R from P.

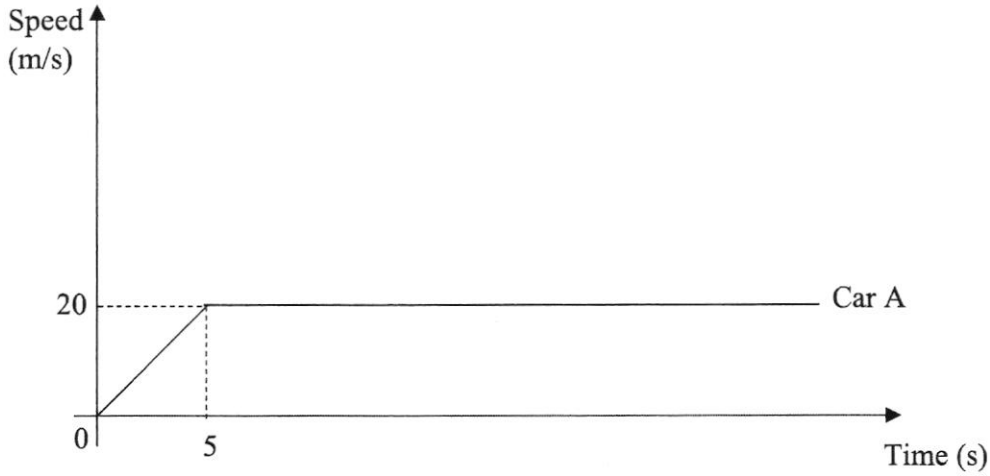
Answer ..... ° [1]

(b) A particle starts from R and moves in the direction of RP. Find the distance travelled by the particle such that it is nearest to Q.

Answer ..... m [3]

20 Car A and Car B travel along the same straight route. Car A accelerates from rest to a speed of 20 m/s in a time of 5 seconds. It then continues at this speed. Car B starts 5 seconds later and accelerates to a speed of 67.5 m/s in 30 seconds after which it continues at this speed.

(a) The speed-time graph for Car A is shown in the diagram. Sketch the speed-time graph for Car B in the same axes.



[1]

(b) Explain, with relevant workings, whether Car B will overtake Car A within the first 20 seconds after Car A starts its journey.

Answer ..... [3]

21(a) L is the point $(4,-2)$. The point M is the result of the translation of point L by $\begin{pmatrix}-8\\-6\end{pmatrix}$.

(i) Find the coordinates of point M.

Answer $M = (\ldots\ldots, \ldots\ldots)$ [1]

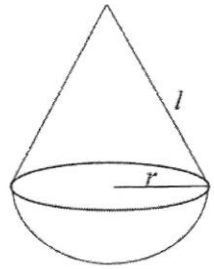
(ii) Find the equation of line LM.

Answer ..... [2]

(b) Explain whether or not $\begin{pmatrix}-3\\8\end{pmatrix}$ and $\begin{pmatrix}9\\-24\end{pmatrix}$ are parallel.

Answer ..... [2]

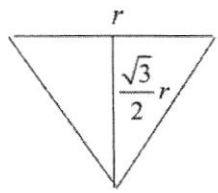
22(a) A solid is made from a cone and a hemisphere. The cone has radius r cm and slant height l cm. The hemisphere has radius r. Write down the total surface area of the solid in terms of r and l.



Answer ..... cm^2 [1]

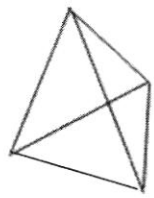
(b) The height and base an equilateral triangle are $\frac{\sqrt{3}}{2}r$ cm and r cm respectively.

(i) Find the area of the equilateral triangle.



Answer ..... cm^2 [1]

(ii) 4 of the equilateral triangles in (i) are used to make a tetrahedron (a right triangular pyramid) shown in the diagram. Find the total surface area of the tetrahedron.



Answer ..... cm^2 [1]

(c) The total surface area of the solid in (a) is equal to the total surface area of the tetrahedron in (b). Find l in terms of r.

Answer $l =$ ..... [2]

23 In supermarket A, water costs \$1.50 per litre, milk costs \$2.40 per litre and cola costs \$1.40 per litre. In supermarket B, water costs \$0.20 more per litre, milk costs \$0.40 less per litre and cola costs \$0.10 less per litre.

This information can be represented by the matrix $\mathbf{P} = \begin{matrix} \\ \text{A} \\ \text{B} \end{matrix}\begin{matrix} \begin{matrix} \text{W} & \text{M} & \text{C} \end{matrix} \\ \begin{pmatrix} 1.5 & 2.4 & 1.4 \\ 0.2 & -0.4 & -0.1 \end{pmatrix} \end{matrix}$.

(a) Andy and John go shopping.
Andy buys 4 litres of water, 2 litres of milk and 3 litres of cola.
John buys 3 litres of water and 4 litres of cola.
Represent their purchases in a (3×2) matrix $\mathbf{Q}$.

Answer (a) $\mathbf{Q} = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$ [1]

(b) Evaluate the matrix $\mathbf{R} = \mathbf{PQ}$.

Answer (b) $\mathbf{R} = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(c) From your answer in (b), write down how much money John would save by shopping in supermarket A.

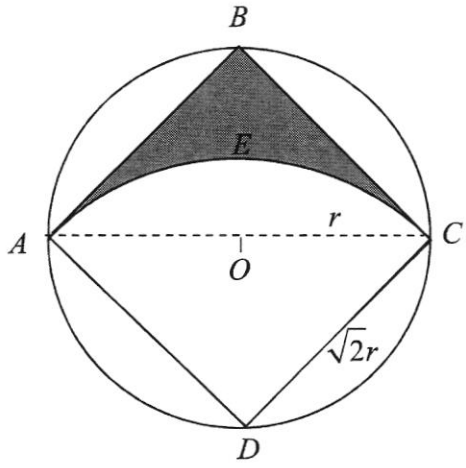
Answer (c) \$..... [1]

(d) Use your answer in (b) to explain whether it is better for Andy to shop at supermarket A or supermarket B.

Answer ..... [1]

24 $ABCD$ is a square of sides $\sqrt{2}r$ cm. Its vertices lie on the circumference of a circle, with centre O and radius r. Arc AEC has centre D.

What fraction of the circle $ABCD$ is **not** shaded? Give your answer in terms of π.



Answer ..... [5]

END OF PAPER

COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018

MATHEMATICS
PAPER 2

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
SECONDARY FOUR NORMAL ACADEMIC (EXPRESS)
4048/2

Tuesday 21 August 2018
08 00 – 10 30
2h 30min

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Name of setter: Mrs Philip

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Factorise completely $x^3 - x - 1 + x^2$. [2]

(b) Express as a single fraction in its simplest form

(i) $\dfrac{y}{x^2 + y^2 - 2xy} + \dfrac{x}{xy - x^2}$, [2]

(ii) $\dfrac{m^2}{4n^4} \div \dfrac{m^5}{\left(6n^3\right)^2}$. [1]

(c) Solve the equation $\dfrac{5}{3-2x} = x - 7$. [3]

(d) **(i)** Express $y^2 + 7y + 5$ in the form $\left(y + a\right)^2 + b$. [1]

(ii) **Hence** solve the equation $y^2 + 7y + 5 = 0$, giving your answers correct to two decimal places. [2]

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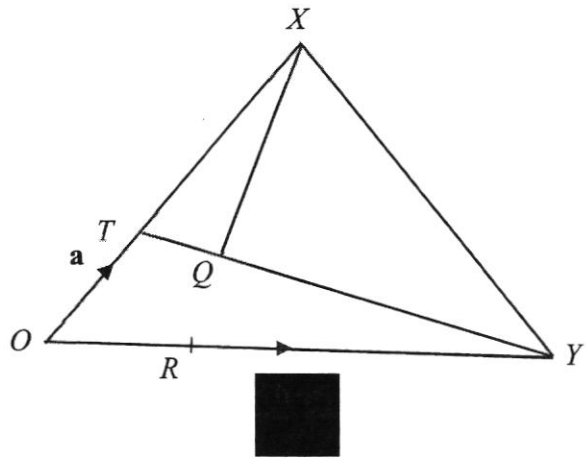
2 The lengths of the sides of a triangle are $(x-1)$ cm, $(x+1)$ cm and $(x+3)$ cm. The largest angle is 120°.

(i) Write down an equation in x to represent this information. [3]

(ii) Solve the equation and find the sides of the triangle. [3]

(iii) Find the **sine** of the smallest angle. [2]

3



In the diagram, $\overrightarrow{OT} = \mathbf{a}$ and $\overrightarrow{OY} = \mathbf{b}$.

X is a point on OT produced such that $OT : TX = 2 : 3$ and Q is a point on TY such that $TQ : QY = 1 : 5$.

(a) Express as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{XY}$, [1]

(ii) $\overrightarrow{TY}$, [1]

(iii) $\overrightarrow{QY}$, [1]

(iv) $\overrightarrow{XQ}$. [2]

R lies on OY such that $\overrightarrow{OR} = \frac{1}{4}\overrightarrow{OY}$.

(b) Express the vector $\overrightarrow{XR}$ in terms of **a** and of **b**. [2]

(c) Show that $\overrightarrow{XQ} = h\overrightarrow{XR}$ where h is a constant. [1]

(d) Write down two facts about X, Q and R. [2]

(e) Find the ratio of the area of triangle XQY to the area of triangle OXY. [2]

4 **(a)** Jack wants to buy a toy from a Japanese web-site for 4650 yen. The Japanese customs charges a 16% VAT for all overseas mail orders. The freight charge for product delivery to Singapore is 2800 yen.

(i) Use the information in the conversion table to calculate the total cost in Singapore dollars, if Jack buys the toy by mail order. [2]

Singapore dollars to 100 units of foreign currency.	Buying	Selling
Japanese Yen	1.2660	1.2798

(ii) The same toy, normally priced locally at S$160.50, exclusive of GST, was sold at a discount of 20% during the Great Singapore Sale. Calculate the selling price of the toy, purchased during the sale inclusive of 7% GST. [2]

(iii) Calculate the percentage Jack saved when buying the toy by mail order. [2]

(b) Amanda invested $ 8000 over a period of 2 years into two different investment plans.
Plan A offers 9.25% per annum of compound interest compounded annually,
Plan B offers 9 % per annum compound interest compounded monthly.

Which plan is a better choice? And why? [3]

(c) The cash price of a Television set is $ 4000.
John purchases the Television set on a hire purchase scheme making n monthly instalments of $90.

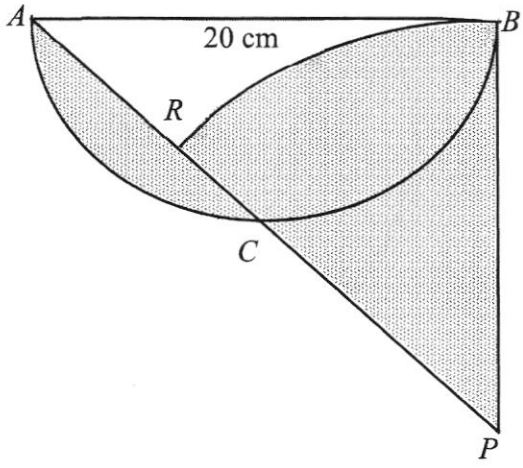
(i) Express, in terms of n,
(a) the total amount payable by hire purchase, [1]
(b) the interest incurred. [1]

(ii) The flat rate for the hire purchase loan is 3% per annum. Express the interest payable in terms of n. [1]

(iii) Form an equation in n and solve it. [1]

(iv) Hence, state the duration of the hire purchase loan in years and months. [1]

5



The diagram shows a semi-circle ACB and a sector PBR of a circle with centre P. It is given that AB is perpendicular to BP and $AB=RP = 20$ cm.

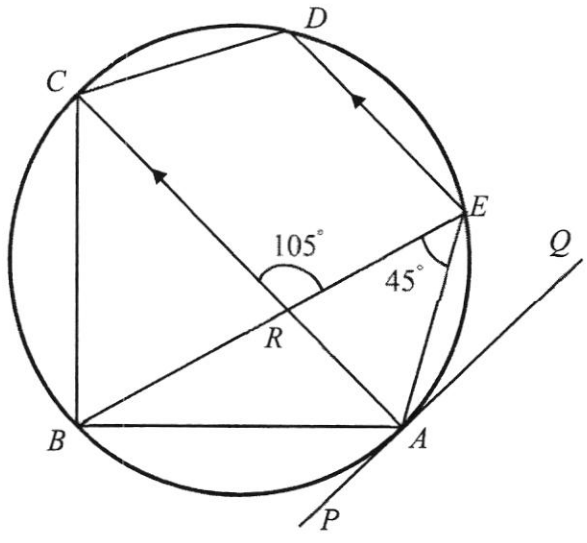
(i) Find, in radians, the angle BPA. [1]

(ii) For the shaded region $ACBRA$, find, correct to one decimal place,

(a) the area, [3]

(b) the perimeter. [2]

6 In the diagram, A, B, C, D and E are points on a circle.
DE is parallel to CA, PQ is a tangent to the circle at A.
$AB = BC$, angle $BEA = 45^\circ$ and angle $CRE = 105^\circ$.



(a) **(i)** Show that AC is the diameter of the circle. [2]

Find, stating reasons clearly,

(ii) angle EAQ, [2]

(iii) angle CDE. [2]

(b) Given that $AR = \frac{1}{4}AC$ and the area of triangle ARE is 90 cm^2, calculate the area of triangle BRC.

[2]

7 The variables x and y are connected by the equation

$$y = \frac{8}{x} - 3x + 8 .$$

Some corresponding values of x and y, correct to one decimal places, are given by the table below.

x	1	2	3	4	5	6	7	8
y	13	6	1.7	–2	–5.4	–8.7	p	–15

(a) Find the value of p. [1]

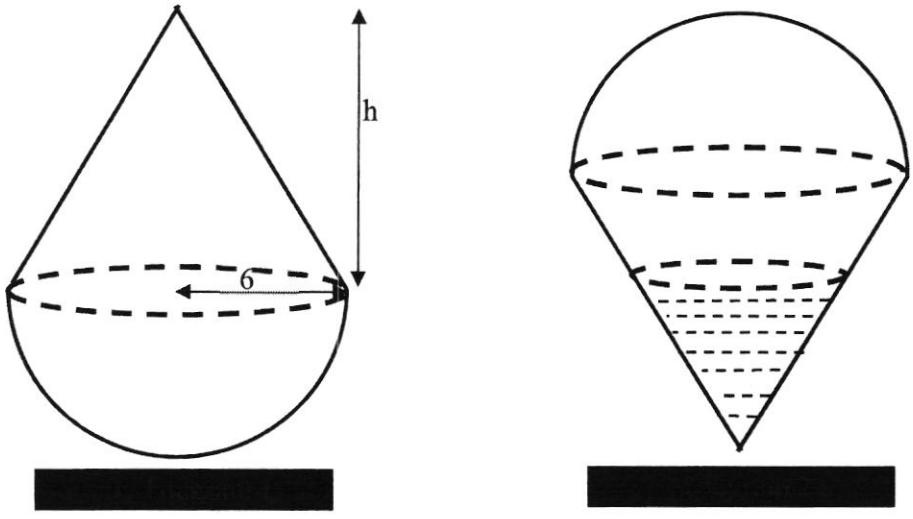
(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for the values of x in the range $1 \leq x \leq 8$. Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for the values of y in the range $-16 \leq y \leq 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = 1.5$. [2]

(d) By drawing a suitable straight line, solve the equation $2x^2 - 6.5x = 4$. [2]

8 A hollow glass container, shown in Diagram 1, is formed by joining a hemispherical base to a cone.
The hemisphere has a radius of 6 cm and the height of the cone is h cm.
The volume of the cone is 980 cm^3.

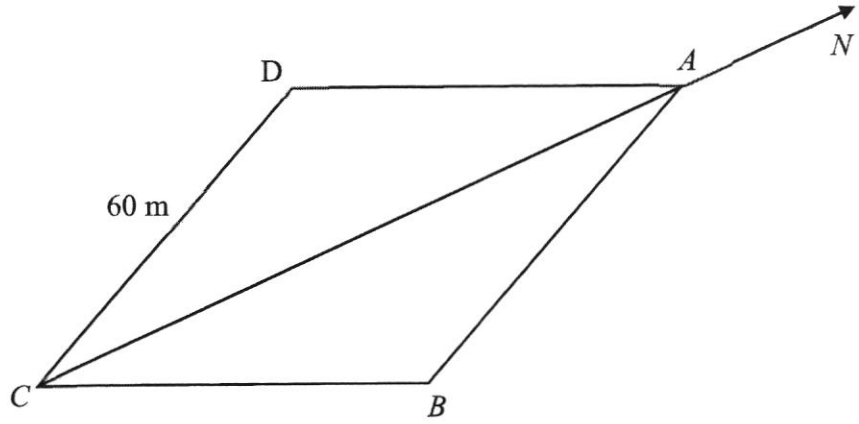


(i) Show that $h = 26.0$ cm. [2]

(ii) Find the surface area, in square metres, of the exterior of container. [3]

(b) The container was half filled with water and then inverted as shown in diagram 2.
Find the height of water level in Diagram 2. [4]

9



In the diagram, A, B, C and D are points on a level field forming a shape of a rhombus with $CD = 60$ m and A is due north of C. The bearing of D from C is 332°.

(a) Find the bearing of D from A. [1]

(b) State the bearing of B from D. [1]

(c) Calculate the distance BD. [1]

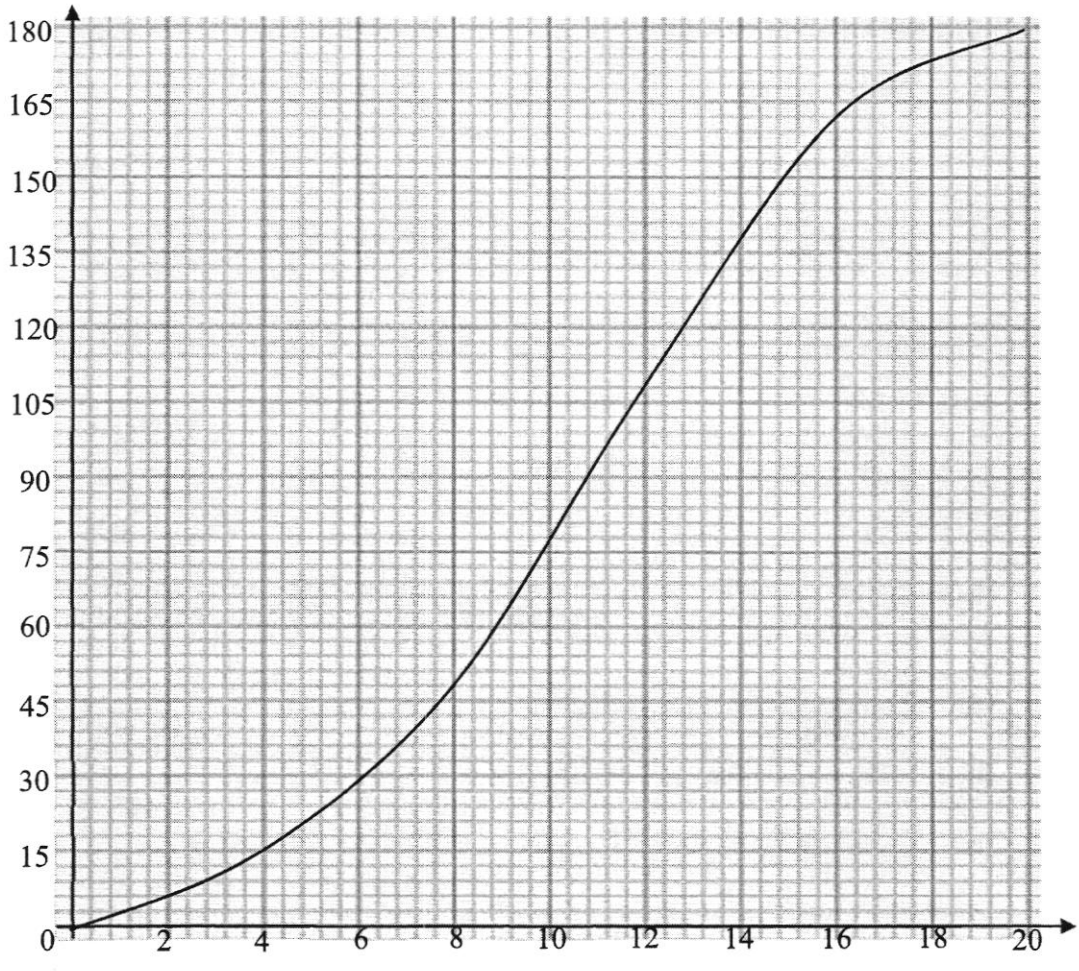
A bird flies horizontally from C to A at the fixed height of 30 m.

(d) Find the greatest angle of elevation of the bird from B as the bird flies above CA. [2]

The field is drawn on a map with scale 1 : 2000.

(e) Find the area of the field $ABCD$ on the map in cm^2. [2]

10 **(a)** 180 alkaline batteries were tested to find out how long they would last.
The cumulative frequency curve below shows the distribution of their life in hours.



(i) Use the graph to estimate

(a) the median time, [1]

(b) the interquatile range of the battery life, [2]

(c) the percentage of the batteries which lasted at least 12 hours. [1]

(ii) The life in hours of 180 lithium batteries has the same interquartile range as the alkaline batteries but a higher median.
Describe how the cumulative frequency curve for the life of lithium batteries differ from the curve for the alkaline batteries. [1]

(b) The table shows the life in hours of another set of 300 batteries that were tested.

Life (x hours)		$2 \leq x < 4$	$4 \leq x < 6$	$6 \leq x < 8$	$8 \leq x < 10$	$10 \leq x < 12$
Frequency	Alkaline	4	22	30	62	10
	Lithium	3	41	52	71	5

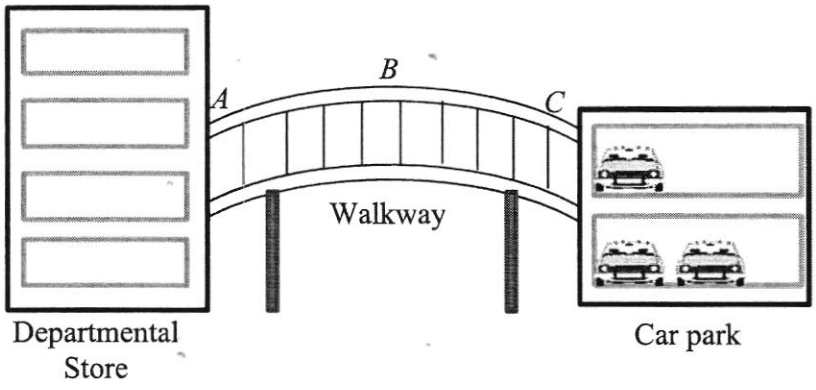
(i) Tom randomly chose one of these batteries.
Find, as a fraction in its lowest terms, the probability that the battery has

(a) a life of 10 hours or more , [1]

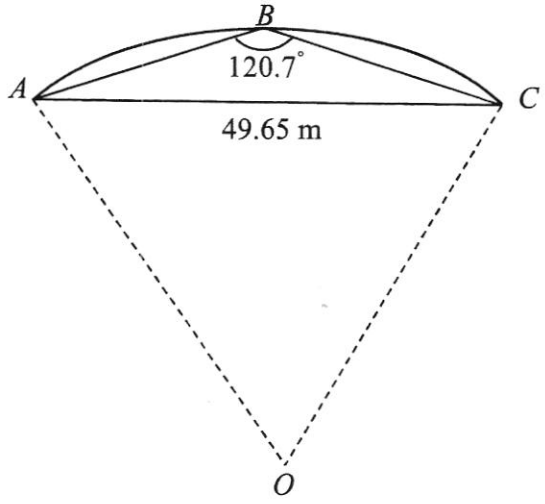
(b) life of under 6 hours. [1]

(ii) Tom randomly chose two of these batteries.
Find the probability that both the batteries chosen are alkaline and both have a battery life of under 8 hours. [2]

11 The figure shows a pedestrian walkway joining a multi-storey car park and a Departmental Store.



To estimate its length the walkway is modelled by the arc ABC as shown in the figure below, where A is the entrance to the department store and C is the exit to the car park.
The arc ABC is part of a sector with centre O.



Given $AC = 49.65$ m and angle $ABC = 120.7^\circ$,

(a) show that $AO = 29$ m. [3]

(b) show that the length of arc ABC is 60 m. [1]

(c) As a safety measure , John is required to conduct an emergency evacuation drill. He planned to conduct the drill on a Thursday and record the time taken by visitors to evacuate the walkway. The table below shows the average walking speed of visitors along the walkway to the Departmental Store at various timings in a day.

Time	Average walking speed in km/h	
	Weekends	Week days
11 00	4.5	5.0
14 00	3.5	4.5
17 00	3.5	3.5

At 12 noon, John will make an announcement for everyone to evacuate the department store via exit *A*.

John predicts each visitor will be able to evacuate the walkway from *A* to *C* in less than a minute.

(i) With clear mathematical working, determine whether John's prediction is accurate. [3]

(ii) State 1 assumption you made in the calculation [1]

End of Paper

2018 Prelim Exam MAP1 Answers

1 $\frac{1}{343} = 7^k$

$\frac{1}{7^3} = 7^k$

$k = -3$

2(a) $\frac{3a}{8} - \frac{a+2}{4}$

$= \frac{3a - 2a - 4}{8}$

$= \frac{a-4}{8}$

(b) $\frac{3a}{8} - \frac{a+2}{4} = 0$

$\frac{a-4}{8} = 0$

$a = 4$

3(a) $-6 < 3 - 2x \leq 9$

$-9 < -2x \leq 6$

$-6 \leq 2x < 9$

$-3 \leq x < 4.5$

(b) $x = 2, 3$

4(a) $3 \notin C$ [1]

(b) $\{1\} \subset (A \cap B)$ [1]

(c) $(A \cup B \cup C)' = \varnothing$ [1]

5 The scale on the vertical axis is not defined so it is not possible to determine the number of customers for Jan and Feb.
A comparison of the number of customers over the three months may lead to a misinterpretation.

6 Let his normal speed by x km/h.

$\frac{105}{60}x = \frac{80}{60}(x+5)$

$\frac{7}{4}x = \frac{4}{3}x + \frac{20}{3}$

$x = 16$

7 Let the normal selling price be \$$x$.

Discounted price = 115% of cost price

$$\frac{80}{100}x = \frac{115}{100} \times 56$$

$$x = 80.50$$

8

Let the height/radius of base of each can be h cm.

$$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$$

$$\frac{40}{90} = \left(\frac{h_1}{h_2}\right)^2$$

$$\frac{h_1}{h_2} = \frac{2}{3}$$

$$\frac{M_1}{M_2} = \left(\frac{2}{3}\right)^3$$

$$M_2 = 1350$$

Since the amount of beans in the bigger can should be 1350g, Jane's workings is incorrect.

9 Let the smallest number be x.

The numbers are x, x, 5, 6, 6, 6.

$$\frac{x + x + 5 + 6 + 6 + 6}{6} = 4.5$$

$$x = 2$$

The numbers are 2, 2, 5, 6, 6, 6.

10(a) $x^2 - 169$

$= x^2 - 13^2$

$= (\boldsymbol{x+13})(\boldsymbol{x-13})$

(b) $731 = 900 - 169$

$= 30^2 - \boldsymbol{13^2}$

$= (30+13)(30-13)$

$= 43 \times 17$

The factors are 17 and 43.

11(a)

$$-5 = 21 - \frac{h}{120}$$

$$h = 3120 \text{ m}$$

(b)

Let the heights of the two aircrafts be h_1 and h_2.

$T_1 = 21 - \frac{h_1}{120}$

$T_2 = 21 - \frac{h_2}{120}$

$T_1 - T_2 = 21 - \frac{h_1}{120} - 21 + \frac{h_2}{120}$

$10 = \frac{h_2 - h_1}{120}$

$h_2 - h_1 = 1200$

The difference is heights is 1200 m.

12(a) $A : B : C : D$

$5 : 9 : 7 : \frac{9}{2}$

C has 2 parts more than A, which is 12 sweets,

25.5 parts is equivalent to 153 sweets.

(b) 4.5 parts is equivalent to 27 sweets.
Dexter gets 27 sweets

13 In 1 hr,

Jane varnishes $\frac{3}{5}$ jar,

Jim varnishes $\frac{2}{3}$ jar;

together they varnish $\frac{3}{5} + \frac{2}{3} = \frac{19}{15}$ jar.

For 20 jars, they need $20 \times \frac{15}{19} = 15\frac{15}{19}$ hr

$= 15$hr 47min

14(a) Let the number of members be y.
$12y$ is divisible by 14 children.

$2^2 \times 3 \times y$ is divisible by 2×7.

y must have 7 as its factor.

Since $50 \le y \le 60$, $\therefore y = 56$

There are 56 members in the club.

(b)

$x = \frac{56 \times 12}{14} = 48$

15(a) P(1st : fail and 2nd : pass)
$= 0.1 \times 0.9$
$= 0.09$

(b) P(at least 2 attempts)
$=1 - $ P(1 attempt)
$= 1 - 0.9$
$= 0.1$

(c) P(1st : fail and 2nd : fail and 3rd : fail .....and nth : fail)
$= (\mathbf{0.1}) \times (\mathbf{0.1}) \times (\mathbf{0.1}) \times \ldots\ldots\ldots\ldots \times (\mathbf{0.1})$
$= (\mathbf{0.1})^n$

(d) P(passes the test in one of the first n months)
$= 1 - (\mathbf{0.1})^n$

16(a) Midpoint of $\boldsymbol{AB} = (\mathbf{0,3})$

(b) Gradient of $AB - -\dfrac{3}{4}$

Sub $(\mathbf{6},\boldsymbol{k})$ into $y = -\dfrac{3}{4}x$

$k = -\dfrac{3}{4} \times 6$

$k = -\dfrac{9}{2}$

17(a)(i) 1 int.$\angle$ +1 ext.$\angle$=180°
6 parts =180°
Each exterior angle = 30°

(ii) No. of sides $= \dfrac{360}{30} = 12$

(b) Let each of the remaining angles be x.
Sum of interior angles $= 8 \times 180° = 1440°$
$6 \times 125° + 4x = 1440$
$x = 172.5°$

18(a) 482 cm^3 per second $= \dfrac{0.482 \text{ litres}}{(1 \div 3600) \text{ hr}}$

$= 1735.2$ litres/hr
$= 1.7352 \times 10^{-3}$ litres/hr

(b) $\%\text{ increase} = \dfrac{2.38\times10^4 - 7.79\times10^3}{7.79\times10^3}\times100\%$

$= 205.52\%$

$= 206\%$ (3 sig fig)

19(a) $\sin Q\hat{P}R = \dfrac{7}{8}$

$Q\hat{P}R = 119.0°$ (to 1 dec pl)

(b) Let the particle travel to a point $\boldsymbol{X}$, nearest to $\boldsymbol{Q}$.

$Q\hat{X}P = 90°$

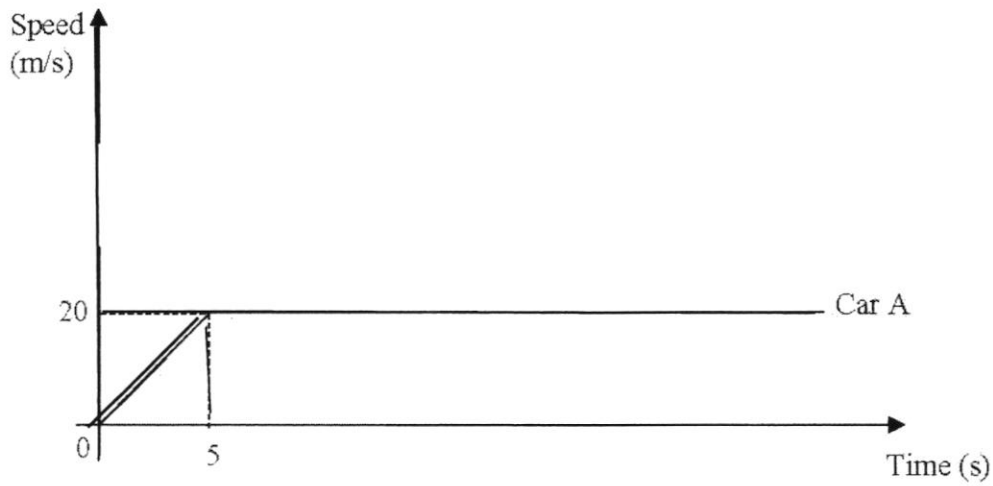
$\sin Q\hat{P}X = \dfrac{7}{8}$

$\dfrac{QX}{3} = \dfrac{7}{8}$

$QX = \dfrac{21}{8}$

$XR = \sqrt{8^2 - \left(\dfrac{21}{8}\right)^2} = 7.56\text{m}$

20(a)



(b) 20 s after A has started,

A has travelled $\dfrac{1}{2}(5)(20)+(15)(20) = 350$ m

For speed of B, v : $\dfrac{v}{(20-5)} = \dfrac{67.5}{30}$

$v = 33.75$

B has travelled $\dfrac{1}{2}(15)(33.75) = 253$ m

So Car B will not overtake Car A within the first 20 s.

21(a)(i) $M=(-4,-8)$

(ii) Gradient $=\frac{3}{4}$

Eqn is $y+2=\frac{3}{4}(x-4)$

$y=\frac{3}{4}x-5$

(b) $\begin{pmatrix}9\\-24\end{pmatrix}=-3\begin{pmatrix}-3\\8\end{pmatrix}$

Since $\begin{pmatrix}9\\-24\end{pmatrix}=\boldsymbol{k}\begin{pmatrix}-3\\8\end{pmatrix}$ where $\boldsymbol{k}$ is a constant,

they are parallel.

22(a) Total surface area $=\pi rl+2\pi r^2$ cm^2

(b)(i) Area of 1 triangle $=\frac{\sqrt{3}}{4}r^2$ cm^2

(ii) Total surface area $=\sqrt{3}r^2$ cm^2

(c) $\pi rl+2\pi r^2=\sqrt{3}r^2$

$l=\frac{r^2\left(\sqrt{3}-2\pi\right)}{\pi r}$

$=\frac{r\left(\sqrt{3}-2\pi\right)}{\pi}$ cm

23(a) $\mathbf{Q}=\begin{pmatrix}4&3\\2&0\\3&4\end{pmatrix}$

(b) $\mathbf{R}=\begin{pmatrix}1.5&2.4&1.4\\0.2&-0.4&-0.1\end{pmatrix}\begin{pmatrix}4&3\\2&0\\3&4\end{pmatrix}$

$=\begin{pmatrix}15&10.1\\-0.3&0.2\end{pmatrix}$

(c) \$0.20

(d) At supermarket B, Andy incurs an additional -\$0.30, which means he pays 30 cents less at supermarket B. So it is better for Andy to shop at supermarket B.

24 Area of shaded region $= \left(\sqrt{2}r\right)^2 - \frac{1}{4}\pi\left(\sqrt{2}r\right)^2$

$= 2r^2 - \frac{\pi}{2}r^2$

Area of circle that is unshaded $= \pi r^2 - 2r^2 + \frac{\pi}{2}r^2$

$= \frac{3\pi}{2}r^2 - 2r^2$

Fraction that is unshaded $= \frac{\frac{3\pi}{2}r^2 - 2r^2}{\pi r^2}$

$= \frac{3\pi - 4}{2\pi}$

Commonwealth Secondary School
Preliminary Examination 2018 (Sec 4E/4NA/5NA)
Solutions to E Math Paper 2

1	(a)	$x^3 - x - 1 + x^2$ $= x^3 - x + x^2 - 1$ $= x(x^2 - 1) + (x^2 - 1)$ $= (x^2 - 1)(x + 1)$ $= (x + 1)(x - 1)(x + 1)$ $= (x+1)^2(x-1)$	
	(b)(i)	$\frac{y}{x^2 + y^2 - 2xy} + \frac{x}{xy - x^2}$ $= \frac{y}{(x-y)^2} + \frac{x}{x(y-x)}$ $= \frac{y}{(x-y)^2} - \frac{x}{x(x-y)}$ $= \frac{y}{(x-y)^2} - \frac{(x-y)}{(x-y)^2}$ $= \frac{2y - x}{(x-y)^2}$	
	(b)(ii)	$\frac{m^2}{4n^4} \div \frac{m^5}{(6n^3)^2}$ $\frac{m^2}{4n^4} \times \frac{(6n^3)^2}{m^5}$ $= = \frac{m^2}{4n^4} \times \frac{36n^6}{m^5}$ $= \frac{9n^2}{m^3}$	
	(c)	$\frac{5}{3 - 2x} = x - 7$ $5 = (x - 7)(3 - 2x)$ $2x^2 - 17x + 26 = 0$ $(2x - 13)(x - 2) = 0$ $x = 6.5 \quad or \quad x = 2$	

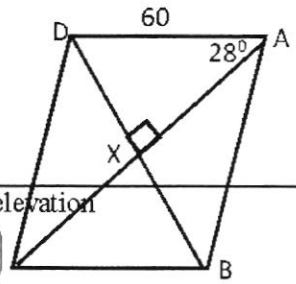
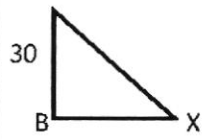
	(d)(i)	Express y^2+7y+5 in the form $(y+a)^2+b$. y^2+7y+5 $=(y+3.5)^2+5-(3.5)^2$ $=(y+3.5)^2-\frac{29}{4}$	
	(ii)	$y^2+7y+5=0$ $(y+3.5)^2-\frac{29}{4}=0$ $(y+3.5)^2=\frac{29}{4}$ $(y+3.5)=\pm\sqrt{\frac{29}{4}}$ $y=-3.5\pm\sqrt{\frac{29}{4}}$ $y=-0.81$ **or** -6.91 (2 *decimal places*)	
2	(i)	Using the cosine Rule , $\cos 120^\circ=\frac{(x-1)^2+(x+1)^2-(x+3)^2}{2(x-1)(x+1)}$ $-\frac{1}{2}=\frac{x^2-6x-7}{2(x^2-1)}$ $x^2-3x-4=0$	
	(ii)	$x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ **or** $x=-1$ For $x=4$, the sides of the triangle are 3 cm, 5 cm and 7 cm.	
	(iii)	**sine** of the smallest angle Let α be the smallest angle. $\frac{\sin\alpha}{3}=\frac{\sin 120^\circ}{7}$ $\sin\alpha=\frac{3\sin 120^\circ}{7}$ $=0.371$ {3 s f}	
3	(a)(i)	$\overrightarrow{XY}=\overrightarrow{OY}-\overrightarrow{OX}$ $=\underset{\sim}{b}-\frac{5}{2}\underset{\sim}{a}$	

	(ii)	$\overrightarrow{TY} = \overrightarrow{OY} - \overrightarrow{OT}$ $= \underset{\sim}{b} - \underset{\sim}{a}$	
	(iii)	$\overrightarrow{QY} = \frac{5}{6}\overrightarrow{TY}$ $= \frac{5}{6}(\underset{\sim}{b} - \underset{\sim}{a})$	
	(iv)	$\overrightarrow{XQ} = \overrightarrow{XY} + \overrightarrow{YQ}$ $= \frac{1}{6}(\underset{\sim}{\boldsymbol{b}} - 10\underset{\sim}{\boldsymbol{a}})$	
	(b)	$\overrightarrow{XR} = \overrightarrow{OR} - \overrightarrow{OX}$ $= \frac{1}{4}\overrightarrow{OY} - \frac{5}{2}\underset{\sim}{a}$ $= \frac{1}{4}\underset{\sim}{b} - \frac{5}{2}\underset{\sim}{a}$ $= \frac{1}{4}(\underset{\sim}{\boldsymbol{b}} - 10\underset{\sim}{\boldsymbol{a}})$	
	(c)	$\overrightarrow{XQ} = \frac{1}{6}(4\overrightarrow{\boldsymbol{XR}}) = \frac{2}{3}\overrightarrow{XR}$	
	(d)	i) X , Q and R are collinear points. ii) $XQ : XR = 2 : 3$	
	(e)	Area of triangle XQY : Area of triangle OXY = 1 : 2	
4	(a)(i)	Total cost in sing dollars $= (\mathbf{1.16 \times 4650 + 2800}) \times \frac{1.2798}{100}$ = \$104.87	
	(ii)	Selling price of toy locally = $1.07 \times 0.8 \times 160.50 = \$137.388$ $= \$137.39 \quad (2\boldsymbol{dp})$	
	(iii)	Percentage Jack saved = $\frac{(137.388 - 104.87)}{137.388} \times 100$ $= 23.7\%$	
	(b)	Plan A $\quad Amount = 8000\left(1 + \frac{9.25}{100}\right)^2$ $= \$9548.45$	

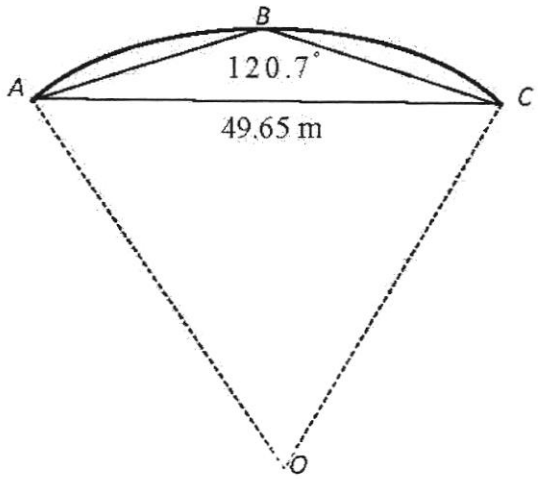
		Plan B $Amount = 8000\left(1+\frac{\frac{9}{12}}{100}\right)^{24}$ $= \$9571.31$ Plan B is better as the plan yields better compound interest for the amount \$ 8000 invested.	
	(c)(i)a	\$ (90 n)	
	(b)	\$(90n – 4000)	
	(ii)	$Interest\ payable = \frac{4000 \times 3 \times \frac{n}{12}}{100}$ $= \$10n$ Note the interest calculated should be simple interest unless otherwise stated.	
	(iii)	90 n - 4000 = 10 n 80 n = 4000 n = 50 months	
	(iv)	50 months = 4 years 2 months.	
5	(i)	$AB=RP$ = BP=20 cm as P is the centre of circle. ⇒ ABR is an isosceles right angled triangle. Angle BPA = $\frac{\pi - \frac{\pi}{2}}{2} = \frac{\pi}{4}$	
	(ii)(a)	Area of region $ACBRA$ = Area of semicircle ACB – Area of unshaded region ARB $= \frac{1}{2}\pi(10)^2$ – (Area of triangle ABP – Area of sector PRB) $= 50\pi - \left(\frac{1}{2}(20)(20) - \frac{1}{2}(20)^2\frac{\pi}{4}\right)$ $= 50\pi - 200 + 50\pi$ $= 114.2\,cm^2\,(1dp)$	
	(b)	Perimeter of $ACBRA$ = Circumfrence of semi-circle ACB + length of arc RB + length of line segment AR. $= \pi(10) + 20\left(\frac{\pi}{4}\right) + (AP - PR)$	

		$= \pi(10) + 20\left(\frac{\pi}{4}\right) + \left(\sqrt{(20)^2 + (20)^2} - 20\right)$ $= 55.4$ cm (1 *dp*)	
6	(a) (i)	Angle *BCA* = Angle *BEA* = 45^0 *(Angles in the same segment)* Angle *BCA* = Angle *BAC* (given AB = BC) Therefore Angle *BCA* = Angle *BAC* = 45^0 Angle *CBA* = $180^0 - 45^0 - 45^0 = 90^0$. As angle in a semi-circle is a right angle and angle CBA = 90^0, Implies AC is a diameter.	
	(ii)	Angle *RAE* = $105^0 - 45^0 = 60^0$. *(In triangle RAE, exterior angle is the sum of interior opposite angles)* Angle EAQ = $90^0 - 60^0 = 30^0$ (AC is the diameter, radius is perpendicular to tangent at A)	
	(iii)	Angle CDE = 180^0 – angle CAE = $180^0 - 60^0 = 120^0$ (In a cyclic quadrilateral CAED the opposite angles are supplementary.)	
	(b)	Note that sides *AR* and *BR* are the corresponding sides of similar triangle *BRC* and triangle *ARC*. By Sine Rule $\frac{BR}{\sin 45^\circ} = \frac{3}{\sin 60^\circ}$ $BR = \frac{3\sin 45^\circ}{\sin 60^\circ}$ $\frac{Area\ of\ triangle\ ARE}{Area\ of\ triangle\ BRC} = \left(\frac{AR}{BR}\right)^2 = \frac{1}{6}$ Therefore Area of triangle *BRC* = $6 \times 90 = 540$ cm^2	
7	(a)	P = -11.9 (3 s f)	
	(c)	Gradient of tangent ≈ 6.55 (3 sf)	
	(d)	$2x^2 - 6.5x = 4$ Dividing by x	

		$2x-6.5=\frac{4}{x}$ *Multiplying by* 2 $4x-13=\frac{8}{x}$ $\frac{8}{x}-3x+8=x-5$ Insert the line $y = x$-5. The point of intersection is the solution to the given equation within the given range. $x \approx 3.78$ (3 sf)	
8	(a) (i)	Given volume of cone = 980 cm^3 $\frac{1}{3}\pi r^2 h = 980$ $h=\frac{3\times 980}{6^2\times\pi}$ $h = 25.9848...$ $h = 26.0\boldsymbol{cm}\,(\boldsymbol{shown})$	
	(ii)	*Surface Area* $= 2\pi r^2 + \pi rl$ $= 2\pi 6^2 + \pi(6)\sqrt{25.9848^2+6^2}$ $= 729.17\ cm^2$ $= 0.0729 m^2$	
	(b)	Volume of water when the container is full $=\frac{2}{3}\pi(6)^3+980$ $=1432.3892$ Volume of water in the container $= 0.5 \times 1432.3892 = 716.1946$ cm^3. Let x be the height of water level required $\frac{716.195}{980}=\left(\frac{x}{26}\right)^3$ x = 23.4 cm (3 sf)	
9	(a)	Bearing of D from $A = 180^0 + 28^0 = 208^0$.	
	(b)	Bearing of B from $D = 090^0$.	
	(c)	$BD = 2\,DX$ (X is the midpoint of the diagonals AC and BD)	
		$= 2\,(60)(sin\ 28^0)$	

		= 56.3 m (3 s f)	
		Note BX = XD CX =XA  DX=60 sin28⁰ AX = 60 cos 28⁰	
	(d)	Greatest angle of elevation $= \tan^{-1}\left(\frac{30}{BX}\right)$ $= \tan^{-1}\left(\frac{30}{60\sin 28^\circ}\right)$ $= 46.8^0$ (1 d p)	Bird  BX=XD
	(e)	Area of the field = 4 (Area of triangle ADX) $= 4\ (0.5)(AX)(DX)$ $= 4\ (0.5)(60)(\sin 62^0)(60\cos 62^0)$ $= 2984.535\ m^2$ Given the scale 1 : 2000 1 cm represents 20 m 1 cm^2 represents 400 m^2 Area on the map = 2984.535 ÷ 400 = 7.46 cm^2 (3 s f)	
10	(a)(i)a	Median time = 10.8 hours	
	(b)	IQR $= Q_3 - Q_1 = 13.6 - 7.6$ = 6 hours	
	(c)	% of battery with life ≥ 12 hours $= \frac{72}{180} \times 100$ = 40%	
	(ii)	The curve shifts to the right of the given curve due to a higher median value.	
	(b)(i)a	P(life of 10 hours or more) $= \frac{150}{300} = \frac{1}{20}$	
	(b)	P(life under 6 hours) $= \frac{70}{300} = \frac{7}{30}$	
	(ii)	P (both batteries have a life under 8 hours) $= \left(\frac{56}{300}\right)\left(\frac{55}{299}\right)$ $= \left(\frac{154}{4485}\right)$	

11



(a) Angle AOC $= 360^0 - 2(120.7^0)$ (Angle at centre is twice angle on circumference)
$= 118.6^0$ Note that AB ≠ BC

Angle OAC = Angle OCA (Triangle OAC is isosceles as OA= OC= R)

$$= \frac{180^\circ - 118.6^\circ}{2}$$

$$= 30.7^\circ$$

Let R be the radius of the sector AOBC.

By Sine Rule

$$\frac{R}{\sin 30.7^\circ} = \frac{49.65}{\sin 118.6^\circ}$$

$$R = \frac{49.65 \sin 30.7^\circ}{\sin 118.6^\circ}$$

$$R = 29.0m \ (3\ sf)$$

(b) $$\textit{Length of arc} = \frac{118.6^\circ}{360^\circ} \times 2\pi R$$

$$= \frac{118.6^\circ}{360^\circ} \times 2\pi(28.871)$$

$$= 60m \ (\textit{shown})$$

(c) Speed = 5 km/h = 5000 m/h

$$Time = \frac{60}{5000} = \frac{3}{250} h = 44 \sec$$

John's prediction is right as it takes less than a minute to evacuate the walkway.

(d) Assuming at 12 noon the walking speed is 5 km/h as it is at 1100 am.