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DUNMAN HIGH SCHOOL Preliminary Examination Year 6

# **MATHEMATICS (Higher 2)**

Paper 1

9758/01

3 hours

September 2018

Additional Materials:

Answer Paper Graph Paper List of Formulae (MF26)

### READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

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Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	3	7	5	6	8	11	11	9	13	13	14	100

For teachers' use:

- 1 If x is a sufficiently small angle, find the first three non-zero terms in the Maclaurin series for  $\sin^7(x+\frac{1}{4}\pi)$ . [3]
- 2 (i) Using appropriate expansions from the List of Formulae (MF26), show that the series expansion of  $\ln(1 + e^{2x})$  in ascending powers of x, up to and including the term in  $x^2$ , is  $\ln 2 + x + \frac{x^2}{2}$ . [3]
  - (ii) Find the set of values of x such that the value for the above expansion for  $\ln(1 + e^{2x})$  is within  $\pm 0.3$  of the value of  $\ln(1 + e^{2x})$ . [2]

(iii) Deduce an expansion for  $\frac{2}{1 + e^{-2x}}$  up to and including the term in x. [2]

- 3 If  $x = 3\cos^2 \theta + 6\sin^2 \theta$ , show that  $6 x = 3\cos^2 \theta$ , and find a similar expression for x 3. By using the substitution  $x = 3\cos^2 \theta + 6\sin^2 \theta$ , evaluate exactly  $\int_3^6 \frac{1}{\sqrt{(x-3)(6-x)}} dx$ . [5]
- 4 Show that the following inequality

 $\tan x + \cot x > 4$  for  $0 < x < \frac{1}{2}\pi$ 

can be simplified to

$$0 < \sin 2x < \frac{1}{2}.$$
[4]

[2]

Hence solve exactly the inequality leaving your answer in terms of  $\pi$ .

5 Functions f and g are defined by

f: x a 
$$1 + \frac{2}{x-1}$$
,  $x \in i$ ,  $x < 1$ ,  
g: x a  $\ln x$ ,  $x \in i$ ,  $0 < x < 1$ .

- (i) Explain why the composite function gf does not exist.
- (ii) Find an expression for fg(x). Hence or otherwise, find  $(fg)^{-1}(0)$ . [4]
- (iii) Find an expression for h(x) for each of the following cases:

$$(a) \quad gh(x) = x, \tag{1}$$

**(b)** 
$$hg(x) = x^2 + 1.$$
 [2]

6 (a) By using the substitution  $y = zx^2$ , find the general solution of the differential equation

$$x^2 \frac{dy}{dx} = 2xy - y^2$$
, where  $x \neq 0$ . [4]

[1]

- (i) Sketch the solution curve that passes through (2, -4), indicating any stationary points and asymptotes clearly. [4]
- (ii) State the particular solution for which y has no turning point. [1]
- (b) A differential equation is of the form  $\frac{dy}{dx} + y = px + q$ , where p and q are constants. Its general solution is  $y = 4x - 1 + De^{-x}$ , where D is an arbitrary constant. Find the values of p and q. [2]

7 (a) Given that  $z^* = \frac{(2i)^3}{(\sqrt{3}+i)^4}$ , find the exact value of |z| and  $\arg(z)$ .

Hence state the smallest positive integer *n* such that  $z^n$  is purely imaginary. [5]

- (b) When the polynomial  $ax^4 + bx^3 + cx^2 + 24x 44$ , where *a*, *b* and  $c \in i$ , is divided by (x-1), (x+1) and (x-2), the remainders are -18, -54 and 0 respectively.
  - (i) Find the values of a, b and c. [3]
  - (ii) The equation  $ax^4 + bx^3 + cx^2 + 24x 44 = 0$ , with the values of *a*, *b* and *c* found in part (i), has a root  $3 (\sqrt{2})i$ . Find the other roots of the equation, showing your working clearly. [3]



The diagram shows the graph of y = f(x). The curve has turning points (-3, 4) and (1, 5) and crosses the *x*-axis at (-1, 0). The curve has asymptotes x = 0, y = 2 and y = x + 2.

Sketch, on separate diagrams, the graphs of

8

(i) 
$$y = f(|x|),$$
 [2]

(ii) 
$$y = \frac{1}{f(x)}$$
, [3]

including the coordinates of the points where the graphs cross the axes, the turning points and the equations of any asymptotes.

Describe a sequence of three transformations which transforms the graph of y = f(x) to the graph of y = f(ax+b), where *a* and *b* are constants such that a < -1 and 0 < b < 1. State the coordinates of the point where the graph of y = f(ax+b) cuts the *x*-axis. [4] 9 The line  $l_1$  passes through the point A with the position vector  $\begin{pmatrix} 1\\3\\2 \end{pmatrix}$  and is parallel to  $\begin{pmatrix} t\\t^2+1\\3 \end{pmatrix}$ , while the cartesian equation of the plane n is given by tr = 2y + z = -3 where t is a real constant.

while the cartesian equation of the plane *p* is given by tx - 2y + z = -3, where *t* is a real constant. It is known that  $l_1$  and *p* have no point in common.

- (i) Show that t = -1. [3]
- (ii) Find the distance between  $l_1$  and p.

[2]

[3]

- (iii) The line  $l_2$  has the cartesian equation 2y = z, x = 3. Show that  $l_2$  lies on p. [2]
- (iv) Given that point B and point C lie on  $l_1$  and  $l_2$  respectively, find  $\overline{BC}$  such that it is perpendicular to both  $l_1$  and  $l_2$ . [3]
- (v) Find the vector equation of the line of reflection of  $l_1$  in p.
- 10 Duncan, an aspiring marathon runner, embarks on the following training regime:
  - 1. On Day 1, he runs 5 km.
  - 2. On Day 2, he runs  $(1 + \alpha)$  times the distance ran in the previous day.
  - 3. On Day 3, he runs  $\alpha$  times the distance ran in the previous day.

You may take  $\alpha$  to be a positive constant.

For each subsequent day, Duncan repeats parts 2 and 3 of the training regime, in that order. Thus on Day 4, he runs  $(1+\alpha)$  times the distance ran on Day 3; and on Day 5, he runs  $\alpha$  times the distance ran on Day 4, and so on.

- (i) The distances that Duncan runs on odd-numbered days follows a geometric progression. State its common ratio in terms of  $\alpha$ . [1]
- (ii) By considering just the distances that Duncan runs on even-numbered days, find the range of values of  $\alpha$  such that the total distance ran on all even-numbered days is finite. [2]
- (iii) By considering the total distances ran on both odd and even numbered days, determine the theoretical maximum total distance, expressing your answer in terms of  $\alpha$ . [2]

Duncan decides to fix  $\alpha = 0.65$ .

(iv) (a) Show that the distance he ran on Day 10 is 10.915 km, correct to the nearest 0.001 km.

[1]

- (b) The distance he ran on Day n first exceeds 42.195 km. Find the value of n. [3]
- (c) Duncan aims to complete his training regime on Day *n*, and instead of following the regime for that day, he plans to only run exactly 42.195 km on that day. Find the total distance that Duncan would have covered at the end of his training regime. [4]

- 11 In the study of Microeconomics, the price p (in thousands) that consumers and producers of a particular product A is willing to pay to consume or produce x quantities (in thousands) of the product is modelled by the following equations:
  - Producers (supply curve):  $x = t + e^t 1$ ,  $p = t^2$ , where t > 0.
  - Consumers (demand curve):  $\frac{xp}{10} + \sin^{-1}\left(\frac{p}{10}\right) = 1$ , where x > 0 and 0 .
  - (i) The price *p* that consumers are willing to pay to consume *x* quantities of the product decreases as *x* increases. Use differentiation to verify this. [2]
  - (ii) On the same diagram, sketch the demand and supply curves where x is the horizontal axis and p is the vertical axis. [3]
  - (iii) Market equilibrium is achieved when the quantity demanded and the quantity supplied are the same at a particular price. Find this price. [2]

Economic surplus, also known as total welfare or Marshallian surplus, refers to two related quantities:

- Consumer surplus is the monetary gain obtained by consumers because they are able to purchase a product for a price that is less than the highest price that they would be willing to pay. Thus the consumer surplus is the area of the region bounded by the demand curve, the *p*-axis and the horizontal line that passes through the equilibrium point.
- Producer surplus is the amount that producers benefit by selling at a market price that is higher than the least that they would be willing to sell for. Thus the producer surplus is the area of the region bounded by the supply curve, the *p*-axis and the horizontal line that passes through the equilibrium point.

The diagram below illustrates an example of the surpluses of another product.



- (iv) The total economic surplus is the sum of the consumer and producer surpluses at the market equilibrium. Find the total economic surplus for product *A*. [4]
- (v) Due to a technological advancement in the manufacturing of product *A*, the supply curve is now translated *a* units in the positive *x*-direction and the new market equilibrium is achieved when x = 5. Find the value of *a*. [3]

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DUNMAN HIGH SCHOOL Preliminary Examination Year 6

# **MATHEMATICS (Higher 2)**

Paper 2

9758/02

3 hours

September 2018

Additional Materials:

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Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	5	9	6	8	12	8	9	9	10	12	12	100

For teachers' use:

(i) Show that the total area of all *n* rectangles,  $S_n$ , is given by

$$S_n = -2 + \frac{3}{n} \sum_{r=1}^n 3^{\frac{r-1}{n}}.$$

Hence evaluate  $S_n$ , leaving your answer in terms of n.

(ii) Find the exact value of  $\lim_{n\to\infty} S_n$ .

The equation of curve  $C_2$  is given by  $y = \frac{x^3}{\sqrt{4 + x^2}}$ .

(iii) The region *R* is bounded by  $C_1, C_2, x = 1$  and x = 2. Find the volume of the solid of revolution formed when R is rotated through 4 right angles about the x-axis, giving your answer correct to 2 decimal places. [3]

DHS 2018 Year 6 H2 Mathematics Preliminary Examination Paper 2

2

- 1 With reference to the origin O, the position vectors of point A and B are **a** and **b**, where **a** and **b** are non-zero and non-parallel vectors.
  - (i) State the shortest distance from *B* to the line passing through *O* and *A*. [1]
  - (ii) Given that  $\mathbf{c} = \mathbf{a} + \mathbf{b}$ , state the geometrical meaning of  $|\mathbf{b} \times \mathbf{c}|$  and show that  $|\mathbf{b} \times \mathbf{c}| = |\mathbf{a} \times \mathbf{b}|$ .
  - (iii) Given that  $\mathbf{a} \times 2\mathbf{b} = \mathbf{d} \times 3\mathbf{a}$ , find a linear relationship between  $\mathbf{a}, \mathbf{b}$  and  $\mathbf{d}$ . [2]
- The equation of curve  $C_1$  is given by  $y = 3^x 2$ . The graph of  $C_1$  is shown in the diagram below. 2 Rectangles, each of width  $\frac{1}{n}$ , where *n* is an integer, are drawn under  $C_1$  for  $1 \le x \le 2$ .



[2]

[4]

[2]

3 A sequence  $u_n$  is given by

$$u_n = \frac{3}{M - n + 1}$$
, where *n* and *M* are positive integers such that  $n < M + 1$ .

(i) Describe the behaviour of the sequence.

Let  $S_n$  denote the sum of the first *n* terms of  $u_n$ .

(ii) Write down  $S_1$ ,  $S_2$  and  $S_3$ . Hence find  $\sum_{n=1}^{M} S_n$  in terms of M. [3]

[1]

- (iii) Show that  $S_n > \frac{3n}{M}$ . [2]
- 4 The diagram below shows the curve y = f'(x). It has turning points at (0, 6), (5, 0) and (7, 7) and intersects the *x*-axis at (-3, 0) and (8, 0).



- (i) State the *x*-coordinates of the stationary points for the curve y = f(x). Hence determine the nature of these stationary points. [3]
- (ii) State the range of values of x such that the curve y = f(x) is concave downwards. [2]
- (iii) A student makes a claim on each of the following statements:
  - (A) If the curve y = f(x) passes through the point (-3, 7), then f(x) > 7 for -3 < x < 5.
  - (B) By using integration by parts,  $\int_{-3}^{8} xf(x) dx > 32f(8) \frac{9}{2}f(-3)$ .

For each of the above statements, explain briefly whether the student is right to make such a claim. [3]

5 (a) A curve has the equation  $(x + y)^2 = 4e^{xy}$ .

(i) Find 
$$\frac{dy}{dx}$$
 in terms of x and y. [2]

- (ii) Given that the curve cuts the positive *y*-axis at point *A*, find the equation of the tangent to the curve at *A*. [2]
- (iii) The tangent to the curve at *A* meets the curve at another point *B*. Find the coordinates of *B*. [3]
- (b) A closed cylinder is designed to contain a fixed volume of  $p \text{ cm}^3$  of liquid such that its external surface area is a minimum. Find the radius of the cylinder in terms of p in cm. [5]

### Section B: Probability and Statistics [60 marks]

- 6 A test consists of 15 multiple choice questions, where each question has *n* possible options, of which only one is correct. A student took the test by randomly choosing an answer to each question. It is known that the probability of answering exactly 3 questions correctly is the same as the probability of answering exactly 4 questions correctly.
  - (i) By forming an equation in terms of *n*, find the value of *n*. [3]

Each correct answer is awarded 3 marks and each incorrect answer carries a penalty of 1 mark. The score is the total marks awarded based on the number of correct and incorrect answers.

- (ii) Find the expected score, *s*, obtained by the student. [3]
- (iii) Find the probability that the score obtained by the student is within 4 marks of s. [2]
- 7 During a symposium, 4 boys and 8 girls are divided into 4 groups of three each for discussion. How many ways are there to divide the 12 participants such that each group consists of exactly 1 boy? [2]

After the discussion, all members of the 4 groups sit at random at a round table.

Find the probability that

- (i) the 3 members in each group are next to each other, [2]
- (ii) every boy is separated from each other by exactly 2 girls. [3]
- For the 12 participants, events *A* and *B* are defined by

*A*: every boy is separated from each other by exactly 2 girls; and *B*: none of the boys are seated next to each other.

[2]

(iii) Determine if the events A and B are independent.

- 8 An interactive simulation ride allows a group of 5 riders to take the ride at a time. The ride time, X minutes, follows a normal distribution with mean  $\mu$  minutes and standard deviation 2 minutes. The ride starts promptly at 10 am daily with no wait time between any groups of 5 riders. There are only 4 scheduled rides every morning. At 10 am on a particular morning, there are already 20 people queuing for the ride. It is assumed that all the people in the queue will take the ride based on the sequence of the queue and the ride times are independent.
  - (i) Show that  $\mu = 14$ , correct to the nearest integer, if  $P(\mu < X < 16) = 0.35$ . [2]

For the rest of the question, use  $\mu = 14$  for your calculations. A ride is considered long if it has a ride time of at least 15 minutes.

- (ii) Find the probability that the 12<sup>th</sup> person in the queue took the ride before 10.30 am on that morning. [3]
- (iii) Show that the probability of having at least 2 long rides on that morning is 0.363. [2]
- (iv) Given that there are at least 2 long rides on that morning, find the probability that none of these long rides are consecutive. [2]
- **9** To study the recent relationship between the property price index, p (in %) and the stock index, s (in thousands), of a particular city, Hilton recorded the readings from each of the past 8 quarters in the table below.

Stock Index, s (thousands)	2.12	2.53	2.63	2.70	2.75	2.83	2.87	2.98
Property Price Index, p (%)	115	130	145	140	146	150	155	170

Hilton realised that he recorded one of the values of *p* incorrectly.

(i) Sketch a scatter diagram for the data and circle the erroneous point X on your diagram. [2]

### [For the remaining parts of this question, you should exclude the point *X*.]

Hilton proposes that *s* and *p* can be modelled by one of the formulae:

$$p = a + b s^5$$
 or  $\ln p = c + d s$ ,

where *a*, *b*, *c* and *d* are positive constants.

- (ii) Determine the better model for the given data, giving a reason for your choice. [3]
- (iii) Assuming that the value of *s* at *X* is correct, estimate the corresponding value of *p*. Give two reasons why you would expect this estimate to be reliable. [4]

Hilton concludes that higher stock index will lead to higher property price index. Comment on his conclusion in the context of the question. [1]

10 The discrete random variable *X* has probability mass function given by

$$P(X = x) = \begin{cases} \frac{ax}{n(n+1)} & \text{for } x = 1, 2, \dots, n, \\ 0 & \text{otherwise,} \end{cases}$$

where *a* is a constant.

- (i) Show that a = 2.
- (ii) Find E(X) in terms of n. You may use the result that  $\sum_{n=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1).$  [3]

Two players  $P_1$  and  $P_2$  play a game with n+1 tokens labelled 1 to n+1. Each player randomly picks one token without replacement and the player who picks the token with the smaller number loses. The amount of money lost by the losing player, in dollars, will be the number on the winning token. For example, if  $P_1$  and  $P_2$  pick the tokens labelled 5 and 3 respectively,  $P_2$  loses \$5 to  $P_1$ .

- (iii) Explain why the probability of  $P_2$  losing a game is 0.5. [1]
- (iv) Given that  $P_2$  loses, find the probability  $P_2$  loses (m+1) in terms of *m* and *n*, where *m* is such that  $1 \le m \le n$ . [3]
- (v) Using the result in part (i), when  $P_2$  loses a game, find the amount that he is expected to lose in terms of n. [3]
- 11 The speeds of cars along a busy stretch of road follow a normal distribution. Studies show that a mean speed of 50 km/h is needed to ensure a smooth flow of traffic. When the mean speed falls below 50 km/h, it may lead to road congestion. If this happens, Wireless Road Pricing (WRP) will be used to charge motorists to discourage them from using the road, hence improving the traffic condition. A random sample of the speeds, x km/h, of 120 cars along the stretch of road is recorded and the data are summarised by

$$\sum x = 5415, \quad \sum x^2 = 351500.$$

(i) Calculate unbiased estimates of the population mean and variance of the speeds of the cars.

[2]

[2]

- (ii) What do you understand by the term 'unbiased estimate'? [1]
- (iii) Test at the 3% significance level whether WRP is needed. [4]

The Road Transport Authority decides to implement WRP on the same stretch of road. After the implementation, the speeds of a second sample of 80 cars are recorded with a mean speed of 60 km/h and a variance of  $1100 \text{ (km/h)}^2$ .

- (iv) A test at the 8% significance level of the second sample suggests that the mean speed has increased beyond  $\mu_0$ . Use an algebraic method to find the maximum value of  $\mu_0$ . [4]
- (v) State one assumption used in obtaining the sample statistics for the second sample. [1]

## 2018 Year 6 H2 Math Prelim Exam (Paper 1) Solution & Markers' Comments

Suggested Solution	Markers' comments
$\begin{aligned} & \sin^{7}(x + \frac{1}{4}\pi) \\ &= (\sin x \cos \frac{1}{4}\pi + \cos x \sin \frac{1}{4}\pi)^{7} \\ &= \left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)^{7} \\ &\approx \frac{1}{8\sqrt{2}} \left[x + \left(1 - \frac{x^{2}}{2}\right)\right]^{7} \\ &= \frac{1}{8\sqrt{2}} \left(1 + \left(x - \frac{x^{2}}{2}\right)\right)^{7} \\ &= \frac{1}{8\sqrt{2}} \left(1 + \left(x - \frac{x^{2}}{2}\right) + \frac{7 \times 6}{1 \times 2} \left(x - \frac{x^{2}}{2}\right)^{2} + \dots\right) \\ &\approx \frac{1}{8\sqrt{2}} \left(1 + 7x + \frac{35}{2}x^{2}\right) \end{aligned}$	<ul> <li>In general, it's easier to use small angle approximation instead of repeated differentiation to find the Maclaurin series.</li> <li>Common mistake is to assume that (x + ¼π) is small. Even though x is small, do note that ¼π is not small. Hence sin<sup>7</sup>(x + ¼π) ≠ (x + ¼π)<sup>7</sup>. See also Assign8 Q2.</li> </ul>
	Total Marks: 3

Suggested Solution	Markers' comments
$\ln\left(1+e^{2x}\right) = \ln\left(1+1+2x+\frac{(2x)^2}{2!}+\right)$ $= \ln\left(2+2x+2x^2+\right)$	• The given instruction is to use appropriate expansions from MF26. Students who differentiated $\ln(1 + e^{2x})$ to obtain
$= \ln \left[ (2)(1 + x + x^{2}) \right]$ = $\ln 2 + \ln \left[ 1 + (x + x^{2} +) \right]$	the Maclaurin series is not given any credit. • Some students wrote
$= \ln 2 + (x + x^{2} +) - \frac{(x + x^{2} +)^{2}}{2} +$	$\ln(1 + e^{2x})$ = $e^{2x} - \frac{(e^{2x})^2}{2} + \dots$
$= \ln 2 + x + \frac{x^{-1}}{2} + \dots$ (shown)	$\approx \left[1 + 2x + \frac{(2x)^2}{2!}\right] - \frac{\left[1 + 2x + \frac{(2x)^2}{2!}\right]^2}{2}$
	Firstly, the RHS will not produce an expression which involves In 2 (Think of a <b>different approach</b> if

		your method does not lead to the correct answer). Secondly, we will not be able to correctly evaluate the constant term for $\ln(1+e^{2x})$ as the constant 1 will appear in every term of the expansion of $e^{2x}$ .
(11)	$-0.3 < \ln (1 + e^{2x}) - \left( \ln 2 + x + \frac{x^2}{2} \right) < 0.3$ Using GC, $\{x \in \mathbb{R} : -1.55 < x < 1.55\}$ y = 0.3 y = 0.3 y = -0.3 $y = \ln (1 + e^{2x}) - \left( \ln 2 + x + \frac{x^2}{2} \right)$	<ul> <li>Quite a number of students did not leave their answer in set notation form as required by the question.</li> <li>The use of GC to solve the inequality <ul> <li>-0.3 &lt; ln (1 + e<sup>2x</sup>) - (ln 2 + x + x<sup>2</sup>/2) &lt; 0.3 is expected since the question did not forbid the use of GC.</li> </ul> </li> </ul>
(iii)	$\therefore \ln (1 + e^{2x}) = \ln 2 + x + \frac{x^2}{2} + \dots$ Differentiate both sides wrt x : $\frac{2e^{2x}}{1 + e^{2x}} = 1 + x + \dots$ $\frac{2e^{2x} / e^{2x}}{(1 + e^{2x}) / e^{2x}} = 1 + x + \dots  (\div \text{ by } e^{2x} \text{ in num \& denom})$ $\therefore \frac{2}{1 + e^{-2x}} = 1 + x + \dots$	<ul> <li>Many students either did not realise or were unable to use the result obtained in part (i) to deduce part (iii). The word deduce means that we need to use the result in part (i) to solve part (iii).</li> <li>[1+(1-2x)+]<sup>-1</sup> ≠ 1+(-1)(1-2x)+<sup>-</sup></li> <li>Likely forms of new expression o Derivative of the original o Integration of the original o Replacement of x by kx</li> </ul>
		Total Marks: 7

2018 Y6 H2 Math Prelim P1 solution & markers' comments

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3 Y6 H2 Math Prelim P1 solution & markers' comments

Suggested Solution	Markers' comments
$x = 3\cos^{2}\theta + 6\sin^{2}\theta$ $6 - x = 6 - 3\cos^{2}\theta - 6\sin^{2}\theta$ $= 6(1 - \sin^{2}\theta) - 3\cos^{2}\theta$ $= 6\cos^{2}\theta - 3\cos^{2}\theta$ $= 3\cos^{2}\theta \text{ (shown)}$	<ul> <li>Most students obtain the 2 marks.</li> <li>Some students tried replacement, which is totally wrong since it is not a change of variable but simple trigonometry.</li> </ul>
$x-3 = 3\cos^{2}\theta + 6\sin^{2}\theta - 3$ = 3(cos <sup>2</sup> \theta - 1) + 6sin <sup>2</sup> \theta = 3(-sin <sup>2</sup> \theta) + 6sin <sup>2</sup> \theta = 3sin <sup>2</sup> \theta	
$\begin{aligned} x &= 3\cos^2\theta + 6\sin^2\theta, \\ \frac{dx}{d\theta} &= \left[6\cos\theta(-\sin\theta) + 12\sin\theta(\cos\theta)\right] \\ &= 6\sin\theta\cos\theta \end{aligned}$ when $x = 3$ , $3\cos^2\theta + 6\sin^2\theta = 3$ $3(\cos^2\theta - 1) + 6\sin^2\theta = 0$ $3(-\sin^2\theta) + 6\sin^2\theta = 0$ $\sin\theta = 0 \Rightarrow \theta = 0$ when $x = 6$ , $3\cos^2\theta + 6\sin^2\theta = 6$ $3\cos^2\theta + 6(\sin^2\theta - 1) = 0$ $3\cos^2\theta + 6\cos^2\theta = 0$ $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$ $\int_{0}^{6} \frac{1}{\sqrt{(x-3)(6-x)}} dx$ $= \int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{(3\sin^2\theta)(3\cos^2\theta)}} (6\sin\theta\cos\theta) d\theta$ $= \frac{6}{2} \left\{ \frac{\pi}{2} \right\} d\theta$	<ul> <li>For definite integral involving substitution, remember to change x = 3 and 6 to the corresponding values in θ.</li> <li>There were a lot of careless mistakes in manipulation.</li> <li>Students should use GC to verify their final answer</li> </ul>
$= 2\left[\theta\right]_{0}^{\frac{\pi}{2}} = 2\left[\frac{\pi}{2} - 0\right]$ $= \pi$	Total Marks: 5
	and a second

On	Suggested Solution	Markers' comments
1	$\tan x + \cot x > 4$ $\tan x + \cot x > 4$ $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} > 4$ $\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} > 4$ $\frac{1}{\frac{1}{2} \sin 2x} > 4$ $\frac{1}{\frac{1}{2} \sin 2x} > 4$ $\frac{1}{\sin 2x} > 2$ $\therefore 0 < \sin 2x < \frac{1}{2} \text{ (shown)}$ So we have $0 < 2x < \frac{1}{6}\pi  \text{or } \frac{5}{6}\pi < 2x < \pi$ $0 < x < \frac{1}{12}\pi  \text{or } \frac{5}{12}\pi < x < \frac{1}{2}\pi$	<ul> <li>Always refer to the result to be shown for hints on how to start. Since the shown result contains only sin 2x, you should convert everything in terms of sin and cos (Note the pairing of trigo functions i.e. sincos, sec-tan, cosec-cot).</li> <li>When dealing with inequalities that are not so straight-forward, it is always good to sketch graphs.</li> </ul>
		Total Marks : 6

Qn	Suggested Solution	Markers' comments
5(1)	$R_{f} = (-\infty, 1) \not\subset D_{g} = (0, 1)$ Thus gf does not exist. $y = f(x)$ $y = 1$ $x = 1$	<ul> <li>R<sub>r</sub> and D<sub>g</sub> need to be written explicitly as R<sub>f</sub> = (-∞, 1) and D<sub>g</sub> = (0, 1) in order to convince that R<sub>r</sub> is not a subset of D<sub>g</sub>.</li> <li>A handful of students did not get the correct R<sub>r</sub> as they did not consider the restriction on the domain of f when it comes to the sketching of the graph of f.</li> </ul>
(ii)	$fg(x) = f(\ln x) = 1 + \frac{2}{\ln x - 1} = \frac{\ln x + 1}{\ln x - 1}$	• This part is generally well done. Students need to know how to use <b>Method 1</b> to obtain the answer.

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	Method 1	
	Let $x = (1g)^{-1}(0)$	
	$\operatorname{Ig}(x) = 0$	
	$\frac{\ln x + 1}{\ln x - 1} = 0$	
	$\ln x + 1 = 0$	
	$\ln x = -1$	
	$x = e^{-1}$	
	$\therefore (fg)^{-1}(0) = e^{-1}$	
	Method 2 (work backwards from final value = 0)	
	Let $x = (fg)^{-1}(0)$	
	$\mathrm{fg}(x)=0$	
	Substitute $f(x) = 0 \implies 0 = 1 + \frac{2}{x-1} \implies x = -1$	
	Substitute $g(x) = -1 \Rightarrow -1 = \ln x \Rightarrow x = e^{-1}$	
	$\therefore$ (fg) <sup>-1</sup> (0) = e <sup>-1</sup>	
	$\{\mathbf{e}_{l}^{-1}\} \xrightarrow{g} \{-1\} \xrightarrow{f} \{0\}$	
	Method 3 (find inverse directly)	
	Let $y = fg(x) = \frac{\ln x + 1}{\ln x - 1}$	
	$y\ln x - y = \ln x + 1$	
	$\ln x(y-1) = y+1$	
	$\ln x = \frac{y+1}{1}$	
	$\mathcal{Y} = 1$	
	$x = e^{\frac{y-1}{y-1}}$	
	<u>x+1</u>	
	$(fg)^{-1}(x) = e^{x-1}$	
	$\therefore$ (fg) <sup>-1</sup> (0) = e <sup><math>\frac{0+1}{0-1}</math></sup> = e <sup>-1</sup>	
1)	$gh(x) = x \Longrightarrow h(x) = g^{-1}(x) = e^x$	• Well-attempted
	Method 1	• We cannot simply post-
	$hg(x) = x^2 + 1$	multiply both sides by $g^{-1}$
	$hg(g^{-1}(x)) = (g^{-1}(x))^2 + 1$	to obtain $h(x)$ . However, we could use Method 1 to
	$\therefore h(x) = (e^x)^2 + 1 = e^{2x} + 1$	replace x by $g^{-1}(x)$ since $g(g^{-1}(x)) = x$ and
		$b(b(x)) = x$ and $a^{-1}(x) = a^{x}$
		g(x) = c.

	Total Marks: 8
$\therefore h(x) = (e^x)^2 + 1 = e^{2x} + 1$	
$h(\ln x) = \left(e^{\ln x}\right)^2 + 1$	
$h(g(x)) = x^2 + 1$	
$hg(x) = x^2 + 1$	
Method 2	

Qn	Suggested Solution	Markers' comments
6(a)	$x^{2} \frac{dy}{dx} = 2xy - y^{2} \dots (1)$ Given $y = zx^{2}$ : $\frac{dy}{dx} = 2xz + x^{2} \frac{dz}{dx} \dots (2)$	• Many students failed to do chain rule to get equation (2), treating z as a constant. Students are
	$dx \qquad dx$ Substitute (2) into (1): $x^{2}\left(2xz + x^{2}\frac{dz}{dx}\right) = 2x(zx^{2}) - (zx^{2})^{2}$ $x^{4}\frac{dz}{dx} = -z^{2}x^{4}$	<ul> <li>reminded that if z is a constant with respect to x, then there is no change of variable involved.</li> <li>There are some careless mistakes, ranging from sign error (which can be checked by simple</li> </ul>
	$\frac{dz}{dx} = -z^2$ $\int \frac{1}{z^2} dz = -\int 1 dx$	differentiation and substituting back) to the following: $\frac{1}{z} = x - C$
	$z = -x + C$ $-\frac{x^2}{y} = -x + C$ $y = \frac{x^2}{x - C}$	$z = \frac{1}{x} - C$ (wrong, should be $z = \frac{1}{x - C}$ )
(i)	Given $(2, -4)$ , $-4 = \frac{2^2}{2-C} \Longrightarrow C = 3$ Hence, $y = \frac{x^2}{x-3} = x+3+\frac{9}{x-3}$	• Many students who has the correct general equation only drew the bottom half of the graph. Before sketching, it would be good to have an idea of what the graph should look like so that you can spot the missing

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Suggested Solution	Markers' comments
y = f(x) $y = x + 3$ $(6, 12)$ $x = 3$	<ul> <li>Students can also use zoom-fit on their GC to see the whole graph.</li> <li>Close to all students did not omit (0, 0), when it is given that x ≠ 0.</li> <li>There are some students who has the correct general equation but evaluated the oblique asymptote incorrectly.</li> </ul>
When $C = 0$ , particular soln is $y = \frac{x^2}{x-0} = x$ which is a straight line and has no turning point.	<ul> <li>Many students didn't answer the question by stating the equation explicitly.</li> <li>Many did differentiation instead or tried to solve <u>dy</u> ≠ 0.     </li> <li>Students are reminded to look at the number of mark(s) given for the time/effort needed.     </li> </ul>
Given $y = 4x - 1 + De^{-x} \Rightarrow \frac{dy}{dx} = 4 - De^{-x}$ $\frac{dy}{dx} + y = (4 - De^{-x}) + (4x - 1 + De^{-x}) = 4x + 3$ $\therefore p = 4, q = 3$	<ul> <li>Many students got this correct.</li> <li>There were some students who attempted to integrate <i>y</i> in terms of <i>x</i> to obtain <i>xy</i>, which is completely incorrect.</li> </ul>
	Total marks : 11

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On	Suggested Solution		Markers' comments
7(a)	* (2i) <sup>3</sup> -8i	Alternative	• Important to remember
	$2 = \frac{1}{(\sqrt{3}+i)^4} - \frac{1}{(\sqrt{3}+i)^4}$	$(2e^{i\frac{\pi}{2}})^3$	complex numbers:
	1-1-[-*]	$z^* = \frac{(21)^4}{(1-1)^4} = \frac{(-1)^4}{(1-1)^4}$	$\checkmark  z  =  z^* $
		$(\sqrt{3}+i)$ $(2e^{i/6})$	$\sqrt{\arg(z)} = -\arg(z^*)$
	-8i 8	$8e^{\frac{3\pi}{2}}$ 1 (3\pi - 4\pi)	Not officient to expand
	$= \frac{1}{(\sqrt{2}+i)^4} = \frac{1}{(\sqrt{2}+i)^2}$	$=\frac{60}{16e^{i\frac{4\pi}{6}}}=\frac{1}{2}e^{i\frac{2}{2}}e^{i\frac{2}{6}}$	the denominator
	$\left( \sqrt{3} + 1 \right) \left( \sqrt{3} + 1^2 \right)$	1 :55	$(\sqrt{3}+i)^4$
	8 1	$=\frac{1}{2}e^{-6}$	The alternative method
	$=\frac{1}{16}=\frac{1}{2}$	$z = \frac{1}{e^{-i\frac{5\pi}{6}}}$	is a useful and efficient
	$\arg(z) = -\arg(z^*)$	2	way of finding the
		$\Rightarrow  z  = \frac{1}{2}, \arg(z) - \frac{5}{6}\pi$	of complicated
	$=-\arg\left[\frac{-81}{(1-1)^4}\right]$		expressions at one go.
	$\left( \left( \sqrt{3}+i \right) \right)$		
	$= -\left[\arg(-8i) - 4\arg(\sqrt{3} + i)\right]$	$z = re^{-i\theta}$ where	
	$- [-1\pi - 4(1\pi)]$	$r =  z  = \frac{1}{2}$	
	$=-\left[-\frac{2}{2}\pi-4\left(6\pi\right)\right]$	$\theta = \arg(z) = -\frac{3}{6}\pi$	• It's wrong to write
	$=\frac{1}{6}\pi$	and the second se	$\frac{1}{6}\pi = -\frac{2}{6}\pi$ . A separate
	$\therefore \arg(z) = \frac{1}{6}\pi - 2\pi = -\frac{5}{6}\pi$		this
	$arg(z^{n}) = rarg(z) = -\frac{5}{2} rz$		
	$\arg(z) = n \arg(z) - \frac{1}{6}nn$	$(2k+1)(1-) k \in \mathbb{Z}$	. On the other hand for
	Since z'' is purely imaginary, $-\frac{1}{6}n\pi =$	$(2k+1)(\frac{1}{2}n), k \in \mathbb{Z}$	complex numbers
	$\Rightarrow n = -$	$\frac{3}{5}(2k+1)$	which are purely real,
	: smallest positive integer $n=3$ (wh	o $k = -3$ )	arguments would take the form $k\pi k \in \mathbb{Z}$
	smanest positive integer n=5 (vin		the form $\kappa\pi, \kappa \in \mathbb{Z}$
(b)(i)	Let $f(x) = ax^4 + bx^3 + cx^2 + 24x - 44$	an da an Anton Africa Constantin Constantin Constantin and a second second second second second second second s	<ul> <li>Instead of long</li> </ul>
	$f(1) = -18 \Longrightarrow a + b + c = 2$		division, it's easier to
	$f(-1) = -54 \Longrightarrow a - b + c = 14$		factor theorems to find
	$f(2) = 0 \Longrightarrow 16a + 8b + 4c = -4$		the 3 equations, before
	From GC : $a = 1, b = -6, c = 7$		solving simultaneously
(11)	$\frac{4}{12}$ ( $\frac{3}{12}$ + $\frac{7}{12}$ + $\frac{24}{12}$ + $\frac{44}{12}$ = 0		It's important to state
(11)	$x^{-} - 6x^{-} + 7x^{-} + 24x^{-} - 44 = 0$ If $3 - (\sqrt{2})i$ is a root. $3 + (\sqrt{2})i$ is also	o a root (since equation has	the reason for existence
	all real coefficients OR by conjugate t	root theorem)	of the conjugate root
	•		$\sqrt{Fan has all real}$
			coeff; or
			✓ Conjugate root
		e a de la companya d	theorem

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Translate the graph by <i>b</i> units in the negative <i>x</i> -direction. Scale the graph by a factor of $-\frac{1}{a}$ parallel to the <i>x</i> -axis. (Also accept scale factor of $\frac{1}{ a }$ . But do not accept scale factor of $\frac{1}{a}$ .) 3. Reflect in the <i>y</i> -axis. From original graph, $f(-1) = 0$ . To find <i>x</i> -intercept of new graph, $f(ax+b) = 0$ so $ax+b=0$ $\therefore x = \frac{-1-b}{a}$ Coordinates of the point where the graph of $y = f(ax+b)$ cuts the <i>x</i> -axis is $(\frac{-1-b}{a}, 0)$ or $(\frac{1+b}{ p }, 0)$ Note that for each transformation, only <i>x</i> or <i>y</i> is to be replaced. Recommended order of transformation of graphs of the form $y = f(x)$ is TSR for <i>x</i> , RST for <i>y</i> while for graphs of the form $g(y) = f(x)$ e.g. $x^2 + y^2 = 1$ is TSR for both <i>x</i> and <i>y</i> . Since $a < 0$ , you need to be careful when performing the scaling. The scale factor should be the positive constant $-\frac{1}{a}$ . Note that for this equation, transformations only involved <i>x</i> , the <i>x</i> -intercept in new graph must result from the transformation of the original <i>x</i> -intercept.		1 0 1
	<ul> <li>Translate the graph by b units in the negative x-direction.</li> <li>Scale the graph by a factor of -1/a parallel to the x-axis.</li> <li>(Also accept scale factor of 1/ a . But do not accept scale factor of 1/a.)</li> <li>Reflect in the y-axis.</li> </ul> From original graph, f(-1) = 0. To find x-intercept of new graph, f(ax + b) = 0 so ax + b = 0 <ul> <li>∴ x = -1-b/a</li> <li>Coordinates of the point where the graph of y = f(ax + b) cuts the x-axis is (-1-b/ a , 0)</li> </ul>	<ul> <li>Note that for each transformation, only x or y is to be replaced.</li> <li>Recommended order of transformation of graphs of the form y = f(x) is TSR for x, RST for y while for graphs of the form g(y) = f(x)</li> <li>e.g. x<sup>2</sup> + y<sup>2</sup> = 1 is TSR for both x and y.</li> <li>Since a &lt; 0, you need to be careful when performing the scaling. The scale factor should be the positive constant - 1/a.</li> <li>Note that for this equation, transformations only involved x, the x-intercept in new graph must result from the transformation of the original x-intercept.</li> </ul>
Total Marks: 9		Original x-intercept.

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T	Concentral Solutions	Markers' comments
	Suggested Solutions $l_1: \underline{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} t \\ t^2 + 1 \\ 3 \end{pmatrix}, \ \lambda \in \mathbb{R}$ $p: tx - 2y + z = -3 \implies \underline{r} \cdot \begin{pmatrix} t \\ -2 \\ 1 \end{pmatrix} = -3$ The direction vector of $l_1$ is parallel to $p$ (i.e., perpendicular to the normal of $p$ ) and the point $A$ does not lie on $p$ . $\begin{pmatrix} t \\ t^2 + 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} t \\ -2 \\ 1 \end{pmatrix} = 0 \implies t^2 - 2t^2 - 2 + 3 = 0 \implies t = -1 \text{ or } 1$	<ul> <li>If the line and the plane have no common point of intersection, the line has to be parallel to the plane and not lying on the plane. Hence have to show that the line is perpendicular to the normal of the plane, and that any point on the line does not lie on the plane.</li> <li>Since it is a "show" question, sufficient working is expected.</li> </ul>

(ii)	$ \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \begin{pmatrix} t \\ -2 \\ 1 \end{pmatrix} \neq -3 \implies t-6+2 \neq -3 \implies t \neq 1 $ Therefore, $t = -1$ (shown) The distance between $l_1$ and $p$ = shortest distance of $A$ to $p$ $ = \frac{ d - \overline{OA} \cdot \underline{n} }{ \underline{n} } $	<ul> <li>This is finding the length of projection of the vector (joining point A and any point on the plane) onto the normal of the plane.</li> </ul>
	$= \frac{\begin{vmatrix} -3 - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 1 \\ \end{vmatrix}}{\sqrt{6}}$ $= \frac{2}{\sqrt{6}}$	<ul> <li>Instead of using OA • n, some students use d<sub>h</sub> • n which is incorrect.</li> </ul>
(iii)	$l_2: 2y = z, x = 3, \implies r = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \mu \in \mathbb{R}$ $\underbrace{\text{Method 1}}_{\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}} = 0 - 2 + 2 = 0$ Therefore, $l_2$ is parallel to $p$ .	<ul> <li>Most students get this part correct.</li> <li>Take note of proper presentation.</li> </ul>
	$\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} = -3$ Therefore, $l_2$ lies on $p$ (shown). $\frac{\text{Method 2}}{\text{Substitute } l_2: 2y = z, x = 3 \text{ into equation of } p:$ LHS of $p = -3 - 2y + 2y = -3 = \text{RHS of } p$	•
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and  $l_2$  respectively, we can

the

 $\overline{BC}$ 

vector

is

formulate

Also,

lines.

your GC.

expression for  $\overline{BC}$ .

since

perpendicular to both lines, it

would be perpendicular to the

to the normal vector of both

the values of  $\mu$  and  $\lambda$  using

director vector of both lines.

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(v)

Method 1 Recognize that  $l_1$  is parallel to Let  $l_1$  be the line of reflection of  $l_1$  in p. the plane, and so the reflection of the line will also Let point B' be the point lying on  $l_1$ ' and is also the be parallel to the plane. reflection of B in p. • Then it suffices to find the point of reflection of any point on  $l_1$  in the plane.  $d_{l_{1}} = d_{l_{1}} =$ 2 because  $l_1$  is parallel to p. • Since point B lies on  $l_1$ , and 3 point C lies on the plane  $\overrightarrow{BB'} = 2\overrightarrow{BC} \implies \overrightarrow{OB'} = 2\overrightarrow{BC} + \overrightarrow{OB}$ (because point C lies on  $l_2$ which lies on the plane), and  $= 2 \begin{vmatrix} -\frac{1}{3} \\ -\frac{2}{3} \end{vmatrix}$ that BC is perpendicular to the plane,  $\overline{BB'} = 2\overline{BC}$ . B' $\frac{8}{3}$  -3 =  $-\frac{13}{2}$ C  $\begin{bmatrix} \frac{8}{3} \\ -3 \end{bmatrix} + \gamma \begin{pmatrix} -1 \\ 2 \end{bmatrix}, \ \gamma \in \mathbb{R}$ Equation of line of reflection  $l_1$  : r = $l_1$ B • It is also possible to find the point of reflection of point A Method 2 using ratio (or mid-point) Let point F be the foot of perpendicular of point A on plane theorem. See Method 2. p. -1) 1  $\overline{OF} = \begin{vmatrix} 3 \\ +\lambda \end{vmatrix} -2$  for some  $\lambda \in \mathbb{R}$ (2) 1 Since point F lies on plane p,  $(1-\lambda)(-1)$  $3 - 2\lambda - 2 = -3$  $2+\lambda || 1$ Solving,  $\lambda = \frac{1}{2}$  $\overrightarrow{OF} = \begin{pmatrix} 1\\3\\2 \end{pmatrix} + \lambda \begin{pmatrix} -1\\-2\\1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}\\\frac{7}{3}\\\frac{7}{3}\\\frac{7}{3} \end{pmatrix}$  $\overrightarrow{OF} = \frac{\overrightarrow{OA} + \overrightarrow{OA'}}{2} \Longrightarrow \overrightarrow{OA'} = 2\overrightarrow{OF} - \overrightarrow{OA}$  $\overrightarrow{OA'} = 2$ 3

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Equation of line of reflection $I_1 : r = \begin{pmatrix} \frac{1}{3} \\ \frac{5}{3} \\ \frac{8}{3} \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \ \gamma \in \mathbb{R}$		(b)	Method 1Common Errors:For the distance on Day n to first exceed 42.195 km, observe that nI. Misinterpretedmust be even, where the distance is larger than the previous day.the question and
	Total marks: 13		$5(1.65)[(1.65)(0.65)]^{\frac{n-2}{2}} > 42.195$ , where <i>n</i> is even inequality
Solution	Markers' comments		From GC,You should argue for the smallest even $n$ ininvolving the total distance. $n$ Distance on Day $n$ the smallest even $n$ in2. Students forgot 
For first Odd-no Days     For first Even-no Days       1     5 $5(1+\alpha)$			10 $11.2075 + 42.195$ your presentation.Influence $50$ $44.25773 > 42.195$ your presentation.formula is only applicable for and applicable for and
$\frac{2}{3} \frac{5(1+\alpha)^{2}(\alpha)}{5(1+\alpha)^{2}(\alpha)^{2}} \frac{5(1+\alpha)^{3}(\alpha)}{5(1+\alpha)^{3}(\alpha)^{2}}$			Method 2ConcludedDistance run on the <i>m</i> th odd-numbered daysmallest $n = 49$ . $= 5 [(1 + 5) (0 + c)]^{m-1}$ 3 Students forget to
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			Distance run on the <i>m</i> th even-numbered day
The common ratio for both sequences = $\alpha (1 + \alpha)$	Majority are able to get this mark.		$= 5(1.65)[(1.65)(0.65)]^{m-1}$ the <i>m</i> th even- numbered day versus finding Day <i>n</i> . A lot
For GP sum to infinity to exist, need $-1 < \alpha (1 + \alpha) < 1$ Since distance for each day is positive, we solve $0 < \alpha (1 + \alpha) < 1$ From GC, $0 < \alpha < 0.618$ (3 sf)	Common Errors: 1. Wrong condition for GP sum to infinity to exist. 2. Forget the		$S[(1.65)(0.65)] > 42.195 \text{ vs } S(1.65)[(1.65)(0.65)] > 42.195$ $m \ge 32 \text{ vs } m \ge 25$ Since <i>m</i> for the even-numbered days is smaller, the distance will first exceed 42.195 on Day $n = 25 \times 2$ , i.e. smallest $n = 50$ .
Since the question did not ask for exact answers, we can just use GC to solve!	2. Forget the condition $\alpha > 0$ . 3. Errors in concluding the correct region in	(c)	Distance for first 50 daysCommon errors:= Distance for first 25 odd-numbered days1. Counting the+ Distance for first 24 even-numbered dayseven- and odd-+ Distance on Day 50numbered days
Theoretical may total distance = $5 + 5(1+\alpha)$	solving inequalities. Refer to 2011 GCE		For first Odd-no DaysFor first Even-no Dayswrongly.15 $5(1+\alpha)$ 2. Didn't take into account the
Theorem and total distance $-\frac{1}{1-\alpha(1+\alpha)} + \frac{1}{1-\alpha(1+\alpha)}$	A Level P1 Q9(ii) for similar phrasing.		$\begin{array}{ c c c c } 2 & 5(1+\alpha)(\alpha) & 5(1+\alpha)^2(\alpha) & \text{distance run on} \\ \hline 2 & & & & & \\ \hline 2 & & & & & \\ \hline \end{array}$
Sum to infinity for the total distance run on $=\frac{3+3(1+\alpha)}{1-\alpha(1+\alpha)}$ Calculation errors are very	Students who applied the sum to infinity formula are		$\begin{bmatrix} 5 & 5(1+\alpha)^{2}(\alpha)^{2} & 5(1+\alpha)^{2}(\alpha)^{2} \\ \hline 4 & 5(1+\alpha)^{3}(\alpha)^{3} & 5(1+\alpha)^{4}(\alpha)^{3} \end{bmatrix}$ 3. Error in applying $S_{n}$ formula.
both even and odd days. $=\frac{10+5\alpha}{1-\alpha-\alpha^2}$ common.	more successful as compared to those		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Distance on Day 10	who derived the sum to infinity from $S_n$ . Students did well for		$\begin{bmatrix} Total & first 25 \text{ odd-numbered} \\ dista & days \\ nce & 5 \left\{ \left[ (1.65) (0.65) \right]^{2^{4}} - 1 \right\} \end{bmatrix} \begin{bmatrix} first 24 \text{ even-numbered days} \\ 5 (1.65) \left\{ \left[ (1.65) (0.65) \right]^{2^{4}} - 1 \right\} \end{bmatrix} \begin{bmatrix} S_{n} = \frac{r(r-r)}{1-r}, \\ n \text{ represents the number of terms that} \end{bmatrix}$
= 5(1.65)[(1.65)(0.65)] = 10.915  km	unis part.		$\frac{2 \left[ \left( 1.65 \right) \left( 0.65 \right) - 1 \right]}{(1.65)(0.65) - 1}$ (1.65)(0.65) - 1 you sum.
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2018 Y6 H2 Math Prelim P1 solution & markers' comments

Y6 H2 Math Prelim P1 solution & markers' comments

## 2018 Year 6 H2 Math Prelim (Paper 2) Solution & Markers' Comments

 Suggested Solution	Markers' comments
Shortest distance = $ \mathbf{b} \times \hat{\mathbf{a}} $ or $\frac{ \mathbf{b} \times \mathbf{a} }{ \mathbf{a} }$	<ul> <li>The key word here is 'state'</li> <li>answer should be quite straightforward.</li> <li>Quite a handful of students omitted the modulus sign, considered dot product instead or did not consider the unit vector for a.</li> </ul>
$ \mathbf{b} \times \mathbf{c} $ is the area of a parallelogram with sides <i>OB</i> and <i>OC</i> . $ \mathbf{b} \times \mathbf{c}  =  \mathbf{b} \times (\mathbf{a} + \mathbf{b}) $ $=  (\mathbf{b} \times \mathbf{a}) + (\mathbf{b} \times \mathbf{b}) $ $=  \mathbf{b} \times \mathbf{a}  =  -(\mathbf{a} \times \mathbf{b})  =  \mathbf{a} \times \mathbf{b} $ (shown) Note that the area of the parallelogram with sides <i>OB</i> and <i>OC</i> is the same as the area of the parallelogram with sides <i>OA</i> and <i>OB</i> (with same base and vertical height)	<ul> <li>Some students wrote  b×c  as the length of a vector perpendicular to both b and c. This is not an interpretation of the geometrical meaning; it is merely translating in words the meaning of modulus of the cross product of b and c.</li> <li>In general,  b×(a+c)  ≠  b×a  +  b×c </li> <li>b×b ≠  b <sup>2</sup> but b • b =  b <sup>2</sup></li> </ul>
$a \times 2b = d \times 3a$ $(a \times 2b) - (d \times 3a) = 0$ $(a \times 2b) + (3a \times d) = 0$ $a \times (2b + 3d) = 0$ Vector a is parallel to 2b + 3d. $a = k (2b + 3d), k \in \mathbb{R}$	<ul> <li>There is no division for vectors. Hence,</li> <li>a×2b = -a×3d ⇒ 2b = -3d</li> <li>3 vectors parallel to a plane (coplanar vectors)</li> <li>⇒ a = λb + μd, λ,μ ∈ ℝ</li> <li>However, the above does not necessarily show a linear relationship as it involves two parameters λ and μ. A linear relationship involves only one parameter e.g. k for this question.</li> </ul>
	Total Marks:

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Qn	Suggested Solution	Markers' comments
2(i)	Total area of <i>n</i> rectangles	• As it is a "show" question
	$ = \frac{1}{n} \left[ (3^{1}-2) + (3^{1+\frac{1}{n}}-2) + (3^{1+\frac{2}{n}}-2) +(3^{1+\frac{n-1}{n}}-2) \right] $ = $\frac{1}{n} \left[ (3^{1}-2) + (3(3^{\frac{1}{n}})-2) + (3(3^{\frac{2}{n}})-2) +(3(3^{\frac{n-1}{n}})-2) \right] $ = $\frac{1}{n} \sum_{r=1}^{n} (3(3^{\frac{r-1}{n}})-2) $ = $-2 + \frac{3}{n} \sum_{r=1}^{n} 3^{\frac{r-1}{n}}$ (shown)	<ul> <li>As it is a show question, the series that show the sum of the area of the rectangles has to be shown clearly.</li> <li>The last term of the series has to be shown. Ending the series with "" means that it is an infinite series, i.e., there are infinite number of rectangles.</li> </ul>
	$-2 + \frac{3}{n} \sum_{r=1}^{n} 3^{\frac{r-1}{n}} = -2 + \frac{3}{n} \left( 1 + 3^{\frac{1}{n}} + \dots + 3^{\frac{n-1}{n}} \right)$	• The width of the rectangles is a constant at $\frac{1}{n}$ , while
	$= -2 + \frac{3}{n} \left( \frac{\binom{3^n}{n} - 1}{\frac{1}{3^n} - 1} \right)$	the breadth increases at $\frac{1}{n}$ for every subsequent rectangle.
	$= -2 + \frac{6}{n\left(\frac{1}{3^{\frac{1}{n}}} - 1\right)}$	<ul> <li>This is obviously a geometric progression (GP)</li> </ul>
		with common ratio $3^{\frac{1}{n}}$ , and so the sum of <i>n</i> terms of a GP is to be used.
(11)	-2	<ul> <li>Clearly, method of difference is not applicable.</li> </ul>
(11)	$\lim_{n \to \infty} S_n = \int_1^2 3^x - 2  dx$ = $\left[ \frac{3^x}{\ln 3} - 2x \right]_1^2$ = $\frac{9}{\ln 3} - 4 - \left( \frac{3}{\ln 3} - 2 \right)$ = $\frac{6}{\ln 3} - 2$	• Students should be able interpret the meaning of $n \rightarrow \infty$ in the context of this question. It means that there is an infinite number of rectangles under the curve, this renders the space between the curve and the rectangles negligible, and so $\lim_{n \rightarrow \infty} S_n$ is the area under
(iii)	Voluma	the curve from $x = 1$ to $x = 2$ .
(***)	$= \pi \int_{1}^{2} (3^{x} - 2)^{2} - \left(\frac{x^{3}}{\sqrt{4 + x^{2}}}\right)^{2} dx$ $= 38.8056$ $38.01 + 13$	• $\pi \int_{1}^{2} \left( 3^{x} - 2 - \frac{x^{3}}{\sqrt{4 + x^{2}}} \right)^{2} dx$ is incorrect because it would be the volume form
	= 38.81 unit"	when the curve

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	$y = 3^{x} - 2 - \frac{x^{3}}{\sqrt{4 + x^{2}}}$ is rotated about the x-axis.
1	rotated about the x-axis.
	• To find the required volume, student has to find the volume generated under
	away the volume generated under curve $C_2$ .
	<ul> <li>It is important to learn how to key in the expression correctly into the GC to get</li> </ul>
	the correct answer.
	Total Marks: 9

Suggested Solution	Markers' comments
The sequence is increasing from $\frac{3}{100}$ to 3.	• Since both <i>n</i> and <i>M</i> are
M	$n < M + 1 \implies \text{last term } n = M.$
	• As the sequence is finite with
	M terms, it's incorrect to say
	that the sequence "converges
	to 3". We use the term
	"converge or diverge" for
	infinite sequence.
3 3	• There is no need to combine
$S_1 = u_1 = \frac{1}{M - 1 + 1} = \frac{1}{M}$	the terms into a single
3 3 3 3	fraction.
$S_2 = u_1 + u_2 = \frac{1}{M} + \frac{1}{M} - \frac{1}{2} = \frac{1}{M} + \frac{1}{M} - \frac{1}{M} - \frac{1}{M}$	
$S_3 = u_1 + u_2 + u_3 = \frac{3}{M} + \frac{3}{M-1} + \frac{3}{M-2}$	
$\sum_{i=1}^{M} S_{i}$	
n=1	
$= S_1 + S_2 + S_3 + \dots + S_M$	
$= (u_1) + (u_1 + u_2) + (u_1 + u_2 + u_3) + \dots + (u_1 + \dots + u_M)$	The second second second share
$= Mu_1 + (M-1)u_2 + (M-2)u_3 + \dots + 2u_{M-1} + u_M$	• It's important to list down the
$= M\left(\frac{3}{M}\right) + (M-1)\left(\frac{3}{M-1}\right) + (M-2)\left(\frac{3}{M-2}\right) + \dots + 3$	enable us to see the pattern of recurring terms
= 3M	
$S_n = u_1 + u_2 + \ldots + u_n$	• Examples :
$=\frac{3}{3}+\frac{3}{3}++\frac{3}{3}$	$M - 1 < M \Rightarrow \frac{3}{1} > \frac{3}{1}$
M $M-1$ $M-n+1$	M - 1 $M$
$> \frac{3}{M} + \frac{3}{M} + \dots + \frac{3}{M} = \frac{3n}{M}$ (shown)	$M-2 < M \Longrightarrow \frac{S}{M-2} > \frac{S}{M}$
	Total Marks: 6
	and more reason on the second of the second s

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Qn	Suggested Solution	Markers' comments
5(a)(i)	$\left(x+y\right)^2 = 4e^{xy}$	Note:
	$2(x+y) = 4e^{yy} \left(x + \frac{dy}{dx}\right) = 4e^{yy} \left(x + \frac{dy}{dx}\right)$ $\frac{dy}{dx} = \frac{2ye^{yy} - y - x}{y + x - 2xe^{yy}}$	<ol> <li>e<sup>xy</sup> ≠ e<sup>x</sup>e<sup>y</sup> = e<sup>x+y</sup></li> <li>ln(4e<sup>xy</sup>) ≠ 4xy. It should be ln(4e<sup>xy</sup>) = ln 4+xy</li> <li>Do not attempt to "sq root" the equation as it will result in</li> </ol>
		$(x+y)^2 = 4e^{xy} \implies x+y = \pm 2e^{\frac{y}{2}}$ You will then have to choose the correct equation based on the point <i>A</i> in (ii). Do not overcomplicate your working.
(ii)	When $x = 0$ , $(0 + y)^2 = 4e^{(0)y}$ $y = 2 (\because y > 0)$ When at $(0, 2)$ , $\frac{dy}{dx} = \frac{2(2) - 2}{2} = 1$ Equation of the tangent to the curve at $(0, 2)$ is y - 2 = 1(x - 0) y = x + 2	
(111)	Substitute $y = x + 2$ into $(x + y)^2 = 4e^{xy}$ , $(x + x + 2)^2 = 4e^{x(x+2)}$ $(2x + 2)^2 = 4e^{x^2+2x}$ $(x + 1)^2 = e^{x^2+2x}$ Using G.C., $x = -2$ or $x = 0$ (reject $\because$ it's point A) $\therefore B(-2, 0)$	Quite a number of students used the y-coordinate of the intersection between the two curves $y = (x+1)^2$ and $y = e^{x^2+2x}$ as the y-coordinate of B which is wrong. Note that you are solving for the x-coordinate of B in that equation and you will need the equation of the tangent i.e. $y = x + 2$ to get the y-coordinate of B.
(b) 3.	Let r and h be the radius and the height of the cylinder respectively. Fixed vol. $p = \pi r^2 h \Rightarrow h = \frac{p}{\pi r^2}$ Surface area, S $= 2\pi r^2 + 2\pi r h$ $= 2\pi r^2 + 2\pi r \left(\frac{p}{\pi r^2}\right)$ $= 2\pi r^2 + \frac{2p}{r}$ $\frac{dS}{dr} = 4\pi r - \frac{2p}{r^2}$	Students need to have a clear picture of which variables are varying. From the qn, it is clear that $p$ is a constant (i.e. it cannot vary). You may vary $r$ but its variation will affect $h$ since they are connected by the volume equation. So for this scenario, you are to vary $r$ such that you can get the minimum surface area.

H2 Math Prelim P2 solution & markers' comments

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For min. S, $\frac{dS}{dr} = 4\pi r - \frac{2p}{r^2} = 0 \implies r = \left(\frac{p}{2\pi}\right)^{\frac{1}{3}}$ $\frac{d^2S}{dr^2} = 4\pi + \frac{4p}{r^3} > 0$ since r and p are positive. $\therefore$ S is minimum when $r = \left(\frac{p}{2\pi}\right)^{\frac{1}{3}}$ cm.	Note: $\frac{d}{dr}(2\pi rh) \neq 2\pi h$ as <i>h</i> varies with <i>r</i> . For students who use the first derivative test, they should present the working as $\frac{dS}{dr} = 4\pi r - \frac{2p}{r^2} = \frac{2(2\pi r^3 - p)}{r^2}$ so that the sign of the derivative can be easily observed by the examiner.
	1 otal Marks: 12





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(iii)	The event "No 2 boys are next to each other" is equivalent to "a least 1 girl between every 2 boys". Thus event $A$ is a subset	Many students did not explain explicitly with mathematical working.
	of event <i>B</i> . Now $P(A B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \neq P(A)$ , as $P(B) < 1$ .	Many students did not indicate $P(B) < 1$ but
	Thus A and B are not mapping and	marks are only deducted for those who computed $P(B)$
		incorrectly. Note that only $P(B) \neq 1$ is
		there is no need to compute $P(B)$ .
		Total Marks: 9

	Markers' comments			
Qn 8(i)	Suggested Solution $P(\mu < X < 16) = 0.35$ $P(X < 16) - P(X < \mu) = 0.35$ P(X < 16) = 0.85 $P(Z < \frac{16 - \mu}{2}) = 0.85$ Using GC, $\frac{16 - \mu}{2} = 1.0364$ $\mu = 13.927 \approx 14$ (shown) The detail time taken by the first two groups	M:	An analytical method of standardization and inverse norm is expected. $P(X < \mu) = 0.5$ $\mu$ The 12 <sup>th</sup> rider belongs to the	
(ii)	Let T be the total time taken by the first two groups of riders. $T = X_1 + X_2 \sim N(14 \times 2, 2^2 \times 2) \Longrightarrow T \sim N(28,8)$ Required Probability = P(T < 30) = 0.76025 = 0.760 (3 sf)	•	third group, and sò for the 12 <sup>th</sup> rider to ride before 10.30 am, the first two groups must complete their rides within 30 minutes. $X_1 + X_2 \neq 2X$ 2X means twice the time taken by one group of riders. $X_1 + X_2$ means the sum of the time taken by two groups of riders.	

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18 Y6 H2 Math Prelim P2 solution & markers' comments

i)	Let Y be the number of long rides out of 4. $Y \sim B(4, P(X \ge 15)) \Rightarrow Y \sim B(4, 0.30854)$ $P(Y \ge 2) = 1 - P(Y \le 1)$ = 1 - 0.63661 = 0.36339 = 0.363 (shown)	۲	Sufficient working is necessary since this is a "show" question.
/)	P(none of these rides are consecutive Y ≥ 2) $= \frac{3[P(X ≥ 15)]^{2}[P(X < 15)]^{2}}{0.363}$ $= \frac{0.13655}{0.363}$ = 0.376	•	With the words "given that", students should recognize that this is a conditional probability question. The numerator is the probability for at least 2 long rides and none of these rides are consecutive. This means the 3 possible scenarios are LSLS, SLSL, LSSL (L: for long ride; S: for short ride).
		1	- Jesse Linder Linder

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p 170 x General trend. x yarkers' comments Common Errors: 1. Wrong labelling of axes. 2. Careless plotting of points: the points are not always accurately plotted, leading to	Suggested Solution p p 170 x 115 2.12 2.98 Since there is no controlled variable, both p on s as well as s on p scatter plots are accepted. $Markers comments Common Errors: 1. Wrong labelling of axes. 2. Careless plotting of points: the points are not always accurately plotted, leading to wrong point X identified are: the first point, the last point or the 4^{th} point.4^{th} point.$			Maylow? comments
p 170 X General trend. Common Errors: 1. Wrong labelling on axes. 2. Careless plotting of points: the points are not always accurately plotted, leading to	p $170$ $x$ $(a)$ $115$ $x$ $(b)$ $115$ $2.12$ $2.98$ Since there is no controlled variable, both p on s as well as s on p scatter plots are accepted. Common Errors: 1. Wrong labelling of axes. 2. Careless plotting of points: the points are not always accurately plotted, leading to wrong point X identified. Popular wrong points identified are: the first point, the last point or the 4 <sup>th</sup> point.	1	Suggested Solution	Warkers comments
115	since there is no controlled variable, both p on s as well as s on p scatter plots are accepted.	n))	Suggested Solution	Markers' comments         Common Errors:         1. Wrong labelling of axes.         2. Careless plotting of points: the points are not always accurately plotted, leading to wrong point X identified.         Popular wrong points identified are: the first point, the last point or the 4 <sup>th</sup> point.
			1	

(ii)	Omitting X, $r = 0.995$ [between p and $s^5$ ] r = 0.976 [between ln p and s] Since the r-value between p and $s^5$ is closer to 1 than that between ln p and s (indicating a stronger linear relationship between p and $s^5$ ), $p = a + bs^5$ is the better model.	Students tend to forget to delete the point X, or they deleted a neighbouring point. You can press 'trace' button to track the points on the scatter plot.
	Note that $\ln p = c + ds \Longrightarrow p = e^c e^{ds}$ curve has the same shape as $p = a + bs^s$ . Thus we are not able to decide the better model purely from the scatter plot.	а К72,43 Ч7154
(iii)	From GC, $p = 101.449 + 0.279507s^5$ When $s = 2.63$ , $p = 136.618 = 137$ (3 sf) Two reasons: 1. The <i>r</i> -value (0.995) is very close to 1, indicating a strong fit of model to the data. 2. $s = 2.63$ lies within data range $2.12 \le s \le 2.98$ , thus interpolation is used. Interpolation is based on the given value (in this case $s = 2.63$ ) lying within data range, not the estimated one.	<ul> <li>Instead of finding an estimate for <i>p</i> when <i>s</i> = 2.63, some students proceeded to find the correct data value for <i>p</i> when <i>s</i> = 2.63. Realise that there should not be a follow up question or reasons for the data value to be reliable.</li> <li>For those who have chosen <i>X</i> to be the firs (or last) point, you should realise by now that the estimate is unreliable because it is an extrapolation.</li> </ul>
(iv)	High correlation <u>does not necessarily</u> imply causation. There could be other factors (e.g. strong economy, increase in population) which can lead to higher property price index. So the higher property price index need not be caused by higher stock index.	Note that both statements need to be stated to give a complete answer in context of the question, which some students were not able to do during the exam.
		Total Marks: 10

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	0.1.4	B.C.	1
i)	Solution $\sum P(Y - x) = 1$	MI:	For any discrete
~	$\sum_{n=1}^{n} \alpha x$		random variable X,
	$\sum_{n=1}^{\infty} \frac{dx}{n(n+1)} = 1$		the sum of
	$a \stackrel{n}{\longrightarrow}$		for all possible
	$\frac{1}{n(n+1)} \sum_{x=1}^{n} x = 1$		values of x is 1.
	$a\left(n(n+1)\right)$		Note that in this
	$\frac{1}{n(n+1)}\left(\frac{1}{2}\right) = 1$		case, a and n are
	a = 2 (shown)		constants so they
			can be factored out
	Alternative		first method
			The resultant sum
	When $n = 1, P(X = 1) = \frac{a}{1(a)} = 1 \therefore a = 2$		is an A.P so apply
	1(2)		the A.P formula.
			(In general, note
			for pure math
			knowledge to
			apply in statistics
			questions)
1	$E(X) = \sum x P(X = x)$	•	This part is
	xeS		done.
	$=\frac{a}{(x-1)}\sum x^2$		
	$n(n+1) = \frac{1}{x-1}$		
	$=\frac{2}{n(n+1)}\left(1^2+2^2+3^2+\ldots+n^2\right)$		
	$2$ $\overline{n}$ $2$		
	$=\frac{1}{n(n+1)}\sum_{i=1}^{n}r^{2}$		
	$2 \begin{bmatrix} n \\ n \end{bmatrix} $		
	$=\frac{1}{n(n+1)}\left[\frac{1}{6}(n+1)(2n+1)\right]$		
	$-\frac{1}{2}(2n+1)$		
	$-\frac{1}{3}(2n+1)$		
)	For this game, the tokens are picked without replacement thus		Some elaboration
/	the two tokens picked by the players will be different. For any		is required here,
	two possible tokens picked, say a and b, there are two equally		beyond stating that
	possible outcomes: $P_1$ picks a and $P_2$ picks b or vice versa, out		there will be no
	of which only one where $P_2$ will lose. Thus the probability of		rephrasing the
	P losing is 0.5		statement that both
	Alternative		$P_1$ and $P_2$ are
	Anomarive		equally likely to
			lose.
		1	



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2018 Y6 H2 Math Prelim P2 solution & markers' comments

1	Let L be the random variable denoting the amount lost for a game if he loses a game. L = X + 1 E(L)	•	First define the random variable <i>L</i> , before trying to link it to <i>X</i> .
	$ = E(X+1)  = \left(\frac{1}{3}(2n+1)\right) + 1 $	•	Note that $0 \le L \le n+1$ and P(L = m+1)
	$=\frac{1}{3}(2n+4)$	•	= P(X = m) Thus $L-1 = X$ .
			Total Marks : 12

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n	Suggested Solution	Markers' comments
(i)	Unbiased estimate of population mean, $\overline{x} = \sum_{x=1}^{\infty} \frac{5415}{5} = 45,125$ (exact)	<ul> <li>Remember to use the exact or 5 s.f. answer for subsequent</li> </ul>
	$\frac{120}{n}$ Unbiased estimate of population variance,	working.
	$s^{2} = \frac{1}{119} \left[ 351500 - \frac{(5415)^{2}}{120} \right] = 900.40 = 900  (3 \text{ s.f.})$	
i)	An estimate is unbiased when the expected value of the estimator $T$ used to obtain the estimate is equal to the value of the population parameter $\theta$ , i.e. $E(T) = \theta$ .	• See Tut19 Q5(ii)
i)	Let X denote the speed a randomly chosen car (in km/h) with population mean $\mu$ .	• Important to define the random variable X.
	To test $H_0: \mu = 50$ vs $H_1: \mu < 50$ Conduct 1-tail test at 3% significance level.	
	Under H <sub>0</sub> , $\overline{X} \sim N\left(50, \frac{900.40}{120}\right)$ . Use the 5 s.f figure for better accuracy	• X normal $\Rightarrow \overline{X}$ normal, no need to use CLT.
	Using a z-test, p-value = $P(\bar{X} \le 45.125) = 0.0376$ (3 s.f.) Since p-value > 0.03, we do not reject H <sub>0</sub> and conclude that there is insufficient evidence at 3% significance level that the mean speed is less than 50 km/h. Hence WRP is not needed.	<ul> <li>Important to answer the question at the end i.e. <u>"WRP is not</u> needed".</li> </ul>

(iv)	Unbiased estimate of population variance, $s^{2} = \frac{80}{79} [1100] = 1113.9 = 1110  (3 \text{ s.f.})$ To test H <sub>0</sub> : $\mu = \mu_{0}$ vs H <sub>1</sub> : $\mu > \mu_{0}$ Conduct 1-tail test at 8% significance level. Under H <sub>0</sub> , $\overline{X} \sim N\left(\mu_{0}, \frac{1113.9}{80}\right)$ If mean speed exceeds $\mu_{0} \Rightarrow H_{0}$ is rejected, $\Rightarrow p$ -value = P( $\overline{X} \ge 60) \le 0.08$	• The sample variance = 1100. We need to find the unbiased estimate by using the formula : $s^2 = \frac{n}{n-1}$ (sample var)
(v)	$P\left(Z \ge \frac{60 - \mu_0}{\sqrt{\frac{1113.9}{80}}}\right) \le 0.08$ $\frac{60 - \mu_0}{\sqrt{\frac{1113.9}{80}}} > 1.4051$ $\frac{\sqrt{1113.9}}{\sqrt{\frac{1113.9}{80}}}$ $\mu_0 < 54.757$ $\therefore \text{ maximum } \mu_0 = 54.7$ Assumption : 1. The speeds of the second sample of cars comes from a random sample, <b>OR</b>	• Qn requires the use of an algebraic method to find max $\mu_0$ as shown on the left. A GC method is not acceptable.
	2. The speeds of the second sample of cars are independent	Total Marks: 12

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